

Self-Stabilization in Hierarchically Structured Energy Markets

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Abstract—The paper investigates the self-stabilization of hierarchically structured markets. We propose a new approach that is motivated by the physical structure of the energy grid and generalizes classical market structures in a natural way. Hierarchical markets have several advantages compared to monolithic markets, i.e., improved reliability and scalability, locality of information, and proximity of energy production and consumption. By simulating scenarios based on real world consumption and production data including households, different renewable energy sources, and other plant types, we present a proof-of-concept of stability of the hierarchical markets in various simulations.

Keywords—smart grid, hierarchical markets, demand response control, agents

I. INTRODUCTION

Control concepts for grid markets in practice are currently completely centralized [2]. Simply spoken, a sensor network all over the energy grid collects information for a central decision making on the producer side. This architecture is in complete antagonism to modern communication and information infrastructures, but an important first step as the recent increase of critical network states and blackouts in today's uncontrolled grids shows [4]. Due to the increasing number of decentralized producers, different storage technologies and renewable energies, alternatives to uncontrolled concepts are eagerly desired. Promising are fundamentally decentralized control approaches, which directly reflect the basic structure of the network. A common way to achieve this is the application of principles from distributed artificial intelligence. This approach is described by several authors.

We think that agents competing in a market are a promising approach for a distributed and automatic control of grids. There are many studies which show that the market price is a means to control energy production and consumption in a natural way. Unlike previous studies, we emphasize on a unified generic modeling of grids at all levels. We propose to describe grids as hierarchical overlapping markets what allows to model households as well as industrial plants, and even the international electricity market. While this market is a real instance of an energy market, our architecture can be seen as a rigorous application of market-control to all levels of a grid.

This work is structured as follows. In the forthcoming section we review classical monolithic markets and discuss

successful auction principles for this setup. Then, in Section III we introduce hierarchical markets and explain the benefits from adopting hierarchical structures to smart grid control scenarios. In Section IV case studies explain the dynamic behavior of the approach and clarify abstract concepts by means of examples. In Section V we explain how hierarchical market scenarios have been implemented and which agent types and strategies have been applied. In specific, strategies to balance adjacent sub-markets are motivated. Section VI describes the behavior of the proposed scenarios. The development of important parameters as price, energy production, and battery level over time are visualized in different scenarios. Section VII concludes the paper and explains possible future research.

II. RELATED WORK

Mascem [16] is a multi-agent system (MAS) to simulate competitive electricity markets. It is intended to provide a means to analyze the impact of market rules and agents' strategies on the system. Mascem consists of one energy market divided into a trading pool, a floor for bilateral contracts, and a range of agents with specific roles. Pippattanasomporn *et al.* [15] propose to use a MAS to control a microgrid. A microgrid is an integrated energy system which can operate in parallel with the grid or in an intentional island mode. Also Li *et al.* [6] developed multi-agent technology for the management and control of distributed energy resources by explicitly coordinating distributed energy resources comprising customer loads and generators to aggregate sufficient distributed capacity to be of strategic value to market participants.

Pricing rules in electricity markets have been experimentally investigated, e.g., by Xiong *et al.* [17]. Of course, these rules have also major influence on the behavior of the agents; different bidding strategies are considered by Kian *et al.* [7] and optimal strategies are known as well [3]. Also classical auction schemes as Vickrey-Clarke-Groves auctions [5] or game theoretic approaches and approaches from computational learning theory have been applied [10]. It should be clear, that there may be large differences between the considered markets if one looks at their inner structure, but they all fit in the scheme described within this paper. The structure discussed in this work leaves market

details open for configuration. A major design goal is to assure that detailed planning is possible for energy suppliers and consumers while maintaining scalability.

III. BENEFITS OF HIERARCHY AND STRUCTURE

The central new concept of our studies is the introduction of an additional hierarchical structure in the market, so that it dissolves into connected sub-markets. This is motivated by the structure of the physical grid, which consists of several hierarchical levels, e.g., a single household, an urban district, or a city. On higher level, the grid is also physically different, i.e., on lowest level there is the low-voltage-grid (400/230V, connected to the medium-voltage-grid, on-site power plants, solar panels), above that the medium-voltage-grid (1-50kV, connected to the high-voltage-grid, small urban power plants, wind parks, solar power plants), there is a high-voltage-grid (110kV, connected to the super-grid, medium-sized power plants, industrial consumers), and the supergrid (220-380kV, nuclear power, coal-fired power plants, hydro-electric power plants, etc.). From a practical perspective it is not very realistic to organize all participants on these different levels of the overall grid in one market. For practical applications of smart grids, this physical structure of the grid has to be introduced to the simulation model in order to investigate a realistic scenario.

A. Modeling the Physical Structure of the Grid

In general the physical structure of the electricity network can be modeled as a graph with nodes $v_1, v_2, \dots, v_n \in V$ and transmission lines $l \in L$, denoted as $G = (V, L)$. Each node is identified with some physical entity which is represented by an agent a_i at node v_i , i.e., there is an agent set $A \equiv V$. The *agents* act on behalf of *entities* of the electric grid. The nodes (agents) can just represent buyers and/or sellers but can also model voltage transformation substations. A modeling of this physical structure of the grid has advantages if the transmission lines and their load are relevant. E.g., in [14] the load of each single transmission line is continuously tracked, which is possible based on methods to calculate the power flow in electricity networks. While the calculation of the full alternating current (AC) power flow model has many pitfalls due to a large number of alternative solutions, which fulfill the nonlinear power balance equations, the directed current (DC) model is accurate enough and can be calculated based on a set of linear equations [13]. The focus of our studies is not the calculation of the flow on transmission lines, because in practice these lines are sufficiently dimensioned also for peak demands. Instead, our research in this paper investigates three central questions:

- How can hierarchical markets with market substructures be controlled?
- What are suitable strategies to balance adjacent markets?

- How is energy production control possible in hierarchical markets?

To investigate these questions we relax the structure of the grid to a tree structure which reflects the logical relationships of agents in the network and their allocation to sub-markets but suppresses details like submission lines. This is described in the following section.

B. Modeling the Logical Structure of the Grid

First, assume that all agents are continuously connected to the internet. Of course, specific security mechanisms have to be implemented to prevent manipulation and particularly important agents should be secured by backup agents, able to replace them in case of failures. These considerations are highly important for the application of a system as proposed, but not topic of this work. We restrict ourselves to develop the self-contained structure and explain its proposed dynamics by different case studies.

Based on these insights we model the logical structure the electric grid $G = (V, L)$ as a tree $G' = (V, T)$ with the same vertex (agent) set but edges $T \subseteq L$. The tree edges T model the relationships among the agents where $(c, p) \in T$ means that p is part of c . Entities are characterized by having the ability to consume, produce, and transmit power. The task of an agent is to control the provisioning of sufficient energy and to reduce overcapacity over appropriate lines on behalf of the entities. These control mechanisms are implemented by using the paradigm of markets.

The whole electric grid is hierarchically divided into *virtual markets* of different granularity. A virtual market or simply market $M = (A', R)$ consists of a number of participating agents $A' \subseteq A$, and a number of market rules R to define its functionality. Each agent $a \in A$ forms a market together with all its children, i.e., $M^a = (\{c \in A \mid (a, c) \in T\} \cup \{a\}, R^a)$. The agent a of a market M^a is called *head* of the market. Agents are allowed to trade with other agents within the same virtual market according to the market rules but not beyond market boundaries. In case of agents being at a leave of the tree a market consists only of that agent itself. As no trading can take place in a single participant market, such markets can be neglected.

IV. CASE STUDIES FOR HIERARCHICAL MARKETS

In this section we explain how the introduced architecture can be applied to enable a smart grid. We present three case studies to highlight the functionality of our architecture on different hierarchical levels.

A. Household

A market for a household $M^h = (A^h, R^h)$ is headed by an agent h representing the house. Its children are electric entities that belong to the household like a boiler, a freezer, an electric car, and a solar panel. A household is considered to be at the bottom of the grid.

Market rules are described by means of products and protocols to buy and sell. The owner of the house has the sovereignty to install rules at his convenience. We propose a market for the household which is very basic and centered around the market head. Because all entities belong to the same owner, any cheating would make no sense and this simple mechanism is sufficient: Agents can buy only from the head. They specify their order in the form $[t_s^r, t_e^r]$, denoting the earliest possible requested time and the latest possible time of power consumption, a duration interval (t_{\min}^d, t_{\max}^d) of power consumption, constraining the duration of necessary power supply, and the amount in kWh. The market head does not charge the agents and has the right to determine when agents are provided with energy within the specified constraints. Energy produced by an entity within the house must be bought by the house agent for the true price of the production which is announced by the respective agent.

Simple entities like a toaster have an empty agent doing nothing at all. They are just consuming energy when turned on. It is the responsibility of the house agent to purchase some kind of product at a higher level, which allows to cover power consumption for such entities.

B. Energy-Mix in a City

Consider the local electricity market of a city, and let an agent c be the head of the market. The actors in this market $M^c = (A^c, R^c)$, represented by an agent each, are households of the township, industrial plants, and other energy consumers, but there may also be energy producers as wind parks, which belong to this city or also biogas plants.

Again the market rules are described by products and protocols of the market. In this exemplary case each participant can *buy* power, where each buy-order consists of a *time window* $[t_s^r, t_e^r]$, denoting the earliest possible time and the latest possible requested time, as well as further information. These are the least possible time of power consumption, a duration interval (t_{\min}^d, t_{\max}^d) of power consumption, an *amount* in kWh, and an *order modifier*, which can be a *maximum price* P or also *market order*. The duration interval is defined as in Section IV-A.

Sell-orders consist of the same items, but P is a minimum price. In principle, the prices will be better for the consumer if it leaves more freedom to be fixed by the supplier. If energy is consumed without buying it in advance, high penalty prices are charged. A result of this construction is that the agents will try to specify demand as specific as possible, and leave as much freedom as possible for the supplier. Note that all orders that have not been executed can be deleted at any time by the ordering party or decay at time t_s . Of course, more complex or specific products are also possible, e.g., for renewable energy.

C. Electricity Market

The highest market level of this model are (national and international) electricity markets. Such organized electricity markets have been established in many countries starting in the 90ies (see [1] as an example), where large amounts of energy are traded by the “big players” in the energy market. The existing electricity market maps quite well to the suggested basic architecture of the system. The target of the energy market today is to reach an economically optimal allocation of energy. This can be generalized to all market levels if an agent-oriented structure as suggested is implemented.

The available products are standardized energy blocks of different *duration* and *amount*, which can be traded continuously or only at specific times, depending on the market. Very important is the temporal availability of energy, i.e., the price reacts immediately to new weather forecasts to satisfy the different demand of the market. Currently, the electricity market consists of different sub-markets. In a *future market* long term transactions are conducted, while short term transactions are conducted at the *spot market*. This covers contracts from several years down to 24 hour blocks, 12 hour blocks, and also one hour blocks to model peak times in the demand. In some markets the blocks go down to the level of a few minutes. This important market is not restricted to electronic agents – also humans participate to control these important extensive trades.

V. SIMULATED SCENARIO

In the following simulated scenario we further formalize the market rules and standardize the available products. The markets are day ahead and divided into trading periods of 30 minutes length each. We consider different scenarios to investigate the questions as discussed in Section III-A.

A. Smart Grid Architecture

We model an electricity grid consisting of three cities. In each city we consider ten representative buildings for our simulation scenario, where some of them are equipped with solar panels¹. One of the cities is equipped with an additional wind generator. Each of these cities is organized in a separate market. The market heads of these markets are organized in a higher market. Additionally, there is a gas power plant, which is also arranged in the higher market. Figure 1 shows the logical structure of the market.

B. Agent Types and their Characteristics

There are various types of agents in the system:

- house agents,
- house agents with solar panel,
- wind plant agents,

¹Experiments have shown that the qualitative behavior of the system does not change for a larger number of agents and the dynamics of large systems cannot be visualized in detail.

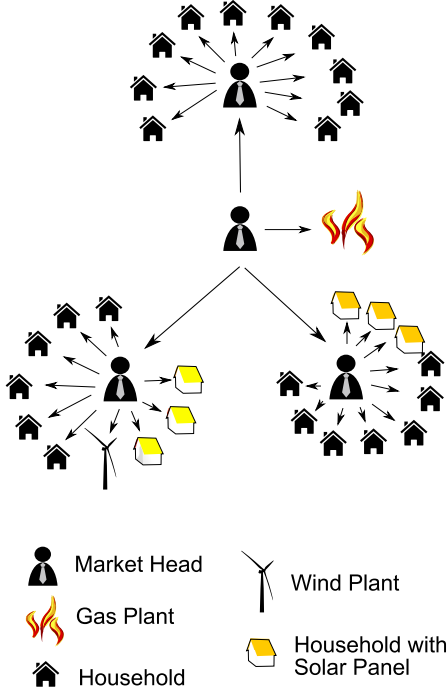


Figure 1. Example for a hierarchical organization of the energy market: The figure shows three cities visualized by ten representative households, each with their market head. In two of the cities a few houses are equipped with solar panels, one of the cities is also equipped with a wind plant. The market heads of the cities are organized in a higher market, which contains also a gas power plant.

- gas power plant agents,
- market agents.

Each agent is equipped with a battery of specific dimension, which is used as energy buffer. This battery models demand which is transferable in time as, e.g., to cool down the freezer, to heat the hot water tank, to recharge the electrical car. By dimensioning the batteries differently large, the influence of shiftable energy demand in the grid can be investigated. Table I shows the battery capacities for the initial experiments.

Table I
BATTERY CAPACITIES AND ENERGY CONSUMPTION

Entity	Battery capacity	Energy consumption
Household	20,000 W	between 0...1,000 W
Solar household	50,000 W	between -3,000...0 W
Gas power plant	500,000 W	unlimited production
Wind plant	200,000 W	between -12,000...0 W

The third column in Table I shows the energy consumption of the different entities in the simulation model. Note, that households which are equipped with solar panels, the gas power plant, and the wind plant have negative consumption, i.e., they produce energy. These figures are based on real world data from the Western Wind Resources Dataset [12],

from the PVWATTS calculator [11], and the Review of Residential Electrical Energy Use Data [9]. The energy consumption the simulations are based on is visualized in Figure 2.

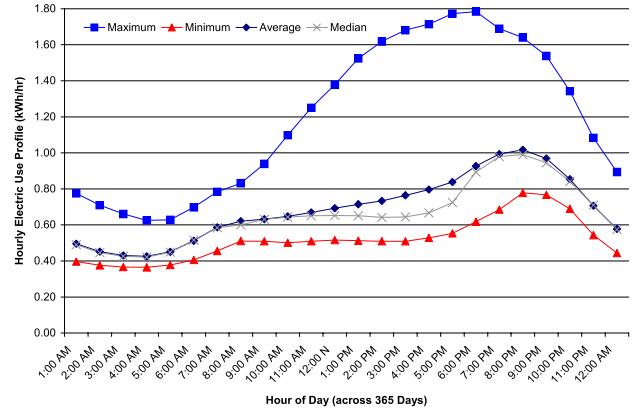


Figure 2. Hourly average residential load profile, averaged over the year (Southern California, Edison territory) [9]

In particular, for the three cities as visualized in Figure 1, the data for Sacramento, San Francisco, and Los Angeles has been utilized. The real production or consumption values are calculated by sampling from a uniform distribution in the given interval in the third column of Table I (instead for the gas power plant, which is regulated price-driven).

C. Agent Strategies

The different agents in the smart grid have different strategies. Most of the agents trade according to a battery-price-strategy, which means, they are sellers if their battery gets too full due to energy production or they buy if their battery gets too empty due to energy consumption. A (future) battery level of 50 percent is used as a reference point to change between these two different behaviors. The farther the level of the battery charging level is from this value, the more aggressively the agent tries to buy or sell energy by choosing an adequate order price.

Depending on the battery load $\ell^{i,t} \in [0, 1]$ at time t , the agent i tries to buy ($\ell^{i,t} < 0.5$) or sell ($\ell^{i,t} > 0.5$) energy to the current market price p^t plus $\Delta^{i,t}$, where

$$\Delta^{i,t} = \begin{cases} (1 - 2\ell^{i,t}) \cdot (p_{\max} - p^t) & \text{if } \ell^{i,t} \leq 0.5 \\ (1 - 2\ell^{i,t}) \cdot p^t & \text{otherwise} \end{cases} \quad (1)$$

for $i = 1, 2, \dots, |A|$ with p_{\max} as maximum price to be paid. The gas power plant produces only power, if the market price is higher than a certain break even value, which marks the price level where the operation of the plant starts to realize profit. As a consequence, in the experiments of the further sections it can be observed that the gas power plant produces power in periods, but not continuously.

D. Load Prediction

As we have seen in Table I, the gas power plant only sells energy, households without solar panel only buy energy, and households with solar panel are able to buy and sell power. The decision to buy or to sell it based on the *future battery level*, which has been introduced but not further described in the previous section. In general, the future battery level is defined as

$$\ell^{i,t+1} = \ell^{i,t} - c^{i,t}, \quad i = 1, 2, \dots, |A|,$$

where $\ell^{i,t}$ is the battery level of agent i at the beginning of period t and $c^{i,t}$ is the predicted energy consumption of agent i in period t (measured as portion of the battery capacity), which can be positive or negative (energy production). The ℓ -value in Equation 1 is equivalent to the value $\ell^{i,t+1}$ at the end of the period t , i.e., the buy or sell orders during period t are adjusted according to the predicted battery level at the end of period t .

This is only possible if the power consumption $c^{i,t}$ in period t can be well predicted. This has been realized by regression methods as described in [8], which is not further stressed here. The quantity of bought or sold energy is then $0.5 - (\ell^{i,t} - c^{i,t})$, i.e., the agents buy a quantity of power so that the predicted energy level of the battery is exactly 0.5 at the end of the period.

E. Trading Rounds

The continuous double auctions are organized in a way that ten trading rounds are executed within one trading period, i.e., each agent has ten opportunities to buy (or sell). One of these trading periods is equivalent to 30 minutes real time. In each round, multiple buyers and sellers are allowed to submit bids and asks and the market clears whenever a bid and an ask matches, in which case the round ends. In any case it is required that a new submitted bid or ask improves on the previously best bid or ask, called the outstanding bid or outstanding ask, respectively. This means bids and asks are not queued in the system and simply erased, if a better one is submitted.

F. Balancing Adjacent Markets

The market heads consist of two traders – a lower market trader and a higher market trader. In the model the market head is an agent who acts altruistic towards the agents in his market, but acts egoistically in the higher market. This means the two trader of the market head act according to different strategies.

In this work we suggest the following setup. Each market head owns an (hypothetic) battery which is used to save excess energy in the own market to be sold in the higher market or to save energy from the higher market to be sold in the lower market. To ensure altruistic behavior towards the agents in the lower market, an average buying price p_a is saved together with the quantity contained in the battery. It

is updated after each trade according to the following rules. Assume that the battery is initially not empty but filled with load $\ell \in [0, 1]$, acquired to the average buying price p_a .

- The agent tries to balance the lower market by conducting transactions in the higher market. To do that, the ratio of unsatisfied asks and bids in the last round of a period is used. If there is an ask overage, the higher market trader tries to sell energy in the higher market to a price at least the average ask price.
- On the other hand, if there is a bid overage, the higher market trader tries to buy energy in the higher market to at most the average buy price in the lower market.
- *Lower Market Transactions*: Sell to a price p_a ; buy to the current market price.
- *Higher Market Transactions*: Buy to a price not higher than the current ask price in the lower market. Sell to a price not lower than p_a .
- All gains and losses of the market agent are shared by the market participants in the lower market according to their average business volume in the last trading period at the end of each trading period.

VI. EMPIRICAL EVALUATION

The proposed smart grid structure is designed according to the rule *as local communication and energy transfer as possible but as global communication and energy transfer as necessary*. This has been realized by a hierarchical structure of the market, which hides on each level existing higher and lower markets to certain extent (only adjacent market levels are visible for the agents).

As a major advantage of this special structure it is conjectured that the communication and energy production overhead in the network can be reduced, because it is closer oriented to the actual energy consumption, which reduces costs. The major new ingredient which is necessary to control these hierarchical market structures is the introduction of *multi-market-trader-strategies*.

To answer the three major questions as posed in Section III-A, the following experiments are carried out:

- Simulation and investigation of the overall market behavior in the described scenario.
- Investigation of the market price development in the different markets. A strategy is considered to be *good*, if the prices in adjacent markets are closely coupled.
- Investigation of the amount of produced energy w.r.t. the amount of consumed energy.

We run the simulation with the setup as previously described. Figures 3 and 4 show the development of the price in the different sub-markets, the energy production, and the battery level of the agents, respectively.

Figure 3 indicates that the strategy as described above couples the markets close together, so that the prices of all sub-markets remain in a narrow band. The same is visualized well for the bid and ask price in the same market.



Figure 3. Price development in the sub-markets

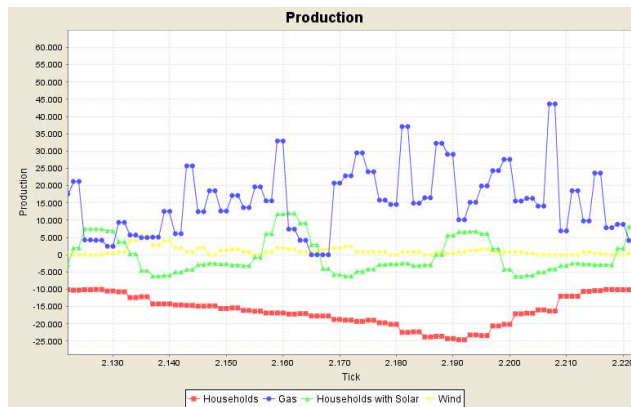


Figure 4. Production development in the sub-markets

Figure 4 shows the energy production behavior of the different types of participants in the smart grid. For the households (red line) it can be seen that the energy consumption changes during the simulation period. A periodic change of the energy consumption/ production can be obtained from the households with solar panels, due to the different day-times and the different solar radiation over the day. The produced wind power is fluctuating during the simulation time. Interesting is the behavior of the gas power plant: Multiple times it is switched on and off, and the energy production is fluctuating in the course of time. Comparing the Figures 3 and 4 shows that especially in times of high prices the energy production gets started or increased.

Furthermore, our experimental analysis has shown that all agents are able to refill their battery to a level of fifty percent after consuming or producing energy in a relatively short period of time (instead for the gas power plant, which battery gets usually recharged to 100 percent). The lowest obtained energy level during the simulation is about 20 percent for a house in market M4, which consists only of consuming households. Concerning the interaction between solar energy

and batteries we have observed an interesting pattern, where an increased production of solar energy fills the batteries of several agents by more than 50 percent. During that time the battery of the gas power plant is completely filled, i.e., no energy from the gas power plant is consumed. This is a typical case of self-regulation behavior of the energy grid – renewable energy is consumed preferentially if it is available.

In our experiments we have analyzed the response to increased solar energy production in the market price: the market price gets significantly decreased when the energy is available. This can be seen as reason for the suspension of the energy production by the gas power plant, see Figure 5. Again, it is well visualized, that abounding energy is transferred to other markets, because the prices in other markets react as well on the increase of available solar energy.

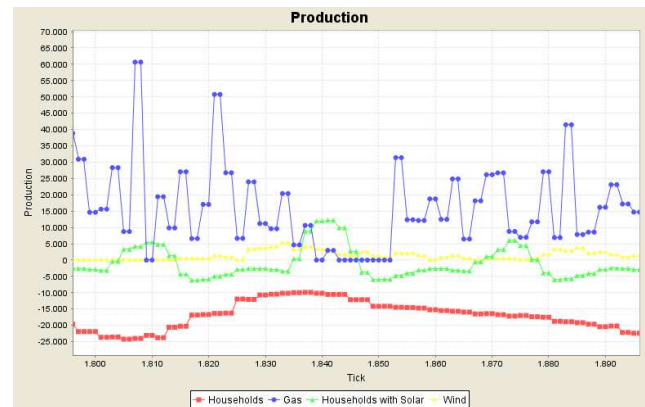


Figure 5. Response in the curves for increased solar energy production

In general it can be obtained, that the control of the energy production by pure price strategies is able to balance energy consumption and energy production well enough that the waste caused by overproduction is negligible.

VII. CONCLUSION

We have demonstrated how to apply principles from distributed computational intelligence to completely remove centralized components from the control mechanisms of a hierarchical grid. The approach is motivated by the physical characteristics of the grid and their participants. If each agent locally maximizes its benefit this leads globally to a robust and self-organizing market. The sub-markets have been shown to couple well, concerning the market price and the transformation of energy between the different markets. The simulated market represents a proof-of-concept for the functionality of the proposed mechanisms. Our approach maps principles that have been known for a long time from monolithic markets to all layers of a hierarchically structured grid. Further research in this direction will be necessary.

It is important to point out that the approach can be applied specifically well if further local storage technologies get available but virtual storages (cooling of the freezer, heating of the hot water tank, charging of electric cars, etc.) can be used as well to shift consumption. The agents would always aim to refill these storage facilities as cheap as possible, like in times of power oversupply. On the other hand, they aim to consume stored power instead of buying power in times of high prices, like in times of insufficient supply. Technologies as on-site power stations would be employed more frequently, if available. There are many model details which have to be investigated more closely in future as, e.g., more sophisticated trading strategies for multi-market-trading.

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