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Excitonic collapse of higher Landau level fractional quantum Hall effect

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The scarcity of the fractional quantum Hall effect in higher Landau levels is a most intriguing fact when contrasted with its great abundance in the lowest Landau level. This paper shows that a suppression of the hard core repulsion in going from the lowest Landau level to higher Landau levels leads to a collapse of the energy of the neutral excitation, destabilizing all fractional states in the third and higher Landau levels, and almost all in the second Landau level. The remaining fractions are in agreement with those observed experimentally.

Electrons in the lowest Landau level exhibit the spectacular phenomenon of the fractional quantum Hall effect (FQHE).[1] Including the several fractions observed in the ultralow temperature measurements in very high quality samples,[2] there now exists evidence for more than 50 fractions in the lowest Landau level. The essential phenomenon is remarkably insensitive to the detailed form of the repulsive interaction, as manifested by the fact that it is quite robust to perturbations arising from Landau level mixing, finite thickness, or the nature of the transverse confinement.

The system of electrons restricted to a higher Landau level (LL) deviates from that in the lowest LL only through the short-distance matrix elements for the Coulomb interaction. Consequently, one would expect the FQHE to be not too sensitive to the Landau level index either. From this point of view, it is astounding that the fractional quantum Hall effect (FQHE) is so rare in higher Landau levels. For example, whereas ten members of each of the sequences $\nu=n/(2n+1)$ and $\nu=n/(2n-1)$ have been observed in the lowest LL, only the first one has been observed in the second LL, and none at all in the third and higher LL's. In fact, the only fractions outside the lowest LL for which clear experimental evidence exists, in the form of reasonably well quantized plateaus, are $\nu=1/3$ and $\nu=1/2$ in the second LL.[2,3] (That the latter has no analog in the *lowest* LL further underscores the striking difference between the lowest and the higher LL physics.) Hartree-Fock variational (Koulakov, Fogler, and Shklovskii[4]), exact diagonalization (Rezayi, Haldane, and Yang[5]), and experimental (Lilly *et al.*, Du *et al.*[6]) studies make a compelling case that a bubble crystal or an anisotropic stripe phase is favored over the FQHE in higher LL's.

The goal of this work is to start from the FQHE end of the problem and ask by what mechanism is the FQHE destroyed in higher Landau levels. Of course, there can be a mundane origin for the disappearance of the FQHE, e.g., sufficiently strong disorder, but of interest to us here is the possible *intrinsic* instability of FQHE in higher Landau levels. To this end, we will start by assuming an incompressible FQHE state and then investigate its stability to quantum fluctuations. A necessary condition for FQHE is that (in the absence of disorder) the energy gap to all excitations remain positive definite. A vanishing of the gap signals an instability of the assumed ground state. It will be shown that the suppression of the short-distance Coulomb matrix elements in higher Landau levels leads to a collapse of the energy of the neutral exciton, destabilizing most of the incompressible FQHE states in higher LL's.

The investigations will be carried out within the framework of the composite fermion (CF) theory[7] of the FQHE, which has proven successful in capturing subtle instabilities of the FQHE. At small filling factors ($\nu\leqslant 1/9$), the neutral composite-fermion exciton becomes gapless.[8] In another example, a recent theoretical study[9] has revealed that the Fermi sea of composite fermions is unstable to Cooper pairing at $\nu=5/2$, but not at $\nu=1/2$, giving insight into the experimental observation of the FQHE at $\nu=5/2$, but not at $\nu=1/2$.[3,2]

A composite fermion, ^{2p}CF, is the bound state of an electron and $2p$ quantum mechanical vortices. Of relevance to experiment are three flavors of composite fermions carrying two, four, and six vortices, namely ^{2}CF's, ^{4}CF's, and ^{6}CF's. According to the composite fermion theory, the interacting electrons at the LL filling factor $\nu_0\equiv n/(2pn+1)$ transform into weakly interacting composite fermions with an effective filling $\nu_0^*=n$. The ground state here corresponds to n filled CF-LL's and the neutral excitation to a particle-hole pair of composite fermions, called the CF exciton. The explicit, parameter-free, lowest-LL form for the microscopic wave functions for the fully polarized CF ground state and the CF exciton can be found in the literature[10] and will not be repeated here. In the lowest Landau level, these wave functions have been found to be accurate in tests against exact diagonalization results available for small systems.[7,11] They are not as good in higher Landau levels for ^{2}CF's, but should still be a reasonable starting point *provided* the ground state is an incompressible state. Interestingly, the wave functions are quantitatively accurate for ^{4}CF's in the second LL.[12]

We will study filling factors $\nu=2s+\nu_0$. Here, the quantity $s=0,1,\ldots$ denotes the Landau level index, with the lowest s LL's having *both* spin states filled, thus contributing $2s$ to the filling factor. As a simplification we will take these $2s$ LL's to be completely inert (neglecting screening by lower Landau levels[13]) and work only with the electrons in the topmost partially filled LL, which will be taken to be fully spin polarized; this is a valid approximation in the limit of high magnetic fields when LL mixing is negligible. Note that only the electrons in the topmost partially filled LL capture vortices to turn into composite fermions; the electrons in the lower, filled LL's remain unchanged.

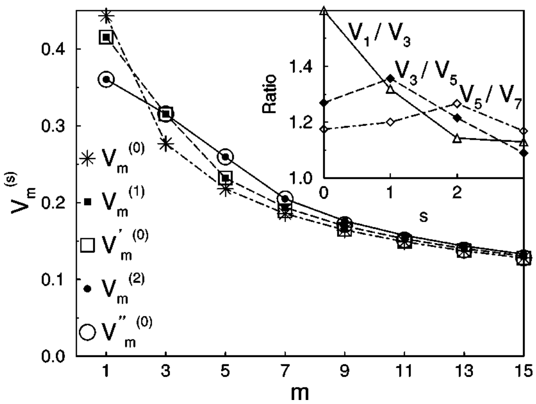

FIG. 1. The Coulomb pseudopotentials for the lowest three Landau levels, $s=0$ (stars) $s=1$ (filled squares), and $s=2$ (filled circles). Also shown are the pseudopotentials of the effective interactions $V'(r)$ and $V''(r)$ in the lowest LL (empty squares and empty circles). The inset shows the ratios V_1/V_3, V_3/V_5, and V_5/V_7 in various Landau levels. The lines are a guide to the eye.

The wave functions Ψ are most easily constructed within the lowest LL. To calculate energies in higher LL's one may promote them to higher LL's by application of the Landau level raising operator. However, this procedure is technically rather cumbersome. We instead proceed by working with an *effective* interaction in the lowest LL which mimics the Coulomb interaction in a higher LL. The interparticle interaction in any given Landau level is completely specified by its Haldane pseudopotentials, $V_m^{(s)}$, defined via $V(r_i-r_j)=\Sigma_m V_m^{(s)} P_m^{(s)ij}$, where $V_m^{(s)}$ is the interaction energy of two particles in the sth LL in the relative angular momentum m state, and $P_m^{(s)ij}$ is the corresponding projection operator.[14] Following Park *et al.*,[15] we map the problem of a given interaction in the $s=1$ Landau level into that of an *effective* interaction $V'(r)$ in the lowest ($s=0$) Landau level, chosen so that the two have the same pseudopotentials, i.e., $V_m^{(1)}=V_m'^{(0)}$. We implement this strategy in an approximate scheme by taking the following convenient form for the effective potential:

$$V'(r)=\frac{e^2}{\epsilon}\left(\frac{1}{r}+a_1'e^{-\alpha_1'r^2}+a_2'r^2e^{-\alpha_2'r^2}\right).$$

The parameters a_1', a_2', α_1', and α_2' are fixed by requiring that the first three to four *odd* pseudopotentials of the effective potential in the lowest LL match exactly the corresponding pseudopotentials of the Coulomb potential in the second Landau level. Only the odd pseudopotentials are relevant due to the antisymmetric form of the spatial part of the wave function. The remaining higher order pseudopotentials are asymptotically correct because $V'(r)\to e^2/\epsilon r$ for large r. The $s=2$ LL can be similarly treated, and the corresponding effective potential will be denoted by $V''(r)$. The pseudopotentials for the Coulomb interaction in the $s=1$ and $s=2$ LL's and for the corresponding effective potentials $V'(r)$ and $V''(r)$ in the $s=0$ LL are shown in Fig. 1, demonstrating that the lowest LL problem with the effective interaction is an excellent approximation to the higher LL problem with Coulomb interaction. With this interaction, we then evaluate the energy of the CF exciton using the quantum Monte Carlo method developed earlier for the lowest LL wave functions.[10] Because the pseudopotentials are matched for the planar geometry, our results in the spherical geometry are meaningful only for sufficiently large systems; unphysical results may be obtained for small systems due to finite size effects. We will work below with systems containing as many as $N=66$ particles, using an efficient updating method discussed earlier.[16]

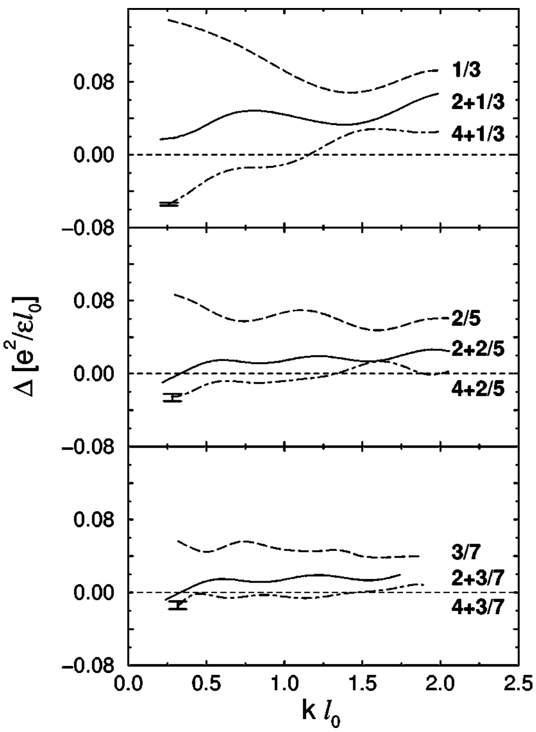

FIG. 2. The dispersions of the composite-fermion exciton in the lowest three Landau levels: $s=0$ (dashed line), $s=1$ (solid line), and $s=2$ (dot-dashed line) at $\nu=2s+\nu_0$, with $\nu_0=1/3$, 2/5, and 3/7. The typical Monte Carlo uncertainty is shown at the beginning of the $s=1$ curves. The energies are given in units of $e^2/\epsilon l_0$ where ϵ is the dielectric constant of the background material, and l_0 is the magnetic length at ν.

We first consider ^{2}CF's, corresponding to FQHE at $\nu=2s+n/(2n+1)$. Figure 2 shows the energy of the CF exciton with $n=1$, 2, and 3 in the three lowest LL's ($s=0$, 1, and 2), for 66 particles. The energies are quoted in units of $e^2/\epsilon l_0$, where $l_0=\sqrt{\hbar c/eB}$ is the magnetic length at ν.

All FQHE is unstable in $s=2$ for ^{2}CF's. We expect that this would remain the case in still higher LL's. The absence of FQHE in $s\geqslant 2$ is consistent with earlier theoretical studies,[4,5,17] which have made a convincing case for either a bubble crystal or a stripe phase in third or higher ($s\geqslant 2$) LL's. At half filling ($\nu_0=1/2$), the wave vector of the stripe phase was estimated[4,5] to be $ql_0\sim 2.4/\sqrt{2s+1}$, which is also approximately equal to the reciprocal lattice vector associated with the bubble crystal away from half filling. Even though the instability occurs for wave vectors below $ql_0\approx 1$, it is unfortunately not possible to determine from the dispersions shown in Fig. 2 a *single* wave vector for the instability, which precludes us from ascertaining from our method the reciprocal lattice vector of the true charge density wave ground state. Note that a comparison between the energies of the 1/3 FQHE state and the Wigner crystal state in the third LL does not indicate a lack of FQHE here,[18] which is understandable in view of the fact that the instability is into a more complicated bubble crystal.

Surprisingly, we find the FQHE to be unstable also in the second LL. The only exception is $\nu_0=1/3$, which corresponds to $\nu=7/3$ in experiments (and, of course, other states related to it by symmetry). We have confirmed that $\nu=7/3$ remains stable in the thermodynamic limit by extrapolating the $kl_0\rightarrow 0$ energy to the $N^{-1}\rightarrow 0$ limit. Given that, by its very design, our approach is expected to *overestimate* the strength of a FQHE state, the results provide strong evidence against FQHE at 12/5 for the pure Coulomb interaction. There is often a minimum observed in ρ_{xx} in the vicinity of $\nu=12/5$,[2,19] which may suggest an incipient FQHE state here (although no plateau has been observed yet). Can a FQHE state here be stabilized by LL mixing or finite thickness effects? LL mixing is a weak effect at large magnetic fields, and is expected to have the opposite effect, because it effectively screens the short range part of the interaction, thereby further weakening a given FQHE state. (It is possible, however, that at relatively low magnetic fields, when the screening is strong, the modification of the interaction due to screening by the lower Landau level[13] might alter this conclusion. We have not investigated this question.) We have considered the effect of the transverse width of the electron wave function by employing the Fang-Howard variational form for the effective interaction,[20] appropriate for the triangular well geometry. The instability is weakened, but not eliminated for typical parameters.

The instability in the second Landau level (Fig. 2) appears to occur at a small wave vector, which suggests a uniform compressible ground state (although a charge density wave state with a large lattice spacing can obviously not be ruled out on account of the finite size of our study). The nature of the compressible state in the second LL is not fully understood at present.

What about the other flavors of composite fermions? There are theoretical indications[18] that these are more stable in higher LL's than the ^{2}CF's. In order to explore this issue further, we have computed the dispersion of the CF exciton for the ^{4}CF's and ^{6}CF's at $\nu=2s+1/5$, $2s+2/9$, $2s+3/13$, and $2s+1/7$ for $s=0$, 1, and 2, shown in Fig. 3. Indeed, these states are more stable in higher Landau levels, with the ^{4}CF states having the largest roton gap (in units of $e^2/\epsilon l_0$) in the second LL and the ^{6}CF states in the third. Unlike for ^{2}CF's, FQHE for ^{4}CF's and ^{6}CF's survives in the second and third LL's. As mentioned earlier, the trial wave function Ψ is quite close to the actual 1/5 ground state in the second LL, in fact more accurate than in the lowest LL,[12] implying that the dispersion shown in Fig. 3 is quantitatively reliable. The 2/7 state is analogous to 1/5: whereas 1/5 is obtained from 1/3 by attachment of two additional vortices, 2/7 is similarly obtained from 2/3. There exists preliminary experimental evidence for both 1/5 and 2/7 in the second LL.[2,19] There is an indication for 1/7 state in the lowest LL,[21] but none yet in higher LL's. The observation of the ^{4}CF and ^{6}CF states in higher LL's is complicated by their rather small energy gaps (because, for a given density,

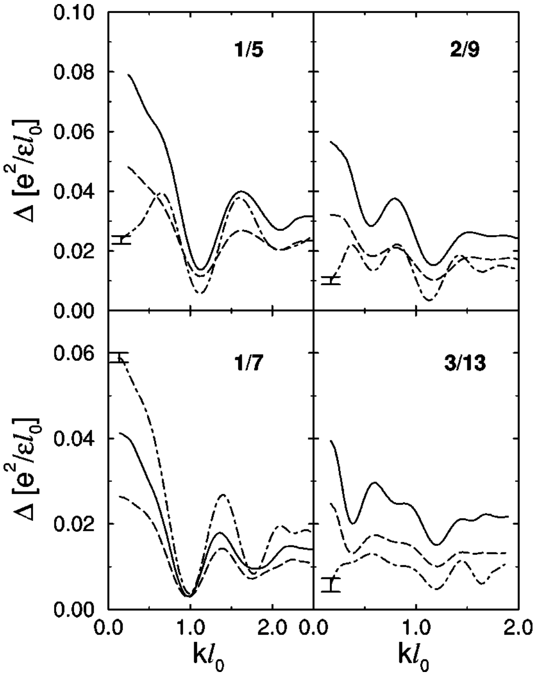

FIG. 3. The same as Fig. 2 but for the fractions with $\nu_0=1/5$, 2/9, 3/13, and 1/7.

$e^2/\epsilon l_0$ is much smaller here than at the corresponding lowest LL fraction), as well as their close proximity to strong integral quantum Hall plateaus.

Why is the FQHE for ^{2}CF's unstable in higher Landau levels but relatively stable to changes in effective interaction within the lowest LL? The reason is that in order for the FQHE to occur, the interaction must not only be repulsive but it must have a sufficiently strong hard-core repulsion at short distances. It is useful to characterize the hard-core nature of the repulsion for a given interaction by the ratio $V_1^{(s)}/V_3^{(s)}$, given in Fig. 1. It appears that for $V_1/V_3\leqslant 1.3$, most FQHE is unstable, with the exception of the 1/3 state which is only marginally stable. In contrast, when finite thickness is taken into account in the lowest LL, all pseudopotentials are suppressed more or less uniformly, and the ratio remains above 1.4–1.5 for typical experimental parameters. For ^{4}CF's the pseudopotential V_1 is ineffective (provided it is not too small to cause an instability) because the wave functions for ^{4}CF's are approximately given by the ^{2}CF wave functions multiplied by a power of Jastrow factor that eliminates the unit relative angular momentum. Similarly, V_1 and V_3 are not relevant for ^{6}CF's. Therefore, the appropriate ratios are V_3/V_5 and V_5/V_7 for ^{4}CF's and ^{6}CF's, respectively. These, however, *increase* in going from the lowest to the higher LL's, as shown in Fig. 1, thus explaining why ^{4}CF's and ^{6}CF's have a qualitatively different LL-index dependence as compared to the ^{2}CF's. There is a close correspondence between the appropriate ratio and the stability, as a comparison of Figs. 1 and 3 shows, with ^{4}CF's (^{6}CF's) being strongest in the $s=1$ $(s=2)$ LL.

In conclusion, our study provides an insight into the paucity of FQHE in higher Landau levels, as well as the qualitative difference between the stabilities of composite fermions of various flavors in different Landau levels. Most FQHE states of ^{2}CF's in higher Landau levels are intrinsically unstable due to a collapse of the neutral excitation, which in turn is triggered by a softening of the short range part of the interaction.

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