Possible Pairing-Induced Even-Denominator Fractional Quantum Hall Effect in the Lowest Landau Level

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We report on our theoretical investigations that point to the possibility of a fractional quantum Hall effect with partial spin polarization at $\nu = 3/8$. The physics of the incompressible state proposed here involves p-wave pairing of composite fermions in the spin reversed sector. The temperature and magnetic field regimes for the realization of this state are estimated.

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The absence of the fractional quantum Hall effect (FQHE) at the simplest even denominator fraction, $\nu = 1/2$, continued to be an enigma for a decade after the discovery of the FQHE [1], but a natural explanation was found [2,3] in the framework of the composite fermion (CF) theory of the FQHE [4,5]. The sequence $\nu = \frac{n}{2n\pm1}$, corresponding to the integral quantum Hall effect of composite fermions carrying two vortices (denoted by ^{2}CFs) at $\nu^* = n$, converges to $\nu = 1/2$ in the limit $n \to \infty$. At least for a model of noninteracting composite fermions, a gapless Fermi sea of composite fermions is obtained here, for which good experimental support exists [6]. A FQHE at $\nu = 1/2$ is not ruled out in principle, though; it may occur if, due to the residual inter-CF interaction, the CF Fermi sea should become unstable into an incompressible state as the temperature is lowered. There is, however, no experimental evidence at present for such an instability at $\nu = 1/2$.

One might expect similar physics at the half filled *second* Landau level (LL), $\nu = 5/2$, but a FQHE is observed here instead [7,8]. The most promising proposal for the physical origin of the FQHE at $\nu = 5/2$ is based on a p-wave pairing of composite fermions [9–15], described by a BCS-like Pfaffian wave function of Moore and Read [9]. In spite of the intuitive appeal and theoretical support of this idea, further experimental tests of its consequences are crucial for its establishment [16]. The difference between $\nu = 1/2$ and $\nu = 5/2$ lies in the interaction matrix elements, i.e., the Haldane pseudopotentials [17]. A strong short-range repulsion between electrons produces a Fermi sea of composite fermions, but when the interaction becomes weakly repulsive at short distances, as in the second LL, it produces an effective *attractive* interaction between composite fermions, causing a pairing instability of the CF Fermi sea [15]. For an attractive interaction between electrons, either a charge-density-wave phase [18] or a spin-singlet FQHE state [19] becomes relevant, depending on parameters. The p-wave pairing between composite fermions thus seems to be favored when the interparticle interaction is *weakly* repulsive at short distances.

An example of weakly interacting fermions is composite fermions themselves. This raises the natural question if they could ever do what electrons do at $\nu = 5/2$, namely, put on (additional) vortices and pair up to produce FQHE. After a careful consideration of a wide range of possibilities, we have concluded that the best candidate is at CF filling of $\nu^* = 1 + 1/2$, when the $0\uparrow$ Landau level of ^{2}CFs is fully occupied and the $0\downarrow$ ^{2}CF Landau level is half filled, as shown schematically in the inset of Fig. 1. This corresponds to a partially polarized state at $\nu = 3/8$. The reason why this system is a good candidate for pairing is because the CF-CF interaction here is weakly repulsive at short distances, the origin of which can be understood intuitively following an argument by Nakajima and Aoki [20]. The Haldane pseudopotentials for composite fermions, $V_m^{\rm CF}$, are expected to be related to the electron pseudopotentials in the lowest LL, $V_m^{\rm elec}$, approximately according to $V_m^{\rm CF} \propto V_{m+2p}^{\rm elec}$, because a capture of $2p$ vortices by

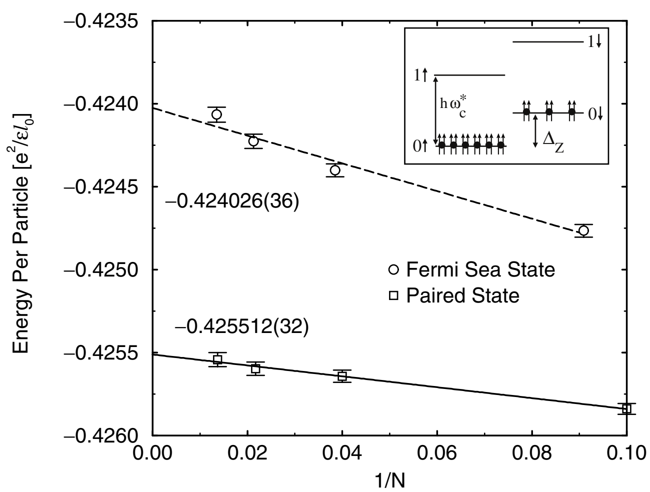

FIG. 1. The composite fermion system is considered at $\nu^* = 1 + 1/2$ with the $0\uparrow$ ^{2}CF LL fully occupied and the $0\downarrow$ ^{2}CF LL half filled, as shown schematically in the inset. The CF LL spacing is $\hbar\omega_c^*$ and the Zeeman splitting is denoted by Δ_Z. This system corresponds to a partially polarized state at $\nu = 3/8$. Two states are considered in which the composite fermion in the $0\downarrow$ ^{2}CF LL forms either a ^{4}CF Fermi sea or a ^{4}CF paired state. The energies of these states are shown as a function of $1/N$, N being the total number of particles, in units of $e^2/\epsilon l_0$, where $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length and ϵ is the dielectric constant of the background material. The error bars reflect the statistical uncertainty in our Monte Carlo calculation.

electrons shifts the relative angular momentum of any pair by $2p$ units. The strong short-range repulsion is thus eliminated when electrons transform into composite fermions [21]. Below we describe our investigations that indeed support the possibility of a p-wave pairing of composite fermions in the spin reversed sector.

The spatial part of the wave function of the electronic state at $\nu = 3/8$ is written as

$$\Psi_{3/8} = J^2 \phi_1^{\uparrow}[\{w_r\}]\phi_{1/2}^{\downarrow}[\{z_i\}], \quad (1)$$

$$J^2 = \prod_{r<s}^{N_\uparrow}(w_r - w_s)^2 \prod_{i<j}^{N_\downarrow}(z_i - z_j)^2 \prod_{i,r}^{N_\downarrow,N_\uparrow}(w_r - z_i)^2, \quad (2)$$

where $w_r = x_r - iy_r$ and $z_j = x_j - iy_j$ refer to the coordinates of the electrons with up and down spins, respectively. The full wave function is written by multiplying by the appropriate spin part followed by antisymmetrization. It has spin polarization $(N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow) = 1/3$ and can be shown to be an eigenstate of the total spin [12,22] with $S = S_z$.

The factor $\phi_1^{\uparrow}[\{w_r\}]$ is the wave function for the completely occupied lowest Landau level. Different states at $\nu = 3/8$ correspond to different choices for $\phi_{1/2}^{\downarrow}[\{z_i\}]$. For the Fermi sea state at $\nu = 3/8$, $\Psi_{3/8}^{\mathrm{FS}}$, we take in Eq. (1)

$$\phi_{1/2}^{\downarrow}[\{z_i\}] = \mathcal{P}_{\mathrm{LLL}} \prod_{j<k}(z_j - z_k)^2 \phi_\infty^{\downarrow}[\{z_i\}], \quad (3)$$

where $\phi_\infty^{\downarrow}$ is the Fermi sea wave function of electrons in zero magnetic field and $\mathcal{P}_{\mathrm{LLL}}$ is the lowest Landau level projection operator. The Pfaffian state, $\Psi_{3/8}^{Pf}$, is obtained with the choice [9]

$$\phi_{1/2}^{\downarrow}[\{z_i\}] = \prod_{j<k}(z_j - z_k)^2 Pf(M^{\downarrow}), \quad (4)$$

where $Pf(M^{\downarrow})$ is the Pfaffian of the $N_\downarrow \times N_\downarrow$ antisymmetric matrix $M^{\downarrow}$ with components $M_{j,k}^{\downarrow} = (z_j - z_k)^{-1}$, defined as $Pf(M^{\downarrow}) \propto A[M_{12}M_{34}\cdots M_{N_\downarrow-1,N_\downarrow}]$, A being the antisymmetrization operator. $Pf(M^{\downarrow})$ is a real space BCS wave function and $\phi_{1/2}^{\downarrow}$ can therefore be viewed as a p-wave paired quantum Hall state of composite fermions.

These states not only have mixed spin, but also an admixture of different flavors of composite fermions [23], those carrying two and four vortices, called ^{2}CFs and ^{4}CFs, respectively. In the first case, ^{2}CFs capture two additional vortices to convert into ^{4}CFs, which effectively experience no magnetic field and form a Fermi sea. In the second case, the ^{2}CFs capture two additional vortices to convert into ^{4}CFs, which form pairs; a gap opens up due to pairing and FQHE results. The wave functions above can be interpreted as describing Fermi sea and paired states of spin-down ^{4}CFs in the background of spin-up ^{2}CFs.

To check which state is energetically superior, we use the explicit analytical expressions for the lowest LL projected wave functions for composite fermions [5,24] and perform $2N$ dimensional integrals using Monte Carlo to obtain their Coulomb energy. We map the states onto the surface of a sphere [17] and calculate the energy of each state in the thermodynamic limit using a least squares fit. Figure 1 shows that while both the paired state and the Fermi sea have energies quite comparale to those of the fully polarized states at 1/3 and 2/5 ($-0.4098e^2/\epsilon l$ and $-0.4328e^2/\epsilon l$, respectively [5,24]), the former is favored over the latter. The energy difference is reduced by approximately an order of magnitude as compared to the analogous difference at 5/2 [12], indicative of a substantially weaker interaction between the composite fermions.

Though consistent with pairing within the spin-down sector, the variational nature of the preceding result does not rule out other ground states. One way to ascertain the validity of $\Psi_{3/8}^{Pf}$ would be to compare it with exact results for small systems. Unfortunately, the smallest system size for this state is ten particles in the spherical geometry, where the Hilbert space is already prohibitively large for an exact diagonalization study. To make progress, we treat the filled $0\uparrow {}^2$CF Landau level as inert, and map the problem of composite fermions at $\nu^* = 1 + 1/2$ into fermions at half filling. The advantage of this method is that only the reversed spin composite fermions in the $0\downarrow {}^2$CF-LL are considered explicitly, which reduces the size of the Hilbert space considerably. Given the strongly correlated nature of the problem, the effective ^{2}CF-^{2}CF interaction is complicated and is expected to contain two, three, and higher body terms. In order to proceed, we neglect all but the two body term; i.e., we assume that the three and higher body terms do not cause any phase transition. This approximate treatment of the effective interaction between composite fermions is the most serious limitation of our model. The physics of the problem suggests the following CF-CF interaction. Consider two electrons in the lowest LL. Attaching $2p$ vortices to each electron modifies the interaction in three ways: (i) The charge is reduced by a factor $(2p + 1)^{-1}$. (ii) The effective magnetic field seen by CFs increases the magnetic length by a factor $(2p + 1)^{1/2}$. (iii) The model pseudopotentials have their relative angular momentum shifted by $2p$. This motivates the following model, similar to the one used previously for an investigation of the spin wave dispersion at $\nu = 1/3$ [20]:

$$\frac{V_{m^*}^{\mathrm{elec}}}{e^{*2}/\epsilon l^*} = \frac{V_m^{\mathrm{CF}}}{e^2/\epsilon l_0}, \quad (5)$$

where $m^* = m + 2p$, $e^* = \frac{e}{2p+1}$, and $l^* = l_0\sqrt{2p + 1}$. A comparison with the pseudopotentials obtained directly from the microscopic wave functions, following the method of Refs. [25,26], demonstrates that the above model is surprisingly accurate. It should be noted that this model is valid only for spin-reversed composite fermions on top of the 1/3 state; in general, the more complicated method of Refs. [25,26] must be used [21].

With the above effective interaction between the spin-down composite fermions, we carry out exact diagonalization to look for pairing correlations within the $0\downarrow {}^2$CF LL.

To begin with, we find a uniform ($L = 0$) ground state at the flux $2Q = 2N_\downarrow - 3$, which corresponds to the Pfaffian wave function, for $N_\downarrow = 8$, 10, 14, and 16 [27]. Next we compare the ground state to the Pfaffian wave function, which is obtained by diagonalizing an interaction (containing three body terms) for which the Pfaffian wave function is the only zero energy eigenstate. The overlaps between the ground state of the $V^{\rm CF}$ interaction (which we identify with the $3/8$ state here) and the Pfaffian wave function are given in Table I; for comparison, the overlaps between the ground state at $1/2$ filling in the second electronic Landau level (identified with $5/2$) and the Pfaffian are also given. While the overlaps are not as decisively large as those between the filled CF-LL wave functions and the exact ground states at the principal filling factors [5], they are significant and overall support the interpretation of the $3/8$ state as a paired state of composite fermions.

To address the issue of the robustness of the $3/8$ paired state, we have investigated its evolution in a model in which the first odd pseudopotential, $V_1^{\rm CF}$, is replaced by $V_1^{\rm model}$. As shown in Fig. 2, the state survives an $\sim$8% change in V_1 in either direction. Based on the previous discussion, one would expect that on the large $V_1^{\rm model}$ side of the "paired" region ($V_1^{\rm model}/V_1^{\rm CF} \approx 1$), the Fermi sea has the lowest energy, whereas the stripe phase is likely on the small V_1 side. We have confirmed this by comparing the energies of the three states following the method of Ref. [26]. This points to the interesting possibility that a transition from the paired state to stripes or Fermi sea may be driven by a change of parameters, as was suggested at $5/2$ as well [14]. It is noted that for the *fully* spin polarized state at $\nu = 3/8$, where one must consider the half filled $1\uparrow$ ^{2}CF Landau level, a similar model for the CF-CF interaction appears to indicate the stripe phase [26].

Another measure of the strength of a FQHE state is the excitation gap. The lowest energy excitations of the $3/8$ state are expected to lie within the spin-down CF LL, because of the reduced effective interaction. Therefore, our effective model containing only spin-down composite fermions is also valid for low-energy excitations. Figure 3 shows the low-energy spectrum for 8, 10, 14, and 16 particles for the effective interaction, $V^{\rm CF}$, obtained by the Lanczos method. The energy gap is on the order of $\sim 0.0004\frac{e^2}{\epsilon l_0}$. While the smallest gap is not identical to the gap for creating a far separated particle-hole pair, we expect both to be of similar magnitude. In an analogous study, Morf [11] estimated the gap at $\nu = 5/2$ to be $0.02\frac{e^2}{\epsilon l_0}$. For a given density, the units $\frac{e^2}{\epsilon l_0}$ differ by a factor of $\sqrt{20/3}$ at $5/2$ and $3/8$, and the theoretical estimates for the $5/2$ and $3/8$ gaps differ by a factor of $\sim$20 in constant units (e.g., mK). Experimentally, the gap at $5/2$ is in the range 200–300 mK [8,28], which would suggest that the gap for the paired state at $3/8$ might be in the range 10–15 mK, which is quite small but above the lowest temperatures where FQHE experiments have been performed [8]. (Given that the $5/2$ gap is a factor of 3–5 smaller than the theoretical value, the number 10–15 mK ought to be taken only as a crude estimate.)

A sufficiently large Zeeman energy will eliminate a partially polarized ground state at $\nu = \frac{3}{8}$. In order to estimate the magnetic field range where the partially polarized state may be viable, let us consider the addition of a single composite fermion to the state in which all states of the $0\uparrow$ CF-LL are occupied. The composite fermion can be added to either the $0\downarrow$ CF-LL or the $1\uparrow$ CF-LL. The former is favorable so long as the Zeeman splitting energy, $\Delta_Z = |g|\mu_B B$, is smaller than the effective cyclotron energy of the composite fermion, $\hbar\omega_c^*$, defined as $\hbar\omega_c^* = \epsilon^{1\uparrow} - \epsilon^{0\downarrow}$, where $\epsilon^{1\uparrow}$ and $\epsilon^{0\downarrow}$ are the Coulomb self-energies for the additional composite fermion in the $1\uparrow$ and $0\downarrow$ CF-LLs. It has been estimated from exact diagonalization as well as CF wave functions [25,29,30] that $\hbar\omega_c^* = 0.028\frac{e^2}{\epsilon l_0}$ for the Coulomb interaction. The condition favoring the addition of the spin reversed CF, $\hbar\omega_c^* > \Delta_Z$, is satisfied for $B < 20$ T for parameters appropriate to GaAs. On the other hand, theoretical calculations [31] indicate that the state at $2/5$ is spin singlet for magnetic fields less than $\sim$5 T, suggesting that the level crossing transition is in fact filling factor dependent, and at $\nu = 3/8$, it may take place somewhere between 5 T

TABLE I. Overlaps of the $3/8$ and $5/2$ states, defined in text, with the paired Pfaffian wave function.

N	5/2	3/8
8	0.87	0.99
10	0.84	0.95
14	0.69	0.87
16	0.78	0.86

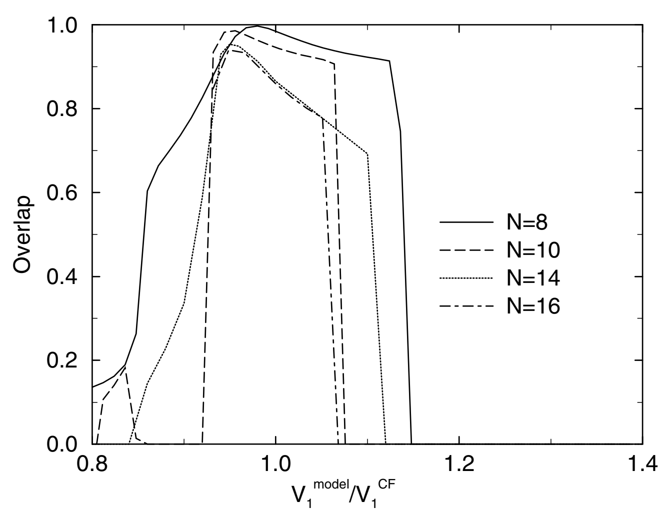

FIG. 2. The overlap between the paired CF wave function of Moore and Read and the exact ground state of the model in which V_1 pseudopotential of the effective CF interaction is varied for $N = 8$, 10, 14, and 16 particles.

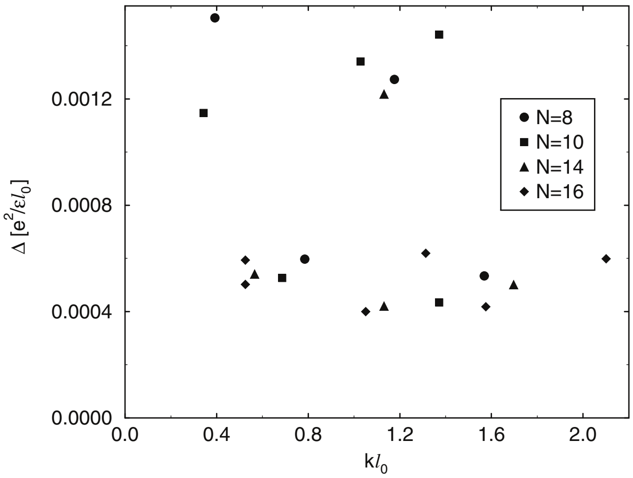

FIG. 3. The low-energy spectrum for the CF model interaction. The energies of all states are measured relative to the ground state for each N. The lowest energy states form a band separated from the continuum.

and 20 T. While it is not possible at the moment to ascertain the crossover B at $\nu = 3/8$ more accurately, these considerations imply that a partially polarized state can occur in presently accessible parameter range. (We note that numerous partially polarized or unpolarized FQHE states have been observed [5].)

Before concluding, it is worth noting how fantastically complex the proposed $3/8$ paired state is when viewed in terms of electrons: first all electrons at $\nu = 3/8$ capture two vortices to become ^{2}CFs at $\nu^* = 1 + 1/2$; then those in the half filled spin reversed ^{2}CF Landau level capture two additional vortices to transform into ^{4}CFs that see no magnetic field; these would normally form a ^{4}CF Fermi sea, but the Fermi sea is unstable to pairing due to a weak residual attraction between the ^{4}CFs; pairing of ^{4}CFs opens up a gap to produce FQHE. An observation of FQHE at this even denominator fraction in the lowest LL, apart from being interesting in its own right, will provide further support for pairing of composite fermions as a valid mechanism for FQHE.

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