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Interacting composite fermions

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Abstract

Even though much of the dramatic physics of two-dimensional electrons in a high magnetic field is explicable in terms of weakly interacting composite fermions (CFs), the inter-CF interaction is responsible for many interesting, non-trivial phenomena. Here, we discuss four examples. (i) At small filling factors, a softening of the roton mode destroys the fractional Hall effect, giving way to the Wigner crystal. (ii) In higher Landau levels, the fractional Hall effect is destroyed due to a collapse of the energy of the neutral exciton. (iii) At $\nu = 5/2$, the Fermi sea of CFs is unstable to Cooper pairing of CFs, thereby opening up a gap and producing a fractional Hall effect. (iv) Prior to the transition into the Wigner crystal, the CF liquid exhibits the Bloch instability into a magnetically ordered, spontaneously broken symmetry phase. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The two-dimensional electron system exhibits absolutely marvelous and unexpected phenomena when exposed to a strong magnetic field [1]. Our understanding of these phenomena relies on the transformation of the strongly correlated liquid of electrons into a weakly interacting gas of composite fermions (CFs) [2–6], where a CF is the bound state of an electron and an even number of quantum mechanical vortices of the many body wave function. The most remarkable property of CFs is that they experience a drastically reduced effective magnetic field.

The CF theory can be summarized by three simple equations:

$$\Psi_\nu = P_{\rm LLL}\Phi_1^{2p}\Phi_{\nu^*}, \tag{1}$$

$$B^* = B - 2p\rho\phi_0, \tag{2}$$

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}. \tag{3}$$

The last two can be derived from the first so in fact, everything stems from a single equation. Here, B is the external magnetic field, ρ the two-dimensional density of particles, $\phi_0 = hc/e$ the fundamental quantum of flux and ν the filling factor of electrons. The even integer $2p$ is the vorticity of the CF, B^* is the magnetic field experienced by CFs, and $\nu^* = \rho/B\phi_0$ is the filling factor of CFs. Φ_{ν^*} is the wave function of non-interacting electrons at ν^*, ψ_ν is the wave function of interacting electrons at ν, and $P_{\rm LLL}$ is the lowest Landau level projection operator. Finally, it is convenient to use as the length scale the natural length unit in the quantum Hall effect, called magnetic length $l_0 \equiv \sqrt{\hbar c/eB}$.

CFs carrying $2p$ vortices are denoted by ^{2p}CF. The state with n filled Landau levels of CFs is denoted by $^{2p}\mathrm{CF}_n$. When the spin degree of freedom is of interest, some (say $n\uparrow$) of the ^{2p}CF-LLs have spin up, and the rest (say $n\downarrow$) have spin down; this state is denoted by $^{2p}\mathrm{CF}_{n\uparrow,n\downarrow}$.

The interaction between the CFs is much weaker than that between electrons, because much of the inter-electron interaction is screened out by the formation of CFs. It is a good first approximation to neglect the interaction between the CFs altogether. The fractional quantum Hall effect (FQHE) is then understood straightforwardly as an integer quantum Hall effect of CFs [2–4], and the compressible state at the half-filled Landau level is well described by a Fermi sea of CFs [7–10]. The weak residual interaction between CFs makes quantitative corrections, but often does not lead to any qualitative changes in the nature of

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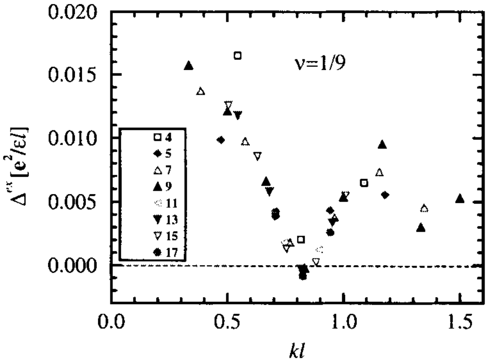

Fig. 1. Exciton dispersion for the FQHE state at $\nu = 1/9$ for systems with several sizes, with the electron number N shown in the figures. Here $l(\equiv \sqrt{\hbar c/eB})$ is known as the magnetic length. All curves assume zero thickness. The error bars are smaller than the symbol sizes. Taken from Jain and Kamilla [12,13].

the state. The model of non-interacting CFs is qualitatively valid in such situations.

For certain parameters, however, the inter-CF interaction is of critical importance because it can drive phase transitions. That is the subject of the present article. We will briefly review here four situations when the predictions of the non-interacting CF model are altered qualitatively by quantum fluctuations caused by the inter-CF interactions. At small ν, the FQHE is destroyed due to a rotonic instability, making way for the Wigner crystal (WC) [11–13]. In hither LLs, the FQHE is destroyed due to an excitonic instability [14]. At $\nu = 5/2$, a Cooper instability in the Fermi sea of CFs signifies the opening of a gap, a necessary condition for FQHE at this filling factor [15]. Finally, at small filling factors and small Zeeman energies, a spontaneous magnetic ordering of the CF state takes place [16]. The instabilities are driven by the residual interaction between the CFs and signify a dramatic breakdown of the model of weakly interacting CFs in the parameter regimes considered.

The theoretical approach for treating the inter-CF interaction employs Jain's wave functions for CFs, given by Eq. (1). Even though these wave functions are motivated by the model of non-interacting CFs, and are related to non-interacting electron states Φ, they are known to be extremely close to the exact solution of the fully interacting electron problem; comparisons with exact diagonalization results for small systems have shown that the energies predicted by Ψ are typically within 0.1% of the exact eigenenergies [12,13,17,18]. Ψ thus incorporates the effects of inter-CF interaction. This is remarkable in view of the fact that Ψ contains no adjustable parameters. We will only give the salient results here; the details of our calculations can be found in the literature [11–16]. In particular, we will not discuss Landau level mixing [19–21] or finite thickness effects [22–24]. These make quantitative corrections, but are not relevant for the qualitative physics considered here.

Related approaches for treating the inter-CF interaction are the Chern–Simons theory [7,25] and Shankar–Murthy's Hamiltonian approach [26,27]. We will discuss below only the results from the microscopic wave function method and refer interested readers to the literature for other approaches.

The results are in striking agreement with experiment. It has been known that the FQHE does not occur at *all* odd denominator filling factors. At very small filling factors, the CF liquid loses to the WC. In higher Landau levels, it loses to charge-density-wave type states. The CF theory provides a natural explanation for the lack of FQHE at small ν and in higher LLs. At $\nu = 5/2$, a FQHE is seen even though one would naively have expected a Fermi sea of CFs. This appears naturally in the CF theory through a Cooper instability of the CF Fermi sea at $\nu = 5/2$. We stress that all of this rather subtle physics is discovered entirely within the tightly constrained, zero-parameter framework of the CF theory.

2. Roton instability at small ν

One method for studying the feasibility of the FQHE is to compare the energies of the variational wave functions of the FQHE state and other candidate states. It is reliable when good guesses exist for the various kinds of states.

The other approach is to start with the assumption of the FQHE state, and investigate its stability. An instability exhibits itself through the softening of certain excitations. When this happens, we know that we started with the wrong state.

Fig. 1 shows the energy of the CF exciton, Δ^{ex}, at $\nu = 1/9$ (measured relative to the energy of the uniform FQHE state) as a function of the wave vector k. At the lowest energy in the dispersion, the exciton is called the roton, by analogy to ^{4}He [28]. The roton mode has been observed for ^2CFs [29], with its energy in good agreement with theory [30,31]. The most striking aspect is that the energy of the CF roton at $\nu = 1/9$ (but not for 1/3, 1/5 or 1/7) falls below that of the uniform FQHE state at approximately $kl \approx 0.85$. The FQHE is thus explicitly demonstrated to be unstable to a spontaneous creation of CF rotons. Similar results are expected for $\nu < 1/9$, ruling out FQHE for $\nu \leq 1/9$. The fact that the wave vector of the instability is close to the reciprocal lattice vector of the WC [32] at $\nu = 1/9$, $k_{WC}l = (4\pi\nu/\sqrt{3})^{1/2} \approx 0.898$ also hints to the nature of the true ground state.

It has been well known that the true ground state at very small filling factors is a WC. In studies [33,34] comparing the energy of Ψ with that of a WC state of electrons, it was predicted that a transition would occur at approximately at $\nu^{-1} = 6.5 \pm 0.5$. Our results demonstrate that this fact can be discovered entirely within the theory of the FQHE. Our results also suggest that the state at $\nu = 1/7$ is likely a FQHE

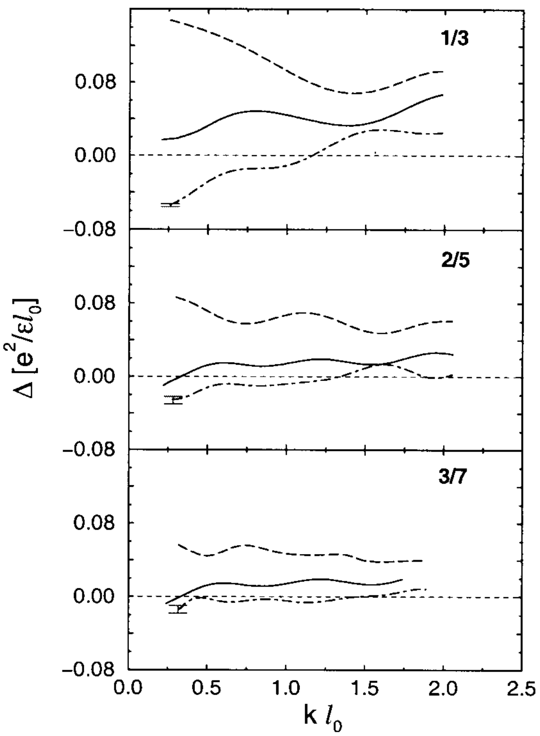

Fig. 2. The dispersions of the CF exciton in the lowest three Landau levels: $s = 0$ (dashed line), $s = 1$ (solid line) and $s = 2$ (dot-dashed line) at $\nu = 2s + \nu_0$, with $\nu_0 = 1/3,\ 2/5$, and 3/7. The typical Monte Carlo uncertainty is shown at the beginning of lowest curve. The energies are given in units of $e^2/\epsilon l_0$ where ϵ is the dielectric constant of the background material, and l_0 is the magnetic length at ν. Taken from Scarola et al. [14].

state, consistent with preliminary experimental observations [35].

3. Excitonic instability in higher Landau levels

There exists evidence for more than 50 fractions in the lowest Landau level. The essential phenomenon is remarkably insensitive to the detailed form of the repulsive interaction, as manifested by the fact that it is quite robust to perturbations arising from Landau level mixing, finite thickness, or the nature of the transverse confinement.

The system of electrons restricted to a higher Landau level deviates from that in the lowest LL only through the short-distance matrix elements of the Coulomb interaction. Consequently, one would expect the FQHE to be not too sensitive to the Landau level index either. From this point of view, it is astounding that the FQHE is so rare in higher Landau levels. The only higher Landau level fractions for which decisive experimental evidence exists, in the form of reasonably well quantized plateaus, are $\nu = 1/3$ and $\nu = 1/2$ (and other fractions related by symmetry) in the second LL [36–38]. No FQHE has been observed at all in the third or higher LLs. Mean-field [39–42], exact diagonalization [43], as well as experimental [44,45] studies suggest that a bubble crystal or a stripe phase is favored over FQHE in higher LLs.

We ask whether the instability of the FQHE can be discovered within the CF theory. We consider filling factors $\nu = 2s + \nu_0$. Here, the quantity $s = 0, 1, \ldots$ denotes the Landau level index, and the factor 2 in $2s$ arises due to spin degeneracy (because a Landau level with both spin states filled contributes 2 to the filling factor). We take the filled LLs to be completely inert and work only with the electrons in the topmost partially filled LL. Furthermore, these electrons will be taken to be fully spin polarized; this is a valid approximation in the limit of high magnetic fields when LL mixing is negligible.

We show here results for ^{2}CFs, corresponding to FQHE at $\nu = 2s + (n/(2n + 1))$. Fig. 2 shows the energy of the CF exciton in the lowest LL at $\nu = 1/3$, 2/5, and 3/7; in the second LL at $\nu = 7/3$, 12/5, and 17/7; and in the third LL for $\nu = 13/3$, 22/5, and 31/7. A system of 66 particles was used for the calculations. The energies are quoted in units of $e^2/\epsilon l_0$, where $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length at ν.

These dispersions demonstrate that the CF theory captures the lack of FQHE in higher LLs through an excitonic instability. All FQHE is unstable in the third ($s = 2$) and higher LLs for ^{2}CFs. In the second ($s = 1$) LL, all states other than $\nu = 7/3$ are unstable for ^{2}CFs.

What about the other flavors of CFs? There are theoretical indications [46] that ^{4}CF and ^{6}CF states are more stable in higher LLs than the ^{2}CF states. In order to explore this issue further, we have computed the dispersion of the CF exciton for the ^{4}CFs and ^{6}CFs at $\nu = 2s + 1/5$, $\nu = 2s + 1/7$, $\nu = 2s + 2/9$, and $\nu = 2s + 3/13$ in the lowest three Landau levels ($s = 0, 1, 2$). No instability was found. Indeed, these states are more stable in higher Landau levels, with ^{4}CF states having the largest roton gap (in units of $e^2/\epsilon l_0$) in the second LL and the ^{6}CF states in the third. There is an experimental indication for the 1/7 state in the lowest LL [35] but none yet in higher LLs. The observation of the ^{4}CF and ^{6}CF states in higher LLs is complicated by their rather small energy gaps (because, for a given density, $e^2/\epsilon l_0$ is much smaller here than at the corresponding lowest LL fraction), as well as their close proximity to strong integral quantum Hall plateaus.

4. Cooper instability at $\nu = (5/2)$

We next turn to the Landau level filling $\nu = 5/2 = 2 + 1/2$, corresponding to a half-filled *second* LL. Again, the fully occupied LL is treated as inert and the electrons in the partially filled second LL as fully spin-polarized. In complete analogy to the half-filled lowest LL, the model of non-interacting CFs would predict a Fermi sea of CFs at $\nu = 5/2$ as well. However, experiments reveal a FQHE state here [36–38]. In fact, 5/2 is the only even-denominator

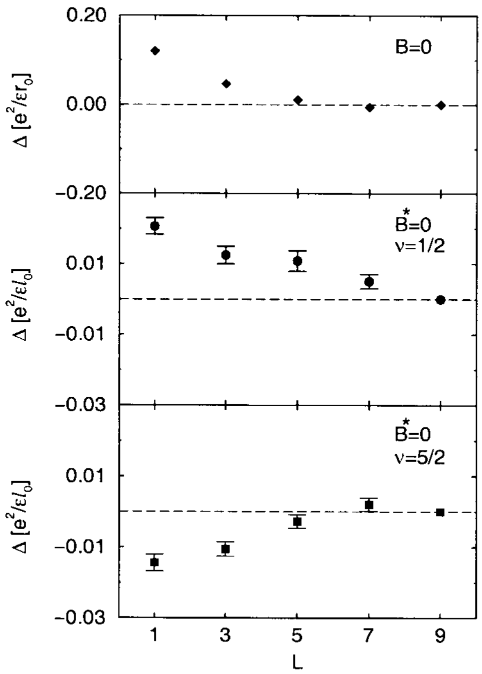

Fig. 3. The interaction energy of a pair of CFs at zero effective flux ($B^* = 0$), which, in the thermodynamic limit, corresponds to $\nu = 1/2$ in the lowest Landau level and $\nu = 5/2$ in the second Landau level. Also shown for comparison is the energy of the analogous electron system at $B = 0$. The results are shown for a system of $N = 27$ particles. The quantity $l_0 = \sqrt{\hbar/eB}$ is the magnetic length, $r_0 = (\pi\rho)^{-1/2}$ is the average interparticle separation, and ϵ is the dielectric constant of the background material. The spherical geometry is used in the calculation. L is the total orbital angular momentum of the pair. The distance between the two partners of the pair increases with L, as can be deduced by the top panel which shows the Coulomb energy of two electrons in an otherwise empty Landau level. Taken from Scarola et al. [15].

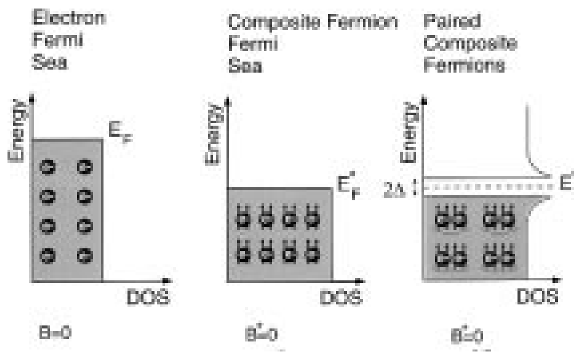

Fig. 4. A schematic depiction of the physics of the Fermi sea of electrons at $B = 0$, the Fermi sea of CFs at $\nu = 1/2$, and pairing of CFs at $\nu = 5/2$. The CFs are shown as electrons carrying two flux quanta. DOS stands for the density of states. Taken from Scarola et al. [15].

fraction to be observed in a single layer system, and its physical origin has been a subject of debate and controversy over the years.

We ask, in analogy to the Cooper problem for ordinary superconductivity: If we begin by assuming a Fermi sea of CFs both at $\nu = 1/2$ and $\nu = 5/2$ and add two CFs at the Fermi surface, will they form a bound state? The energy of the pair is computed as a function of the distance between the two CFs. As shown in Fig. 3, the interaction between the CFs is repulsive at $\nu = 1/2$, but *attractive* at $\nu = 5/2$, indicating that the CF-Fermi sea is unstable to a pairing of CFs at $\nu = 5/2$. The difference between the physics of 1/2 and 5/2 is shown schematically in Fig. 4.

It is stressed that, unlike BCS theory, we do not assume any attractive interaction, phonon-mediated or otherwise. The only interaction in the problem is the repulsive Coulomb interaction between electrons. However, it translates into a weak attractive interaction between CFs at $\nu = 5/2$.

The appearance of pairing may seem surprising in a model with strong repulsive interaction. The Coulomb repulsion is overcome through the formation of CFs, which screens out the Coulomb interaction quite effectively, to the extent that a total neglect of the interaction between CFs is valid for many purposes. Furthermore, the screening takes place in a topologically rigid manner, independent of the interaction strength or the Landau level index, through the binding of precisely two vortices to each electron. Therefore, it is plausible that it may sometimes cause an overscreening of the Coulomb interaction, producing an effectively attractive interaction between CFs. Why is there an attraction at $\nu = 5/2$ but not at $\nu = 1/2$? The matrix elements for the Coulomb interaction in the second Landau level are weaker than in the lowest LL because of the greater spread of the electron wave function in the former, especially at short distances. This slight softening of the inter-electron repulsive interaction in the second LL is sufficient to push the inter-CF interaction into weakly negative territory.

The pairing in this system has a novel, topological origin, and occurs in spite of strong repulsive interaction between electrons; the repulsion is circumvented because the objects that pair up are not electrons but CFs. We speculate that a fundamental reorganization of the state, e.g. creation of new quasiparticles, must happen in any system in order for pairing to ensue starting from repulsive interactions alone. This is indeed the case in several theoretical models of high temperature superconductivity, where also the pairing is believed to be caused by repulsive interactions.

There has been earlier work on pairing of CFs. Greiter et al. [47] argued for p-wave pairing of CFs at $\nu = 1/2$ and 5/2 within a Chern–Simons formulation of CFs [25]. The Chern–Simons method, however, is quantitatively unreliable for this question because of its inadequacy in describing the energetics or the short-distance behavior. Even within this approach, Bonesteel [48] has noted that a pair breaking

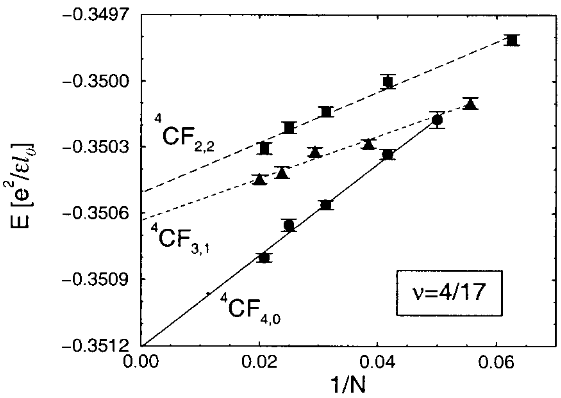

Fig. 5. Thermodynamic extrapolations for the energies of ${}^4CF_{2,2}$, ${}^4CF_{3,1}$, and ${}^4CF_{4,0}$, the variously spin-polarized states of ^{4}CFs at 4/17. The energies are quoted in units of $e^2/\epsilon l$, where $l = \sqrt{\hbar c/eB}$ is the magnetic length and ϵ is the dielectric constant of the background material. The lines show the best straight line fits. Taken from Park and Jain [16].

term not considered in Ref. [47] may potentially alter its conclusion. Greiter et al. further suggested that the paired CF state may be described in terms of a Pfaffian wave function written by Moore and Read [49]. Recent exact diagonalization [50,51] and variational [52] studies have provided support for the validity of a Pfaffian-like wave function at $\nu = 5/2$.

5. Bloch instability

The system of electrons in a uniform, positively charged background is a widely used and studied model in condensed matter physics. It was suggested by Bloch [53] in 1929 that, due to exchange effects, the Fermi sea of electrons is susceptible to a spontaneous polarization of the electron spin at low densities, when the interaction becomes strong relative to the kinetic energy. Another competing phase at low-densities is the WC, a lattice of electrons. A phase diagram for interacting electrons as a function of density has been the subject of much theoretical study and controversy. The usual perturbative approaches, e.g. the Hartree–Fock or random-phase approximation, are not useful at low densities; sophisticated quantum Monte Carlo calculations [54] indicate that a transition into the WC state occurs at $r_s \approx 37$, preceded by a ferromagnetic phase [55]. The ferromagnetic Bloch phase has not yet been observed, though.

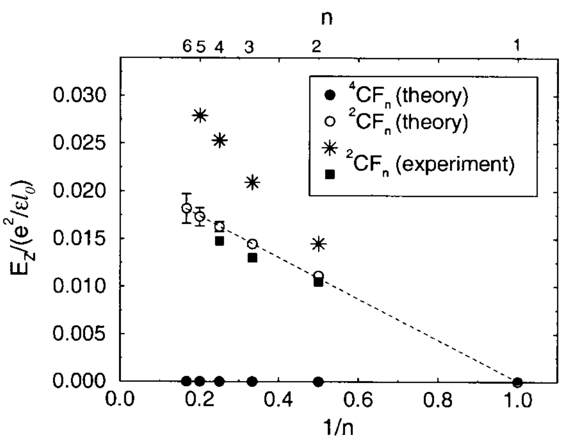

Fig. 6. Critical Zeeman splittings above which the CF state is fully polarized. The empty and filled circles are predictions of theory, and stars and squares are taken from the experiments of Du et al. [59] and Kukushkin et al. [60]. The figure is taken from Park and Jain [16].

Motivated by these observations, we have searched for Bloch's magnetization of CFs prior to Wigner crystallization. Of interest here is the *intrinsic* magnetic ordering caused by the exchange interaction, and not the trivial magnetization due to the Zeeman coupling of the electron magnetic moment to the external magnetic field. The Zeeman coupling will therefore be set to zero in what follows. Since there is experimental evidence for the WC phase on both sides of ${}^4CF_1$ (i.e. $\nu = 1/5$) [56,57], we focus on ^{4}CFs and evaluate the energies of ${}^4CF_{n\uparrow,n\downarrow}$.

The weakly interacting CF model works well for ^{2}CFs, relevant in the filling factor range $2/3 > \nu > 1/3$. The theoretical phase diagram [58] of the spin polarization of ${}^2CF_n$ as a function of the Zeeman energy computed with the help of Ψ is in reasonably good quantitative agreement with the experimentally determined phase diagrams [59,60]. For ^{2}CFs, the model of independent CFs is successful in predicting various qualitative features, namely the possible spin polarizations as well as the energy ordering of the differently polarized states; in particular, the ground state in the absence of the Zeeman energy is the least polarized state, as expected for weakly interacting fermions. With a single mass parameter (the polarization mass of the CFs [58]) the non-interacting CF model provides a reasonable semi-quantitative fit to our results over the relevant range of filling factors. Further insight into the success of the free CF-model is obtained in the Hamiltonian approach [61]. Interestingly, there is experimental evidence [60] that the transition between the fully polarized and unpolarized state CF states does not occur directly but through an intermediate state with a partial spin polarization. Murthy [62] has proposed that this state is a Hofstadter lattice of CFs, and has half the maximum possible polarization.

Now let us go back to the main topic of this section: ^{4}*CFs*. Because we are interested in thermodynamic phases, all energies are obtained by a careful extrapolation to the thermodynamic limit, $N^{-1} \rightarrow 0$, as shown in Fig. 5 for ${}^4CF_{4,0}$, ${}^4CF_{3,1}$ and ${}^4CF_{2,2}$. The principal result is that for ^{4}CFs, the fully polarized state is the ground state even at zero Zeeman energy. The model of independent CFs thus fails dramatically for ^{4}CFs at small Zeeman energies, indicating that the inter-CF interaction is sufficiently strong to cause a spontaneous polarization of the ^{4}CF liquid. The phase diagram of the spin polarization of ^4CF_n is contrasted with that of ^2CF_n in Fig. 6, which also gives the experimental results for ^{2}CFs.

We thus predict that the CF liquid exhibits a broken symmetry magnetic phase prior to a transition into the WC. This predictions ought to be experimentally verifiable. The transitions between QHE states of different polarizations have been seen in transport experiments [59], and the polarization itself has been measured in optical luminescence studies [60] and also by NMR [63,64]. Since the magnetization we are predicting is to be distinguished from that caused by the Zeeman coupling, it is hoped that our work will motivate polarization measurements under hydrostatic pressure, which can be used to tune the g factor through zero [65–67]. Our results would imply an absence of any transition at finite Zeeman energies at $n/(4n + 1)$, and a finite jump in the degree of polarization when the g factor changes sign.

Acknowledgements

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References

[1] D.C. Tsui, H.L. Stormer, A.C. Gossard, Phys. Rev. Lett. 48 (1982) 1559.
[2] J.K. Jain, Phys. Rev. Lett. 63 (1989) 199.
[3] J.K. Jain, Phys. Rev. B 41 (1990) 7653.
[4] J.K. Jain, Physics Today April (2000) 42.
[5] O. Heinonen (Ed.), Composite Fermions World Scientific, New Jersey, 1998.
[6] S. Das Sarma, A. Pinczuk (Eds.), Perspectives in Quantum Hall Effects Wiley, New York, 1997.
[7] B.I. Halperin, P.A. Lee, N. Read, Phys. Rev. B 47 (1993) 7312.
[8] R.L. Willett, et al., Phys. Rev. Lett. 71 (1993) 3846.
[9] V.J. Goldman, et al., Phys. Rev. Lett. 72 (1994) 2065.
[10] W. Kang, et al., Phys. Rev. Lett. 71 (1993) 3850.
[11] R.K. Kamilla, J.K. Jain, Phys. Rev. B 55 (1997) 13 417.
[12] J.K. Jain, R.K. Kamilla, Int. J. Mod. Phys. B 11 (1997) 2621.
[13] J.K. Jain, R.K. Kamilla, Phys. Rev. B 55 (1997) R4895.
[14] V.W. Scarola, K. Park, J.K. Jain, PRB (2000) in press.
[15] V.W. Scarola, K. Park, J.K. Jain, Nature 406 (2000) 863.
[16] K. Park, J.K. Jain, Phys. Rev. Lett. 83 (1999) 5543.
[17] G. Dev, J.K. Jain, Phys. Rev. Lett. 69 (1992) 2843.
[18] X.G. Wu, G. Dev, J.K. Jain, Phys. Rev. Lett. 71 (1993) 153.
[19] V. Melik-Alaverdian, N.E. Bonesteel, Phys. Rev. B 52 (1995) R17032.
[20] V. Melik-Alaverdian, N.E. Bonesteel, Phys. Rev. Lett. 79 (1997) 5286.
[21] R. Price, S. Das, Sarma, Phys. Rev. B 54 (1996) 8033.
[22] K. Park, N. Meskini, J.K. Jain, J. Phys.:Condens. Matter 11 (1999) 7283.
[23] F.C. Zhang, S. Das, Sarma, Phys. Rev. B 33 (1986) 2903.
[24] M.W. Ortolano, S. He, S. Das, Sarma, Phys. Rev. B 55 (1997) 7702.
[25] A. Lopez, E. Fradkin, Phys. Rev. B 44 (1991) 5246.
[26] R. Shankar, G. Murthy, Phys. Rev. Lett. 79 (1997) 4437.
[27] G. Murthy, R. Shankar, in: O. Heinonen (Ed.), Composite Fermions, World Scientific, New Jersey, 1998.
[28] S.M. Girvin, A.H. MacDonald, P.M. Platzman, Phys. Rev. Lett. 54 (1985) 581.
[29] A. Pinczuk, B.S. Dennis, L.N. Pfeiffer, K. West, Phys. Rev. Lett. 70 (1993) 3983.
[30] V.W. Scarola, K. Park, J.K. Jain, Phys. Rev. B 61 (2000) 13 064.
[31] K. Park, J.K. Jain, Phys. Rev. Lett. 84 (2000) 5576.
[32] L. Bonsall, A.A. Maradudin, Phys. Rev. B 15 (1977) 1959.
[33] P.K. Lam, S.M. Girvin, Phys. Rev. B 30 (1984) 473.
[34] Hangmo Yi, H.A. Fertig, Phys. Rev. B 58 (1998) 4019.
[35] V.J. Goldman, M. Shayegan, D.C. Tsui, Phys. Rev. Lett. 61 (1988) 881.
[36] R.L. Willett, et al., Phys. Rev. Lett. 59 (1987) 1779.
[37] W. Pan, et al., Phys. Rev. Lett. 83 (1999) 3530.
[38] W. Pan, et al., Physica E 6 (2000) 14.
[39] A.A. Koulakov, M.M. Fogler, B.I. Shklovskii, Phys. Rev. Lett. 76 (1996) 499.
[40] R. Moessner, J.T. Chalker, Phys. Rev. B. 54 (1996) 5006.
[41] M.M. Fogler, A.A. Koulakov, Phys. Rev. B 55 (1997) 9326.
[42] E. Fradkin, S.A. Kivelson, Phys. Rev. B 59 (1999) 8065.
[43] E.H. Rezayi, F.D.M. Haldane, K. Yang, Phys. Rev. Lett. 83 (1999) 1219.
[44] M.P. Lilly, et al., Phys. Rev. Lett. 82 (1999) 394.
[45] R.R. Du, et al., Solid State Commun. 169 (1999) 389.
[46] A.H. MacDonald, S.M. Girvin, Phys. Rev. B 33 (1986) 4009.
[47] M. Greiter, X.G. Wen, F. Wilczek, Nucl. Phys. B 374 (1992) 567.
[48] N.E. Bonesteel, Phys. Rev. Lett. 82 (1999) 984.
[49] G. Moore, N. Read, Nucl. Phys. B 360 (1991) 362.
[50] R. Morf, Phys. Rev. Lett. 80 (1998) 1505.
[51] E.H. Rezayi, F.D.M. Haldane, Phys. Rev. Lett. 84 (2000) 4685.
[52] K. Park, et al., Phys. Rev. B 58 (1998) R10167.
[53] F.Z. Bloch, Z. Phys. 57 (1929) 545.
[54] B. Tanatar, D.M. Ceperley, Phys. Rev. B 39 (1989) 5005.
[55] F. Rapisarda, G. Senatore, Aust. J. Phys. 49 (1996) 161.
[56] H.W. Jiang, et al., Phys. Rev. Lett. 65 (1990) 633.
[57] V.J. Goldman, et al., Phys. Rev. Lett. 65 (1990) 2189.
[58] K. Park, J.K. Jain, Phys. Rev. Lett. 80 (1998) 4237.
[59] R.R. Du, A.S. Yeh, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 75 (1995) 3926.
[60] I.V. Kukushkin, K.V. Klitzing, K. Eberl, Phys. Rev. Lett. 82 (1999) 3665.
[61] R. Shankar, Phys. Rev. Lett. 84 (2000) 3946.
[62] G. Murthy, Phys. Rev. Lett. 84 (2000) 350.
[63] S. Melinte, et al., Phys. Rev. Lett. 84 (2000) 354.
[64] A.E. Dementyev, et al., Phys. Rev. Lett. 83 (1999) 5074.
[65] D.R. Leadley, et al., Phys. Rev. Lett. 79 (1997) 4246.
[66] W. Kang, et al., Phys. Rev. B 56 (1997) R12776.
[67] H. Cho, et al., Phys. Rev. Lett. 81 (1998) 2522.