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Quick Estimation of Rectangular Patch Antenna Dimensions Based on Equivalent Design Concept

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***Abstract*—This paper presents a new method for quickly estimating the physical dimensions of a rectangular microstrip antenna (RMSA). This is based on the newly introduced concept of "Equivalence of Design" for RMSA. Two designs are said to be equivalent if they result in the same resonance frequency. This is an outcome of Bhatnagar's postulate. It relates the classical extension in length ΔL with the length and width of the patch apart from the substrate thickness. This paper emphasizes the importance of substrate thickness normalized with respect to guided wave length. This is termed as the 'H' parameter. For RMSA this is the key parameter rather than the individual parameters – Dielectric Constant (ε_r), Substrate Thickness (h) or Resonance Frequency (f_0). A new parameter – the scaling factor (Ψ) has been introduced and defined. Based on these, transformation laws have been put forward. These can be used for quickly estimating the RMSA design parameters from a "known good design". The laws have been verified by estimating physical parameters of RMSA and then calculating its resonance frequency. This has been repeated for several hundred designs. The matching has been excellent. Simulation and measurement results of a known good design (Design1) and one of the transformed designs (Design3) are also presented. The results of the transformed design are in good agreement with those of Design1 considering fabrication and measurement tolerances.**

I. Introduction

Several books are available on microstrip antenna (MSA) design [1], [2], [3]. Several formulae are in use for decades. Generally these formulae provide initial design values which may be required to vary for desired performance of the antenna. Enormous data is available on the design of microstrip antenna. Yet a person has to start *abinitio* each time a new design is to be made. To minimize such efforts the idea of transformation of design was recently mooted out [4]. This means that if a design is ready for one set of materials and it is required to fabricate antenna on another set of materials then the new design parameters may be obtained by suitable scaling of the proven design. For rectangular patch antennas, given substrate material means the dielectric constant (ε_r) and substrate thickness (h) are known. The problem then is to estimate the physical length (L_p) and physical width (W_p) of the patch for the desired resonant frequency (f_0). For the sake of completeness the classical formulae for these estimations are reproduced below:

Physical Width of the radiating patch W_p is given by

$$W_p = \frac{\lambda_0}{2\sqrt{\varepsilon_r}} = 0.5\lambda_g \tag{1}$$

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where $\lambda_0 = \frac{c}{f_0}$ is the free space wavelength, c being the velocity of light. $\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$ is the wave length in the dielectric medium. It is often called guided wave length. Effective dielectric constant ε_{re} is given by [?]

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-0.5} for \frac{w}{h} \geq 1 \tag{2}$$

Electrical length of the patch is given by [?]

$$L_e = \frac{\lambda_0}{2\sqrt{\varepsilon_{re}}} \tag{3}$$

Increase in patch length due to fringing fields is 2ΔL, where ΔL is expressed as [?]

$$\Delta L = 0.412h\left[\frac{\varepsilon_{re} + 0.3}{\varepsilon_{re} - 0.258}\right]\left[\frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.813}\right] \tag{4}$$

Physical length of the patch is $L_p = L_e - 2\Delta L$

For the same parameters other formulae are also in use. Some of these are [3]:

$$W_p = \frac{\lambda_0}{2\sqrt{\frac{\varepsilon_r + 1}{2}}} \tag{5}$$

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 10\frac{h}{W}\right]^{-0.5} \tag{6}$$

$$\Delta L = \frac{h}{\sqrt{\varepsilon_{re}}} \tag{7}$$

Above process is to be repeated each time a MSA is to be designed. Earlier designs, whether good or bad, have no bearing on the new design. The study presented in this paper is an effort to correlate the new values with earlier design and to estimate the new values quickly.

II. Bhatnagar's Postulate and "Equivalent Design" Concept

The idea of "Equivalent Designs" and "Transformation of Designs" was mooted out by one of the authors of this paper [4]. As this reference is perhaps not easily available, its salient points have been recast here. It has been postulated that "For a rectangular antenna, extension (d) in the physical length of the patch is directly proportional to the thickness (h) of the antenna substrate and the electrical length (L_e) of the patch and is inversely proportional to its width (W_p). The constant of proportionality (β) is independent of the resonant frequency (f_0), thickness (h) and dielectric constant (ε_r) of the substrate". This is Bhatnagar's Postulate.

$$d \propto \frac{h \times L_e}{W_p} \tag{8}$$

$$d = \beta \frac{h \times L_e}{W_p} \tag{9}$$

where β is the constant of proportionality and has been termed as Bhatnagar constant. For a rectangular patch its value is unity.

Classically, $d = 2\Delta L$ and $W_p = \left(\frac{\lambda_g}{2}\right)$. This gives

$$\Delta L = \beta \times H \times L_e \tag{10}$$

where, $H = \frac{h}{\lambda_g}$, and, for a rectangular patch,

$$\Delta L = H \times L_e \tag{11}$$

The central idea put forward is that the normalized thickness 'H' of the dielectric substrate is the key parameter that governs the microstrip antenna. The parameter H embodies the effect of h, ε_r and f_0. With the help of the new concept it is possible to transform one design into another without going through elaborate calculations. Any variation in h, ε_r or f_0 will change H and will thus be reflected in concerned formulae. The important thing that comes up is that any change in either h or ε_r or f_0 can be offset by suitably changing the other parameters so that H remains constant. All other quantities will then remain unchanged. This is the uniqueness of H. This makes H the basic parameter. We then have a new tool 'H' to manipulate MSA designs. All these observations are based on well proven classical formulae which are empirical and approximate. These are applicable for electrically thin substrates having semi-infinite ground plane. All the limitations that apply to these classical formulae are also applicable to the new ones.

A. Transformation of one design into another

Two designs of a MSA will be called as equivalent if they result in the same resonance frequency. Let the proven ready design be called Design1. Then the parameters f_1, ε_{r1}, h_1, L_{p1}, and W_{p1} refer to the corresponding parameters of Design1 and are known. This is to be transformed into an equivalent design (say Design2) on a material with dielectric constant ε_{r2}. Then one needs to estimate L_{p2}, and W_{p2} so that $f_2 = f_1$. Similar problem was faced by VLSI design experts. As the fabrication technology advanced, minimum feature size was reduced. To cope with the problem equivalent design concept was evolved, scaling factors and laws of transformation were developed. A similar exercise needs to be done for the case of MSA. Let H be constant between the two designs. Then, $H_2 = H_1 = H$, which gives,

$$\frac{1}{c} \times h_2 \times f_2 \times \sqrt{\varepsilon_{r2}} = \frac{1}{c} \times h_1 \times f_1 \times \sqrt{\varepsilon_{r1}} \tag{12}$$

$$h_2 = \Psi h_1 \tag{13}$$

where

$$\Psi = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \tag{14}$$

Ψ will be called as scaling factor. For quick estimation of design parameters the following equations can be easily derived from the basic definitions of these parameters.

$$W_{p2} = \Psi W_{p1} \tag{15}$$

$$L_{e2} = \Psi L_{e1} \tag{16}$$

$$\Delta L_2 = \Psi \Delta L_1 \tag{17}$$

$$L_{p2} = \Psi L_{p1} \tag{18}$$

Equations (14) to (18) form the laws of transformation with the condition that $h_2 = \Psi h_1$. For all these investigations the width of the rectangular patch (W_p) was taken to be $\lambda_g/2$ (as used in classical formulae). However, in practical cases W_p is much larger than this. The rule of thumb is $1 \leq \frac{W_p}{L_e} \leq 2$. Laws of transformation should be generalized to take this into account. Let $W_p = n(\lambda_g/2)$, where n is any real number. Bhatnagar's postulate then gives: $\triangle L = \beta H L_e/n$, this equation will replace equation (10). For a rectangular patch $\beta = 1$. Therefore, $\triangle L = H L_e/n$. This equation will replace equation (11).

B. Generalization of laws of transformation

Laws of transformation as proposed above are based on constant H values. By definition resonance frequency f_0 is constant. As the material changes, ε_{r1} is changed to ε_{r2}. This means that h_2 must have the restricted value Ψh_1 so that H remains constant. However, the dielectric sheets are available in discrete thickness values. Therefore, the parameter h can have only these discrete values unless the material is specially produced in desired h value. The laws of transformation should be generalized to meet this situation. Let the design with the given material be called Design3 and all the corresponding parameters are associated with the subscript 3. W_p is equal to $n0.5\lambda_g$ and is independent of h. As h changes, ε_{re} and ΔL also change. Therefore L_e and L_p also change. However if $\frac{h}{W_p}$ remains constant then ε_{re} and L_e will not change for a given resonance frequency. Therefore W_p should be changed in such a manner that $\frac{h_3}{W_{p3}} = \frac{h_2}{W_{p2}}$. This gives

$$W_{p3} = \frac{h_3 W_{p2}}{h_2} = \frac{W_{p1}}{\phi} \tag{19}$$

where $\phi = \frac{h_1}{h_3}$ is the second scaling factor. Then the problem that remains is to evolve a procedure to find L_p when the value of h is changed.

C. Investigation of Physical Length of the Patch as a function of Substrate Thickness

To solve the above mentioned problem, effect of h on L_p has been studied. Initially f_0 and ε_r have been kept constant. The study has been repeated for various combinations of these two parameters. Classical formulae have been used to estimate L_p for various values of h for pairs of f_0 and ε_r. The formulae are valid for thin substrates. Therefore the investigations have been restricted to the range $0.002 \leq H \leq 0.020$. A new parameter L_{pp} has been introduced. $L_{pp} = L_p + \frac{h}{\varepsilon_r}$. Values of this parameter have been computed for a range of values of h, H and ε_r for resonance frequencies from 1 GHz to 10 GHz.

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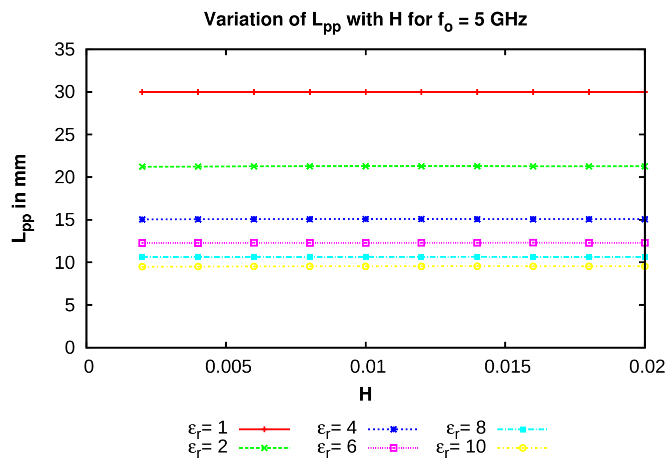

Figure 1. L_{pp} v/s H

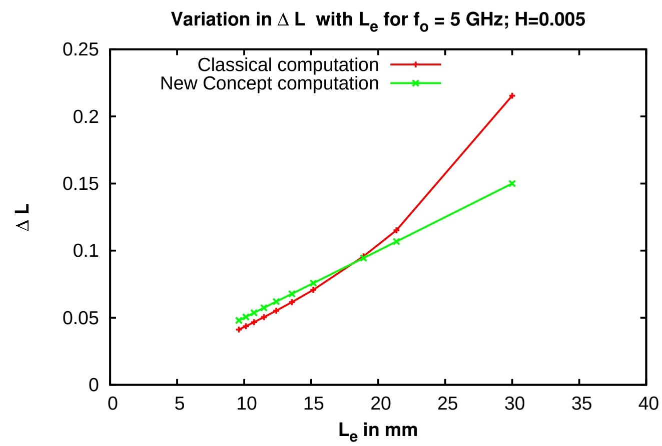

Figure 2. Variation of ΔL vs L_e for H=0.005

Fig. 1 depicts variation of L_{pp} with H for different values of ε_r. The results lead to the conclusion that L_{pp} is constant in the range of normal h values. This leads to the conclusion that

$$L_{p1} + \frac{h_1}{\varepsilon_{r1}} = L_{p2} + \frac{h_2}{\varepsilon_{r2}} \tag{20}$$

If the two materials are same $(\varepsilon_{r2} = \varepsilon_{r1})$, then

$$L_{p2} = L_{p1} + \frac{(h_1 - h_2)}{\varepsilon_{r1}} \tag{21}$$

If L_{p2} corresponds to h_2 and L_{p3} corresponds to h_3 for a set of f_0 and ε_r, then

$$L_{p3} = L_{p2} + \frac{h_2 - h_3}{\varepsilon_{r2}} \tag{22}$$

as $\varepsilon_{r3} = \varepsilon_{r2}$.

This then is a solution to the problem.

D. Transformation procedure

The entire procedure is recapitulated below. A known good design is available for resonance frequency f_0 in a material of thickness h_1 and dielectric constant ε_{r1}. This known Design1 is to be transformed into a new design (Design3) in a material with dielectric constant ε_{r3} and substrate thickness h_3 so as to achieve the same resonance frequency f_0. Proceed as follows:

- Calculate the scaling factor $\Psi = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r3}}}$ and the second scaling factor $\phi = \frac{h_1}{h_3}$
- Then, the desired new design parameters are

$$H_3 = \frac{H_1}{\Psi\phi} \tag{23}$$

$$W_{p3} = \frac{W_{p1}}{\phi} \tag{24}$$

$$L_{e3} = \Psi L_{e1} \tag{25}$$

$$\Delta L_3 = \frac{\Delta L_1}{\phi} \tag{26}$$

$$L_{p3} = \Psi L_{p1} + \frac{\Psi h_1 - h_3}{\varepsilon_{r3}} \tag{27}$$

III. Validation of the New Approach

A. Variation in ΔL with Electrical Length L_e

Variation of ΔL with L_e has been studied over a very wide range of resonance frequency from 1 GHz to 10 GHz, ε_r = 1 to 10 and H = 0.002 to 0.020. ΔL has been calculated from the new postulate as well as from classical formulae. The new concept relates ΔL directly with L_e. According to the new postulate ΔL varies linearly with L_e and predicts that ΔL = 0 for L_e = 0. Slope of the line is βH. This is logical and straight forward. This is not the case with classical thinking. The classical formula is empirical and does not give ΔL = 0 for L_e = 0. However the match between classical formula and the new postulate is quite good. Fig. 2 depicts the variation of ΔL with L_e over a very wide range. Only one of the graphs is presented here. One surprising result has been obtained. In all these graphs the curve drawn with classical calculations and the straight line drawn with estimations based on the new concept cross at ε_r = 2.56. Below this value the classical prediction is higher than the new one. For dielectric constant greater than this value both the estimations are nearly equal, the classical one being always slightly lower than the new concept. ε_r = 1 is the dielectric constant of air. This material is above the patch in all cases. This may have some bearing on the results. This needs further investigations. It may however be noted that except for foam material, all other commercially available material have $\varepsilon_r > 2$.

B. Verification of Transformation Laws

To Verify the laws of transformation, a rectangular patch antenna has been designed for a material with dielectric constant ε_r = 3.0 and substrate thickness h = 0.25 mm to resonate at 5 GHz. Classical formulae gave L_{e1} = 17.48 mm, L_{p1} = 17.23 mm and $W_{p1} = n(\lambda_g/2)$ = 25.98 mm for n = 1.5. This has been taken as known good design and termed as Design1. Using the newly developed transformation laws, this design has been transformed in equivalent designs for materials with dielectric constant ε_{r3} varying from 1 to 10,

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Table I
ESTIMATIONS BASED ON TRANSFORMATION LAWS, f_0 = 5 GHz

ε_r	h	n	Ψ	Φ	W_{p3}	L_{p3}	L_{e3}	Nf_0
1	2	1	1.73	0.13	204.56	28.28	30.27	4.95
2.94	0.4	1	1.01	0.64	40.91	17.35	17.65	5
3	0.25	1.5	1	1	25.98	17.23	17.48	5
4	0.8	1	0.87	0.32	81.82	14.77	15.13	5.01
5	1.2	1	0.77	0.21	122.72	13.14	13.54	5.01
6	1	1.5	0.71	0.25	102.28	11.16	11.44	5.01
8	0.9	1	0.61	0.28	92.06	10.45	10.70	5.01
9	0.5	2	0.58	.51	51.14	9.9	10.09	5.01
10	0.2	1	0.55	1.27	20.45	9.43	9.57	5.02

Table II
SIMULATION AND MEASURED RESULTS

ε_r	h	n	W_p	L_p	f_0_sim	f_0_meas
3	0.25	1.5	25.98	17.23	4.96	4.97
2.94	0.4	1	40.9	17.35	4.91	4.88

thickness h_3 varying from 0.2 mm to 2.0 mm and n varying from 0.8 to 2. Values of W_{p3} and L_{p3} have been estimated using the transformation laws.

In Table 1 each row shows a transformed design. 1^{st} and 2^{nd} columns give the dielectric constant and substrate thickness of the material for which MSA design has been made. W_{p3} and L_{p3} obtained by transformation are given in columns 6^{th} and 7^{th}. For the estimated values of patch width and length, the resonance frequency has been found and is given in the last column of Table-1 under the heading Nf_o . It can be seen that in all the cases the target value (5 GHz) has been achieved. More than 100 estimations have been made and the laws have been verified. Table-2 presents verification of Design1 and one of the Design3 by simulation and measurement. Fig. 3 shows simulated and measured return loss plots for Design1 and Design3 and Fig. 4 shows their comparison. It is observed that for Design1 measured value of resonance frequency achieves target value with -0.6% error and for the transformed design (Design3) this error is -1.8% compared to the measured value of Design1. Measurements have been performed on Agilent's PNA N5224A.

IV. DISCUSSIONS AND CONCLUSION

For the fundamental TM_{10} mode, the length L_e should be slightly less than that given by equation (3). Some authors [3] have taken $\varepsilon_{re} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+10\frac{h}{W}\right]^{-0.5}$ while others [4] have taken $W_p = \frac{\lambda_0}{2\sqrt{\frac{\varepsilon_r+1}{2}}}$. Thus there are many formulae in use to get values quickly. These are approximate formulae only. Compared to all these the proposed laws are easy to use. Scaling factor Ψ is a crucial parameter. It provides the crucial link between any two designs. It leads to a new way of looking at MSA world. Individual values of ε_{r1} and ε_{r2} do not matter. Important thing is the square root of their ratio. Thus increase or decrease in ε_{r1} can be offset by increasing or decreasing ε_{r2} such that Ψ remains constant. This provides another degree of freedom in MSA design. The new simplified

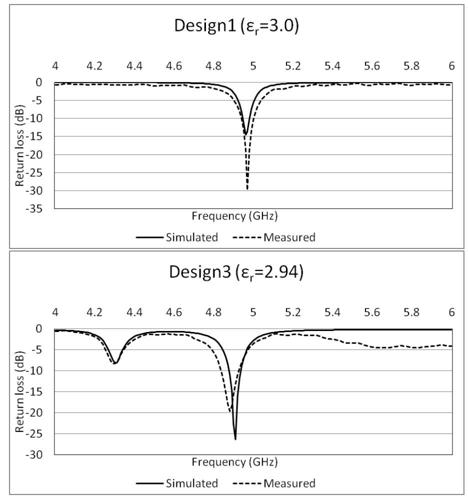

Figure 3. Simulation and Measured Return Loss Plots

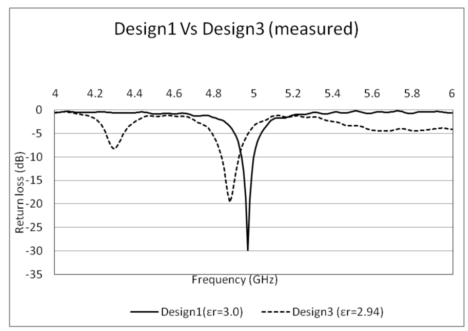

Figure 4. Comparison of Design1 with Transformed Design3

approach of designing rectangular microstrip antennas shall help the designers with a new tool that should be easy to use and provide results with better accuracy. As per the new concept H is the basic parameter of MSA. It may be noted that (i) the new approach gives the same results as the classical one and (ii) for given f_o and ε_r, $L_{pp}(= L_p + \frac{h}{\varepsilon_r})$ is constant over normal range of h. This can be used to estimate any change in L_p due to change in h if f_o and ε_r are constant. It may also be noted that microwave laminates are supplied in ε_r values of 2 or greater and these are in small thicknesses. For this thickness range L_{pp} is constant to a first approximation as shown in Fig. 1.

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