A NOVEL DESIGN AND MATHEMATICAL MODEL FOR SENSITIVITY OF A MEMS BASED PIEZOELECTRIC ACOUSTIC SENSOR

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Abstract- **Piezoelectric microelectromechanical systems(MEMS) has been a growing area of research and interest in past decades, in which ferroelectric films are combined with silicon technology for a variety of applications such as accelerometers, pressure sensors etc. and they are used in many fields such as defense ,medical sciences. Optimum thickness of piezoelectric layer and silicon diaphragm are necessary for desired bandwidth and applications. Residual stress plays a key role in design and sensitivity of these sensors. Thickness of nearly 2 µm for ZnO was found optimum for a Si diaphragm of 25 µm. This paper reports the design and mathematical model of MEMS based acoustic sensor with a 3µm thick piezoelectric layer sandwiched between two pairs of aluminum. electrodes, one at the centre and one along the periphery on a 25 µm thick Si diaphragm 3x3 mm in dimension. The sensor have a resonance frequency of 41.8 KHz. Sensitivity was found to be 334.7µv/pa without the effect of residual stress and 221.6µv/pa with residual stress.**

I. Introduction

Due to high coupling coefficient and low dielectric coefficient , ZnO has been extensively utilized in Microelectromechanical systems (MEMS), microactuato-rs, micro-sensors[1], as an excellent material to be used as a transducer. In this paper, ZnO is used in d_{31} mode to convert mechanical loads to electrical signals. However, the voltage sensitivity of the sensor is limited due to the low piezoelectric coefficient of ZnO [2]. Fortunately, this low dielectric coefficient of ZnO solves the problem. Also, complementary potentials produced across the two top electrodes can improve the sensitivity[3] and decrease in capacitance by variations in dimensions of the electrodes can further enhance sensitivity.

An acoustic sensor operating in d_{31} was designed and mathematical model of the same was discussed. Stress produced in the diaphragm results in generation of potential across the electrodes. By estimating the stress/strain generated across the ZnO layer, sensitivity can be found. Many different shapes for diaphragms are possible but square diaphragms are mostly preferred as they are easy to fabricate and result in maximum stress generation compared to circular or rectangular diaphragm [4], [5]. Schematic diagrams of the side of the designed piezoelectric acoustic sensor are shown in Fig.1. Pressure applied on the diaphragm produces stress gradient across the length of the ZnO layer. The maximum stress produced is at the centre of each side of the diaphragm and is given as[6],[11]:-

$$\sigma_x = \frac{6}{h^2}M_x \text{ where } M_x = -D\left(\frac{d^2w}{dx^2} + v\,\frac{d^2w}{dy^2}\right)$$

And $D=\frac{Eh^3}{12(1-v^2)}$

where, M_x, D, w, *E*, *h*, *v* and w are x component of the bending moment produced due to the applied pressure, flexural rigidity, out of plane (z-component) deflection of diaphragm, young's modulus of Si, thickness and Poisson's ratio of the Si-diaphragm.

For zinc oxide, with the c-axis oriented in the +*z* direction, the induced polarization [7] in the *z* direction can be expressed as:

$$P_z = d_{31}(\sigma_x + \sigma_y) = \frac{-6d_{31}}{h^2}(Mx + My) \qquad (1)$$

Maximum deflection is at the centre of the diaphragm and is given by [6]:-

$$P=\frac{3.41h^2\sigma w_o}{a^2h} + \frac{4.13Eh^4w_o}{a^4(1-v^2)h} \qquad (2)$$

where σ, w_o and *a* are residual stress, maximum deflection and half length of diaphragm. Maximum stress intensity is produced at the centre of each side of the diaphragm and is given as [6]:-

$$\sigma_{max} = \frac{1.76p}{\frac{3{,}41\sigma}{p} + \frac{4.54h^2}{a^2}} \qquad (3)$$

Dimensions of inner Al electrode are 1.5mm*1.5mm. Outer electrodes are along the periphery, their dimensions are 0.45mm*3mm.

II. Design

According to the layout shown in Fig.1, the dimensions of the various layers are as given in table I. A simplified model, in which very thin insulating oxide layers were neglected, was used to carry out simulations. Simulations of the structure were carried in ANSYS tool using finite element method (FEM) to calculate stress, maximum deflection and potential generated across the electrodes by applying a pressure of 400 Pa. Despite of the imperfections of the simplification, the resulting simplified model provides quite satisfactory results which agree well with results obtained through mathematical model.

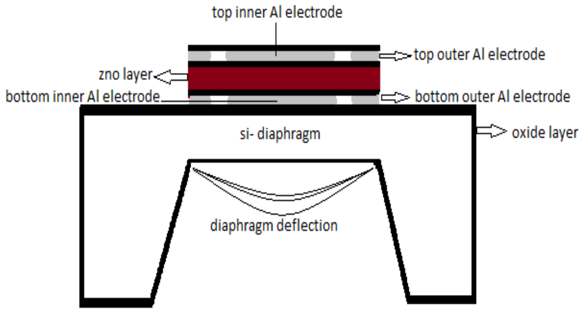

Fig.1

Table I

S. NO.	Name	Thickness
1	Silicon diaphragm	25 micron
2	Insulating oxide layer	0.5 micron
3	Bottom aluminum electrodes	1 micron
4	PECVD oxide layer	0.1 micron
5	Piezoelectric ZNO	3 micron
6	PECVD oxide layer	0.1 micron
7	Top aluminum electrodes	1 micron
8	Insulating oxide layer	0.2 micron

TABLE II

Parameters	Si	Sio_2	Al
E (Gpa)	135	65	70
$\rho(Kg/m^3)$	2330	2648	2700
v	0.42	0.17	0.35

Since Si and SiO_2 are almost isotropic, a 3-D element with isotropic properties, SOLID45, was used for them. SOLID92 was used for the Al electrodes and 3-D coupled field solid element, SOLID98, was used to model piezoelectricity of ZnO layer. Piezoelectric coefficient and compliances of ZnO are listed in Table III [2], and the other material properties are in Table II [15].

III. Simulations and FEM results

To solve the stress distribution in the ZnO film and find the diaphragm deflection, the static structural analysis was carried out in ANSYS. Fig.2 shows the vertical displacement of the diaphragm. Although the diaphragm displaces downwards, it experiences different stresses in different areas. A 3-D plot of deformation by applying pressure on the square diaphragm was also drawn and shown in Fig. 3 [10].

TABLE III

Parameters	Values	Units
ρ	5.675	10^3 Kg/m^3
$\varepsilon_{11}/\varepsilon_0$	8.5	-
$\varepsilon_{33}/\varepsilon_0$	10.2	-
d_{33}	12.4	10^{-12}C/N
d_{31}	-5.0	
d_{15}	-8.3	
e_{33}	1.57	C/m²
e_{31}	-0.36	
e_{15}	-0.36	
c_{11}	210	10^9 N/m^2
c_{12}	121	
c_{13}	105	
c_{33}	211	
c_{44}	43	

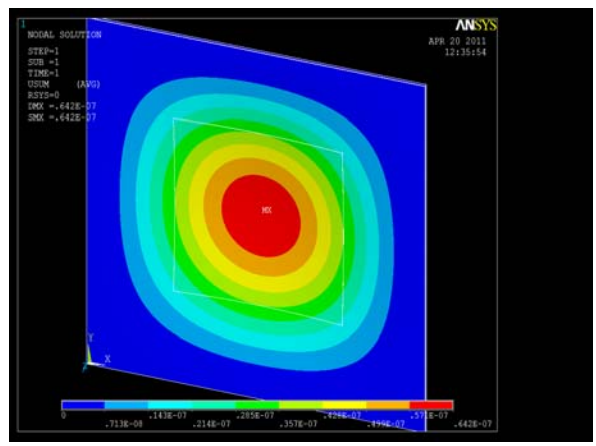

Fig.2

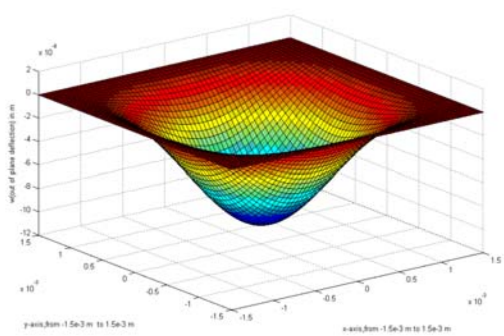

Fig.3

Fig. 4 shows the in-plane components of stress. Stress produced is tensile at the boundaries and compress- ive at the centre of the diaphragm; hence potential develo- ped at the boundaries is complementary to the potential developed at the centre of the ZnO layer. For maximum sensitivity, thickness of the Si diaphragm and the ZnO layer should be so adjusted that the net residual stress is nearly zero.Graphs were plotted for variations of maximum stress and maximum deflection v/s length of diaphragm at various values of residual stress in Si diaphragm. Fig.5 and Fig.6 show the plots obtained in MATLAB. At a half length of 1.5 mm, values of maximum deflection and maximum stress are almost similar to the results of ANSYS. Besides voltage sensitivity, resonance frequency is another impor-tant consideration of acoustic sensors. The resonance frequency, calculated using ANSYS modal analysis and harmonic analysis depends on the length of the diaphragm and thickness of various layers. Fig.7 shows the result of harmonic analysis obtained at a pressure of 400 Pascal.

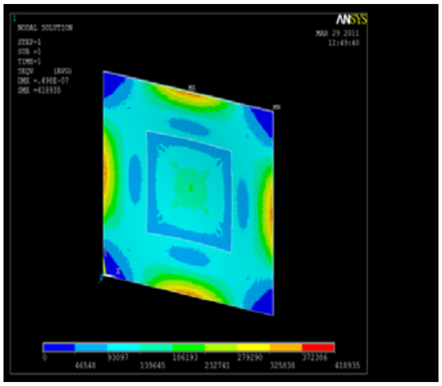

Fig.4

The sensor shows a first mode of resonance frequency of 41.8 KHz. Fig.8 shows the variations of resonance frequency versus the width of the diaphragm. Mathematical equation for resonance frequency of thin square diaphragms is given by [8]:

$$f_r = \frac{1.654 c_p h}{a^2} \qquad (4)$$

where, $c_p = \sqrt{E/\rho_0(1-v^2)}$
and ρ_0 is the density of the substrate.

As expected by equation (4), large diaphragm size and small thickness lead to low resonance frequency. Thickness of supporting layers such as SiO_2, changes the resonance frequency in the same manner as ZnO. Sensitivity of square shaped sensors, as discussed later, increases by increasing length of the diaphragm. Hence, resonance frequency can be increased at the cost of low sensitivity by increasing layer thickness or decreasing layer dimensions.

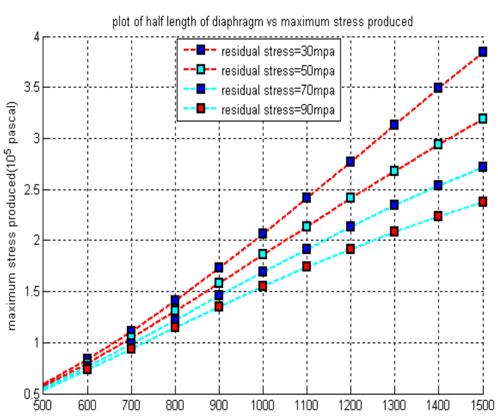

Fig.5

However these sensors are best suited for low frequency operations since for low frequencies, large dimensions are required which result in higher sensitivity. Values of resosnance and results of harmonic analysis help in finding the cut- off frequency and bandwidth. Bandwidth and cut-off frequency depends on the dimensions of the sensor and are controllable to some extent. Larger dimensions increase the maximum stress developed but they decrease the resonance frequency.

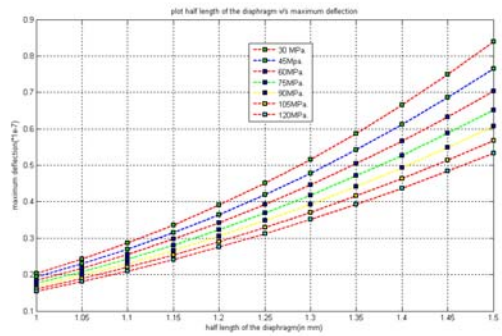

Fig.6

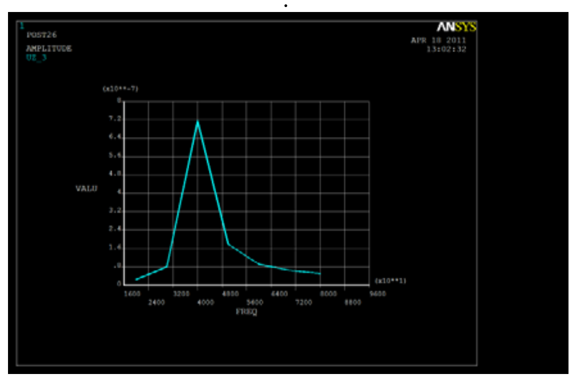

Fig.7

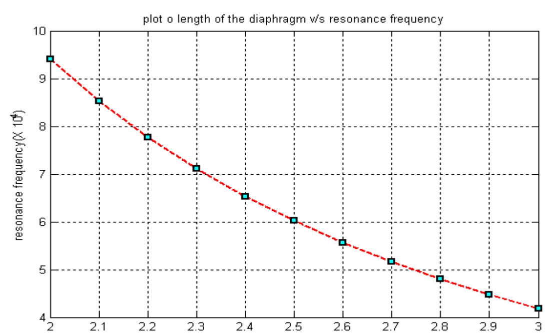

Fig.8

IV. Mathematical Model for Sensitivity

The acoustically induced deflections in a clamped square diaphragm are described by the solution to the classical plate equations [11]. For a square-shaped diaphragm piezoelectric acoustic sensor, the expression for generated polarization across the piezoelectric layer can be calculated by combining the solution of the plate equation with the material constitutive equations [13]. Fig. 9 shows the diagrammatic representation for clamped square diaphragm with central electrode extending from -L/4 to L/4 and outer electrode extending from 7L/10 to L along the periphery where L is the full length of the diaphragm. For zinc oxide with the c-axis oriented in the $+z$ direction, the induced polarization in the z direction is given by (5), where σ_x and σ_y are the x and y component of stress, $s^{E11} and\ s^{E12}$ are elastic compliance coefficients with constant electric field, and w is out of plane deflection.

$$P_z = d_{31}(\sigma_x + \sigma_y) = \frac{-d31\, z}{s^{E11}+s^{E12}}\left(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}\right) \qquad (5)$$

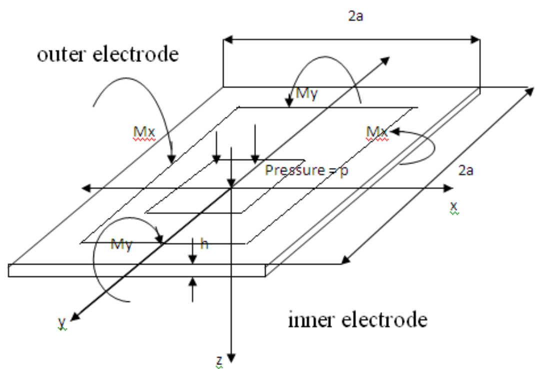

Fig.9

The acoustically induced voltage between the pair of electrodes can be calculated by integrating the average induced polarization across the electrode area and then dividing it by the capacitance of the dielectric ZnO layer. By making a simplifying assumption that the Poisson's ratio is constant along the x-y plane for different materials, the average induced voltage can be expressed as [7]:-

$$V_o = \frac{z_o - z_i}{A_o \varepsilon_{33}} \int_{A_o} \int_{z_o}^{z_i} \frac{P_z(x,y,z)}{z_o - z_i}\, dz\, dA_o \qquad (6)$$

$$= \frac{z_o - z_i}{A_o \varepsilon_{33}} \int_{A_o} \frac{-d31}{s^{E11}+s^{E12}} \left(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}\right) dA_0$$

$$\cdot \int_{z_o}^{z_i} \frac{z}{z_o - z_i}\, dz$$

$$= \frac{z_o - z_i}{A_o \varepsilon_{33}} \left(\frac{-d31}{s^{E11}+s^{E12}}\right) S_o \left(\frac{z_o + z_i}{2}\right)$$

Where z_o and z_i are the distance from the neutral plane to the outer and inner zinc oxide surfaces respectively, A_o is the electrode area, ε_{33} is the z-direction permittivity component of the zinc oxide, φ is the applied pressure, S_o is the dimension integration constant for an electrode pattern on a square plate given by:

$$S_o = \int_{Ao} \left(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}\right) dA_o$$

For the value of partial differentials, first w has to be computed using the equations for clamped square diaphragms. The solution to w is given by[9]:

$$w = \frac{w_o}{4}\left(1 + \cos\frac{2\Pi x}{L}\right)\left(1 + \cos\frac{2\Pi y}{L}\right) \quad \text{Where } w_0 = \frac{p\left(\frac{L}{2}\right)^4}{c_b 12D};$$

c_b=4.06 and L/2 is the half length of diaphragm and can also be denoted by a. On solving for S_0 and substituting its value in (6), a sensitivity of 334.7µv/pa is obtained.

V. Residual Stress Effects

Residual stress although undesired, is inevitably present in all structures due to flaws in fabrication steps or due to internal property of materials. Residual stress plays a very important role in optimizing the thickness of ZnO layer such that the net residual stress is zero in the resultant diaphragm which is made up of many layers of materials. The effect of residual stress on resonance frequency is given as follows [7]:

$$\text{fs} = f_0\sqrt{\left(1 + \frac{N(2a)^2}{D*5.37*3.14^2}\right)} \qquad (7)$$

where f_0 is the resonance frequency without residual stress and N is the force per unit length along the edge of the plate. N can be expressed as:

$$N = \sigma_a(h_1 + h_2 + \ldots) = \int_{h_1} \sigma_1(z)dz + \int_{h_2} \sigma_2(z)dz + \ldots$$

Where σ_a is the average residual stress, $\sigma_1(z)$ and $\sigma_2(z)$ are the in plane stresses in individual layers[12] of thickness- h_1, h_2 respectively. Approximate value of residual stress in ZnO is 1GPa (compressive) [7] and in Si it is 65MPa (tensile) [6]. Residual stress shifts the distribution of acoustically induced stress. If tensile stress becomes excessively large, the behavior of the diaphragm becomes like that of a membrane. Acoustic sensors can be treated as one degree of freedom oscillator [14], with spring constant as a function of residual stress. Since the sensors output voltage is proportional to deflection, we can manipulate the simple oscillator equation to relate sensors sensitivity, s to sensitivity of the sensor without residual stress, s_r.

$$\frac{s_r}{s} = \left(\frac{f_0}{f_s}\right)^2 = F$$

Value of F obtained by substituting values is 1/1.51. So, sensitivity with residual stress s_r is:

S_r = 334.7/1.51 µv/Pa =221.6 µv/Pa.

VI. Thickness of Piezoelectric layer for Sensitivity Saturation.

Simulations with different values of thickness of ZnO layer were done keeping thickness of Si diaphragm constant. The results obtained are shown in Fig.10. It is seen that sensitivity gets saturated for a thickness of nearly 12 µm. Residual stress also plays a key role in thickness of ZnO layer for sensitivity saturation. Fig.11 shows the results obtained through mathematical model for thickness of ZnO layer for sensitivity saturation with and without residual stress. Variations of residual stress with thickness of ZnO layer helps in finding the optimum thickness of ZnO layer. For maximum sensitivity, thickness of piezoelectric layer should be 2µm for a Si diaphragm of 25µm. Furthermore, it seen that sensitivity is maximum when residual stress is zero. The plot for variations of sensitivity and residual stress with thickness of ZnO layer is shown in Fig.12.

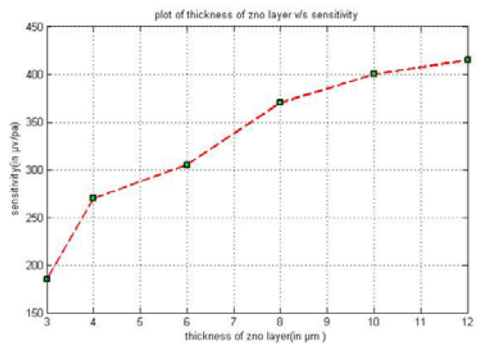

Fig.10

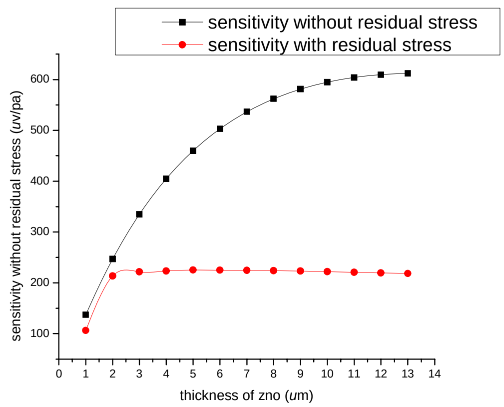

Fig.11

Sensitivity and residual stress change with changes in the thickness of Si diaphragm. Fig.13 depicts the variations in the sensitivity with thickness of Si diaphragm and Fig.14 exhibits the variations of residual stress with thickness of Si diaphragm.

VII. Conclusion

In this paper a piezoelectric acoustic sensor was designed and mathematically modeled. Static analysis was done in ANSYS design tool in order to find stress-strain distribution and maximum deflection. Harmonic analysis was done to find bandwidth and resonance frequency. Dimensions of the Si diaphragm and other layers are important for controlling stress, strain, sensitivity etc.

Resonance frequency also depends on dimensions and residual stress and has a value of 41.8 KHz. Bandwidth of the sensor lies in the range where we get a linear response for membrane deflection versus frequency. Residual stress affects stress, maximum deflection and sensitivity of the acoustic sensors. Sensitivity with residual stress was found to be 221.6 µv/Pa. Optimum thickness of the piezoelectric ZnO layer, for which residual stress was zero and thus sensitivity was maximum, was 2 µm for a 25 µm thick Si diaphragm. Thickness of ZnO layer for sensitivity saturation was 12µv/Pa if we ignore the effect of residual stress. Variations of sensitivity with thickness of Si diaphragm were also exhibited.

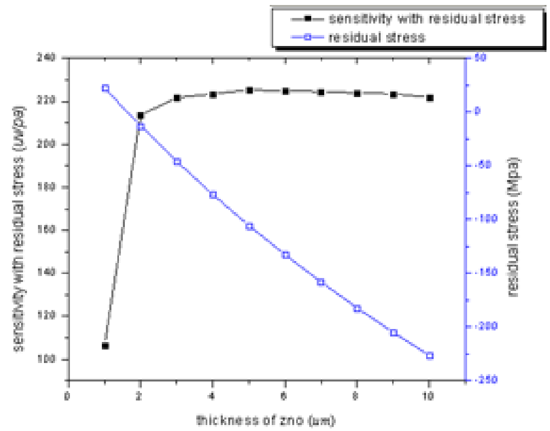

Fig.12

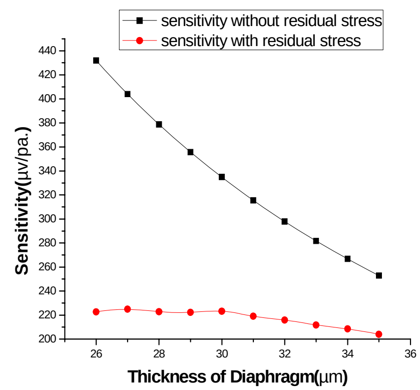

Fig.13

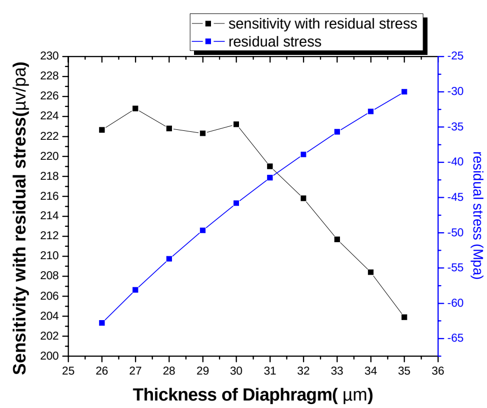

Fig. 14

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