

Selecting the Probability Distribution of Monthly Maximum Temperature of Dhaka (Capital City) in Bangladesh

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Abstract

Rising temperature in the atmosphere causes sea level rise and affects low lying coastal areas and deltas of the world. The last decade of the twentieth century was globally the hottest since the beginning of worldwide temperature measurement during the nineteenth century. Many PDFs have been proposed in recent past, but in present study Weibull, Lognormal, Gamma, GEV, etc are used to describe the characteristics of maximum temperature. This paper attempts to determine the best fitted probability distribution of monthly maximum temperature. To identify the appropriate probability distribution of the observed data, this paper considers a data set on the monthly maximum temperature of Dhaka over the period January, 1972 to September, 2015.

Goodness-of-fit criteria like the Kolmogorov-Smirnov (KS), the R^2 , the χ^2 and the Root Mean Squared Error (RMSE) were used in this paper to check the accuracy of the predicted data. According to the goodness-of-fit criteria and from the graphical comparisons it can be said that Generalized Skew Logistic distribution (GSL) provided the best fit for the observed monthly maximum temperature data of Dhaka station among the probability distribution considered in this paper.

Keywords: Temperature, Probability Distribution, Model Selection, Bangladesh

1. Introduction

Global warming is an important issue, with a variety of influences on agriculture, water, health and economy. It is now recognized that climate variability and extreme events affect society more than changes in the mean climate (Intergovernmental Panel on Climate Change (IPCC), 2001). It is broadly recognized that Bangladesh is more vulnerable to these changes. Bangladesh is highly vulnerable because it is a low-lying country located in the deltaic plain of the Ganges, the Brahmaputra and the Meghna and densely populated. Its national economy strongly depends on agriculture and natural resources that are sensitive to climate change and sea-level rise. The impact of higher temperature and more extreme weather events such as floods, cyclone, severe drought and sea-level rise are already being felt in South Asia and will continue to intensify (Huq et al., 1999; Ali, 1999). In a study at the International Rice Research Institute, Peng *et al.* (2004) found a 10% decrease in rice yield per 1°C increase in

growing season night temperature. Donat and Alexander (2012) shown that the distribution of global daily temperatures has indeed become “more extreme” since the middle of the 20th century.

In light of the recent climate trends and current predictions for the twenty-first century, climatic change is becoming a major concern for scientists and society in general. There is an increasing interest in different parts of the world in research on extreme temperatures and their variation. Temperature extremes are an important aspect of any climate change because ecosystems and societal responses are most sensitive to them. Mearns *et al.*, (1984) and Hansen *et al.*, (1988) concluded that relatively small changes in the mean temperature could produce substantial changes in the frequency of temperature extremes. The last decade of the twentieth century was globally the hottest since the beginning of worldwide temperature measurement during the nineteenth century. Alexander *et al.*, (2006) showed that annual trends in the lowest and highest daily minimum and maximum temperatures in the latter half of the twentieth century increased at many locations throughout the world.

In the past, a number of studies have been carried out on trend of climate change in climatic parameters over Bangladesh. Chowdhury and Debsharma (1992) and Mia (2003) pointed out that temperature has been changed by using historical data of some selected meteorological station. Parathasarathy *et al.*, (1987) and Divya and Mehritra (1995) reported mean annual temperature of Bangladesh has increased during the period of 1895-1980 at 0.31°C over the past two decades. Karmakar and Shrestha (2000) using the 1961-1990 data for Bangladesh projected that annual mean maximum temperature will increase to 0.4°C and 0.73°C by the year of 2050 and 2100 respectively. Rahman and Alam (2003) found that the temperature is generally increasing in the June-August period. Average maximum and minimum temperatures show an increasing trend of 5°C and 3°C per century, respectively. On the other hand, average maximum and minimum temperatures of December-February period show, respectively, a decreasing and an increasing trend of 0.1°C and 1.6°C per century. Regional variations have also been observed around the average trend (SMRC, 2003). In a recent study, Climate Change Cell (2009a) has analyzed the temperature and sunshine duration at all BMD stations of Bangladesh. Islam and Neelim (2010)

analyzed the maximum and minimum temperatures of four months (January, April, May and December) and two seasons only. The two months of April-May were considered as the summer season and the two months of December-January as the winter season in the study. The study found in general an increasing trend in both summer and winter temperatures. There were some research works about the temperature of Dhaka city. Dhaka is the capital of Bangladesh and one of the largest and densely populated mega cities of the world. Hossain *et al.*, (2014) analyses the pattern of change of temperature of Dhaka city using the data of maximum and minimum monthly temperature over the period 1995 -2010. They have shown that minimum average monthly temperature is showing significantly increasing pattern. Impacts of climate change would result from changes in variability and extreme event occurrence rather than from an increase in mean temperature (Houghton *et al.*, 1996; Watson *et al.*, 1996; Parmesan *et al.*, 2000). Temperature is the important variable. Impacts of extreme events are more serious when extreme weather conditions prevail over extended periods. That is why this study tries to fit the appropriate probability distribution of monthly maximum temperature. The Extreme-value data can be characterized by theoretical probability distributions. For the probability estimates of annual temperature extremes, the two-parameter Gumbel distribution and the three-parameter generalized extreme value (GEV) distribution are commonly applied (Fragó and Katz, 1990; Brown and Katz, 1995; Zwiers and Kharin, 1998; Kharin and Zwiers, 2000). This paper attempts to fit and select the best fitted probability distribution of monthly maximum temperature of Dhaka Station and also check the accuracy of the fitted probability distributions using the goodness-of-fit criteria.

2. Materials and Methods

Data

This paper considers a secondary data set on the monthly maximum temperature of Dhaka stations over the period January, 1972 to September, 2015. The required data was collected from the website of Bangladesh Agricultural Research Council (BARC) and Tutiempo Network.

Probability Distributions

Many PDFs have been proposed in recent past, but in present study Weibull, Lognormal, Gamma, and GEV are used to describe the characteristics of maximum

temperature. Parameters defining each distribution function are calculated using maximum likelihood method.

Weibull (W) Distribution

The probability density function (PDF) of the Weibull distribution with two parameters is given by (Weibull, W., 1951):

$$f(v; k, c) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right]; \quad 0 < v < \infty, k > 0, c > 0$$

where, ' k ' and ' c ' are the shape and scale parameters respectively.

Lognormal (LN) Distribution

Lognormal distribution is a probability distribution of a random variable whose logarithm is normally distributed. The probability density function (PDF) of the Lognormal distribution is given by (Johnson, Norman L., *et al.*, 1994):

$$f(v; \mu, \delta) = \frac{1}{v\sqrt{2\pi\delta^2}} \exp\left[-\frac{(\ln(v) - \mu)^2}{2\delta^2}\right]; \quad 0 < v < \infty, \mu \in R, \delta > 0$$

where, ' μ ' and ' δ ' are the mean and standard deviation of the normal random variable $\ln(v)$ respectively and ' Φ ' is the standard cumulative distribution function.

Gamma (G) Distribution

Lancaster (1966) quotes from Laplace (1836) in which the latter obtains a Gamma distribution. The probability density function (PDF) of the gamma distribution is given by:

$$f(v; a, b) = \frac{v^{a-1}}{b^a \Gamma(a)} \exp\left(-\frac{v}{b}\right); \quad v > 0, a > 0, b > 0$$

The parameters ' a ' and ' b ' are the shape and scale parameters respectively.

Generalized Extreme Value (GEV) Distribution

GEV distribution is a flexible model that combines the Gumbel, Frechet and Weibull maximum extreme value distributions (Ying, A. and Pandey, M. D., 2007)]. For, GEV the Probability density function is given by:

$$f(v; \mu, \sigma, \xi) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{v - \mu}{\sigma}\right)\right]^{(-1/\xi)-1} \exp\left\{-\left[1 + \xi \left(\frac{v - \mu}{\sigma}\right)\right]^{-1/\xi}\right\}; \quad \xi \neq 0$$

again, for $v > \left(\mu - \frac{\sigma}{\xi}\right)$ in the case $\xi > 0$, and for $v < \left(\mu - \frac{\sigma}{\xi}\right)$ in the case $\xi < 0$

where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ is the shape parameter.

Three Parameters Lognormal (3-P LN) Distribution

The lognormal distribution derives its name from the relationship that exists between random variables V and $Y = \ln(V - a)$. If Y is distributed normally (b, c) , then V is lognormal (a, b, c) . Accordingly, the probability density function of V may be written as (A. Clifford Cohen and Betty Jones Whitten, 1980):

$$f(v; a, b, c) = \frac{1}{(v-a)c\sqrt{2\pi}} \exp\left[\frac{-[\ln(v-a)-b]^2}{2c^2}\right]; c^2 > 0, a < v < \infty.$$

Skewed t (ST) Distribution

There are different parameterizations for the skewed generalized t distribution and the skewed generalized t distribution was first introduced by Panayiotis Theodossiou in 1998 (Theodossiou, P., 1998). For details about Skewed t distribution please visit: https://en.wikipedia.org/wiki/Skewed_generalized_t_distribution#skewed_t_distribution. Also, Indhumathy, *et al.*, (2014) and Davis (2015) used the following probability density function (pdf) for Skewed t distribution:

$$f_{STD}(v) = \frac{2}{2\theta_1\theta_2^{1/2}\beta\left(\frac{1}{2}, \theta_2\right) \left[1 + \frac{|v-\theta_0|^2}{\theta_2\{1+\text{sgn}(v-\theta_0)\theta_3\}^2\theta_1^2}\right]^{1/2+\theta_2}}; -\infty < v < \infty$$

where, θ_0 and θ_1 are the location and scale parameters respectively θ_2 and θ_3 are the degrees of freedom and shape parameters respectively.

The parameters of the skewed t distribution are obtained using the maximum likelihood method which maximizes the logarithm of likelihood function is given by

$$LL = \ln \prod_{i=1}^n \{f_{STD}(v_i)\} = \sum_{i=1}^n \ln \{f_{STD}(v_i)\}$$

where, v_i is the maximum temperature in time step i and n is the number of data points.

Generalized Skewed Logistic (GSL) Distribution

Perks (1932) first introduced the generalized logistic distribution. Achim Zeileis and Thomas Windberger (2014) wrote a R Package ‘glogis’ for Fitting and Testing Generalized Logistic Distributions and they used the following probability density function (pdf) of the generalized skew logistic distribution:

$$f_{GSLD}(v) = \frac{\gamma}{\sigma} \frac{e^{-\left(\frac{v-\mu}{\sigma}\right)}}{\left(1 + e^{-\left(\frac{v-\mu}{\sigma}\right)}\right)^{\gamma+1}} ; \quad -\infty < v < \infty$$

where, μ , σ and γ are the location, scale and shape parameters respectively.

The parameters of the generalized skew logistic distribution are obtained using the maximum likelihood method which maximizes the logarithm of likelihood function is given by

$$LL = \ln \prod_{i=1}^n \{f_{GSLD}(v_i)\} = \sum_{i=1}^n \ln \{f_{GSLD}(v_i)\}$$

where, v_i is the maximum temperature in time step i and n is the number of data points.

Goodness-of-fit Tests

Goodness-of-fit tests are used to check the accuracy of the predicted data using theoretical probability function. To evaluate the goodness-of-fit of the PDFs to the monthly maximum temperature data, the KS, the R^2 , the χ^2 and the RMSE were used.

Kolmogorov-Smirnov (KS) Error Test

The KS test computes the largest difference between the cumulative distribution function of the model and the empirical distribution function. The KS test statistic is

$$\text{defined as } D = \max_{1 \leq i \leq n} |F_i - \hat{F}_i|$$

where, \hat{F}_i is the predicted cumulative probability of the i^{th} observation obtained with the theoretical cdf and F_i is the empirical probability of the i^{th} observation are obtained with the Cunnane (1978) formula: $F_i = \frac{i-0.4}{n+0.2}$ where, $i = 1, \dots, n$ is the rank for ascending ordered observations.

R² Test

The R^2 test is used widely for goodness-of-fit comparisons and hypothesis testing because it quantifies the correlation between the observed cumulative probabilities and the predicted cumulative probabilities of a monthly maximum temperature distribution. A larger value of R^2 indicates a better fit of the model cumulative probabilities \hat{F} to the observed cumulative probabilities F . The R^2 is defined as:

$$R^2 = \frac{\sum_{i=1}^n (\hat{F}_i - \bar{F})^2}{\sum_{i=1}^n (\hat{F}_i - \bar{F})^2 + \sum_{i=1}^n (F_i - \hat{F}_i)^2}; \bar{F} = \frac{\sum_{i=1}^n \hat{F}_i}{n}$$

Chi-Square Error Test

Chi-Square test is used to assess whether the observed probability differs from the predicted probability. Chi-Square test statistic is defined as $\chi^2 = \sum_{i=1}^n \frac{(F_i - \hat{F}_i)^2}{\hat{F}_i}$.

Root Mean Squared Error (RMSE) Test

Root mean square error (RMSE) provides a term-by-term comparison of the actual deviation between observed probabilities and predicted probabilities. A lower value of RMSE indicates a better distribution function model. Root mean square error (RMSE)

is defined as $RMSE = \left[\frac{\sum_{i=1}^n (F_i - \hat{F}_i)^2}{n} \right]^{1/2}$.

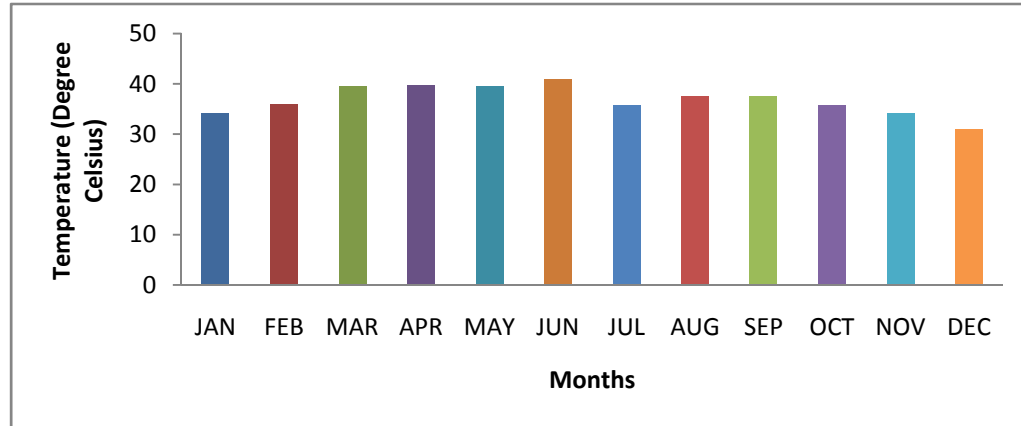
3. Results and Discussion

The minimum of the monthly maximum temperature of Dhaka city was 24.2 on January, 2014 and the maximum was 40.8 on June 1987. Table 1, shows the frequency of the observed monthly maximum temperature of Dhaka Station over the study period.

Table 1: Frequency distribution of monthly maximum temperature of Dhaka Station

Temperature (Degree Celsius)	Frequency	Percent
< 25	3	0.57
25-28	24	4.57
28-31	77	14.67
31-34	149	28.38
34-37	220	41.90
37-40	51	9.71
40 and above	1	0.19

It is observed that about 5 percent month's temperature was less than 28 degree celsius and only 10 percent month's has temperature more than 37 degree celsius. About 42 percent month's temperature was lies between 34 and 37 degree celsius in Dhaka Station. Also 70 percent month's temperature was lies between 31 and 37 degree celsius (Table 1). Figure 1 shows the monthly maximum temperature of Dhaka city. It may be concluded from Figure 1 that the maximum temperature was recorded on June whereas the minimum was on December over the study period.

**Figure 1:** Maximum temperature of different months of Dhaka city

The estimation of parameters of all the PDFs considered in this study were carried out using maximum likelihood method and estimated parameters of different PDFs used in this study are presented in Table 2.

Table 2: Estimated parameters of different PDFs considered in this study

PDF	Parameters	Estimated value of parameters
W	Shape (k)	13.8612
	Scale (c)	34.9288
LN	Mean (μ)	3.511602
	Standard Deviation (δ)	0.091013
G	Shape (a)	123.90
	Scale (b)	0.2714
	Shape (α)	-0.3774836
GEV	Scale (β)	3.1269918
	Location (μ)	32.7177221
3-P LN	Shape (c)	0.01395
	Scale (a)	5.360
	Threshold (b)	-179.10
ST	Location (θ_0)	37.062
	Scale (θ_1)	4.522
	Degrees of Freedom (θ_2)	68409.844
	Shape (θ_3)	-3.101
GSL	Location (μ)	35.93695
	Scale (σ)	0.9585
	Shape (γ)	0.34993

The statistical parameters for fitness evaluation of PDFs currently analyzed are presented in Table 3. Considering K-S error, χ^2 error and RMSE, the probability functions Weibull, Lognormal, Gamma, Generalized Extreme Value, three parameters Lognormal and Skewed t have large errors indicating their inadequacy in modeling monthly maximum temperature of Dhaka. The higher value of R^2 and the lower values of K-S error, RMSE and chi square error of Generalized Skew Logistic

distribution (GSL) indicate that the best fit among the probability distribution considered in this paper.

Table 3: Values of Statistical tests for different distributions of Dhaka Stations

PDF	Values of different statistical tests			
	$K - S$	R^2	χ^2	$RMSE$
W	0.0446219	0.9950808	1.163366	0.02034866
LN	0.1125662	0.9584065	5.685295	0.05929675
G	0.1073298	0.9632692	5.164221	0.05582154
GEV	0.07519518	0.9816559	2.346937	0.0383749
3-P LN	0.09561822	0.9719812	4.374118	0.04878234
ST	0.04442208	0.9951503	1.043893	0.02016918
GSL	0.04100315	0.9977198	0.9359856	0.01404306

The graphical comparisons of different probability distribution considered in this study and the histogram of the observed monthly maximum temperature Dhaka station are presented in Figure 2. As seen from Figure 2 and different statistical tests from Table 3, Generalized Skew Logistic distribution (GSL) provided the best fit for the observed monthly maximum temperature data of Dhaka station.

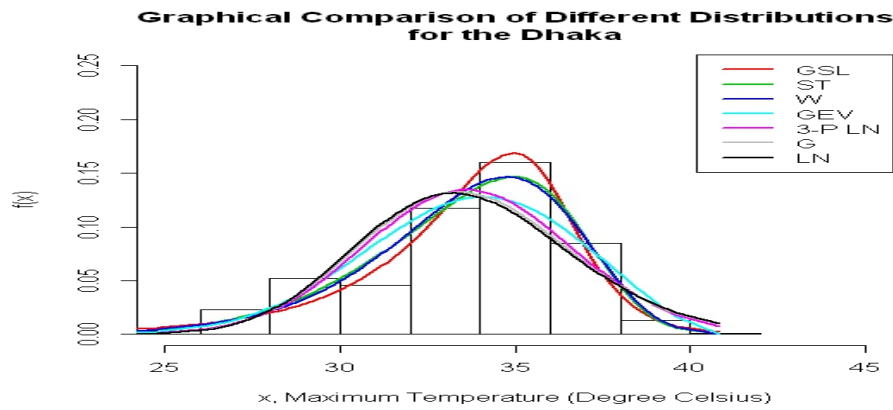


Figure 2: Graphical Comparison for different distribution of Dhaka Stations

4. Conclusions

This paper identified that the Generalized Skew Logistic distribution (GSL) provided the best fit for the observed monthly maximum temperature data of Dhaka among the probability distribution considered in this paper.

References

- A. Clifford Cohen and Betty Jones Whitten, (1980): Estimation in the Three-Parameter Lognormal Distribution, *Journal of the American Statistical Association*, 75: 399-404.
- Achim Zeileis and Thomas Windberger, (2014): Fitting and Testing Generalized Logistic Distributions, Package ‘glogis’, CRAN.
- L. V. Alexander, X. Zhang, T. C. Peterson, J. Caesar and Others (2006): Global observed changes in daily climate extremes of temperature and precipitation, *J. Geophys. Res.*, 111: D05109.
- A. Ali (1999): Climate change impacts and adaptation assessment in Bangladesh, *Climate Research*, 12: 109-16.
- B. G. Brown and R. W. Katz (1995): Regional analysis of temperature extremes: Spatial analog for climate change? *J. Climate*, 8: 108–119.
- M. H. K. Chowdhury and S. K. Debsharma (1992): “Climate change in Bangladesh – A statistical review”, *Report on IOC-UNEP Workshop on Impacts of Sea Level Rise due to Global Warming, NOAMI*, held during 16-19 November 1992, Bangladesh.
- Climate Change Cell, (2009a): Characterizing long-term changes of Bangladesh climate in context of agriculture and irrigation, Department of Environment, Dhaka.
- C. Cunnane (1978): Unbiased plotting positions – a review. *J. Hydrol*, 37(3–4): 205–222.
- C. Davis (2015): *The Skewed Generalized T Distribution Tree Package Vignette*. (Available at <https://cran.r-project.org/web/packages/sgt/vignettes/sgt.pdf>). Accessed on 18 April, 2016.
- Divya, R. Mehrotra (1995): Climate Change and hydrology with emphasis on the Indian subcontinent, *Hydrologic Sciences Journal*, 40: 231-241.
- Donat, M. G., and L. V. Alexander, (2012), The shifting probability distribution of global daytime and night-time temperatures, *Geophys. Res. Lett.*, 39, L14707.
- T. Faragó and R. W. Katz, R. W. (1990): *Extremes and Design Values in Climatology*. WMO/TD-No 386.
- J. I. Hansen, I. Fung, A. Lacis, D. Rind, S. Lebedeff (1988): Global climate changes as forecast by Goddard Institute for space studies three-dimensional model, *J. Geophys. Res.*, 93: 9341–9364. doi:10.1029/88JD00231.

- Hossain Mohiuddin, Md. Musfiqur Rahman Bhuiya, Mustofa Mahmud Al Mamun, (2014): An analysis of the temperature change of Dhaka city, *Proceedings of 5th International Conference on Environmental Aspects of Bangladesh*, pp. 46-48.
- J. T. Houghton, L. G. Meira Filho, B. A. Callander, N. Harris, A. Kattenberg and K. Maskell (eds.), (1996): *Climate Change 1995. The Science of Climate Change*. Published for the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, pp. 572.
- S. Huq, Z. Karim, M. Asaduzzaman and F. Mahtab (Eds) (1999): *Vulnerability and Adaptation to Climate Change in Bangladesh*, Kluwer, Dordrecht, p. 147.
- D. Indhumathy, C. V. Sessaiah and K. Sukkiramathi (2014): A Study on Wind Speed Distributions, *International Journal of Research in Engineering and Technology*, 3(4): 579-582.
- Intergovernmental Panel on Climate Change (2001): *Climate Change 2001: The scientific Basis*, Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change, Houghton, J.T., Ding. Y., Griggs, D.J., Noguer, M., van der Linden, P.J., Dai, X., Maskell, K., Johnson, C.A. (Eds), IPCC, Cambridge University Press, Cambridge, p. 881.
- T. Islam and A. Neelim (2010): *Climate Change in Bangladesh: A closer look into temperature and rainfall data*, The University Press Limited.
- Johnson, L. Norman, Kotz, Samuel, and N. Balakrishnan (1994): 14: Lognormal Distributions, *Continuous univariate distributions. Vol. 1*, Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics (2nd ed.), New York: John Wiley & Sons.
- S. Karmakar and M. L. Shrestha (2000): Recent climate change in Bangladesh, SMRC No.4, SMRC, Dhaka.
- V. V. Kharin and F. W. Zwiers (2000): Changes in the extremes in an ensemble of transient climate simulations with a coupled atmosphere-ocean GCM. *J. Climate*, 13: 3760–3788.
- H. O. Lancaster (1966): “Forerunners of the Pearson Chi-square.” *Australian Journal of Statistics*, 8: 117-126.
- P. S. Laplace (1836): “*Theorie Analytique des Probabilities.*”
- L. O. Mearns, R. W. Katz, and S. H. Schneider (1984): Extreme high temperature events: changes in their probabilities with changes in mean temperature, *J. Clim. Appl. Meteorol.*, 23: 1601–1608.

- N. M. Mia (2003): Variations of temperature of Bangladesh, *In Proceedings of SAARC Seminars on Climate Variability In the South Asian Region and its Impacts*, SMRC, Dhaka.
- B. Parthasarathy, N. A. Sontake, A. A. Monot, and D. R. Kothawale (1987): Drought-flood in the summer monsoon season over different meteorological subdivisions of India for the period 1871-1984, *Journal of Climatology*, 7: 57-70.
- S. Peng, J. Huang, J. E. Sheehy, R. E. Laza, R. M. Visperas, X. Zhong, G. S. Centeno, G. S. Khush, and K. G. Cassman, (2004): Rice yields decline with higher night temperature from global warming, *P. Natl. Acad. Sci.*, 101: 9971-9975.
- W. F. Perks (1932): On some experiments in the graduation of mortality statistics, *Journal of the Institute of Actuaries*, 58: 12-57.
- A. Rahman and M. Alam (2003): Mainstreaming adaptation to climate change in Least Developed Countries (LDCs), Working Paper 2: Bangladesh Country Case Study, IIED, London, UK.
- SMRC. (2003): The vulnerability assessment of the SAARC coastal region due to sea level rise: Bangladesh case, SMRC Publication No. 3, SAARC Meteorological Research Centre, Dhaka.
- P. Theodossiou (1998): Financial Data and the Skewed Generalized T Distribution, *Management Science*, 44: 1650–1661. [doi:10.1287/mnsc.44.12.1650](https://doi.org/10.1287/mnsc.44.12.1650).
- R. T. Watson, M .C. Zinyowera, R. H. Moss and D. J. Dokken (eds.), (1996): *Climate Change 1995: Impacts, Adaptations, and Mitigation of Climate Change: Scientific-Technical Analyses*. Published for Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, pp. 878.
- W. Weibull (1951): A statistical distribution function of wide applicability, *J. Appl. Mech. Trans.*, ASME, 18 (3): 293–297.
- A. Ying and M. D. Pandey (2007): The r largest order statistics model for extreme wind speed estimation, *J. Wind Eng. and Aerodyn.*, 95(3): 165–182.
- F. W. Zwiers, and V. V. Kharin (1998): Changes in the extremes of the climate simulated by CCC GCM2 under CO2 doubling. *J. Climate*, 11: 2200–2222.
- C. Parmesan, T. L. Root and M. R. Willig (2000): Impacts of Extreme Weather and Climate on Terrestrial Biota, *Bulletin of the American Meteorological Society*, 81(3): 443-450.