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Revenue Forecasting using Holt–Winters Exponential Smoothing

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Abstract

A lot of time series demonstrate seasonal behavior with trend, such type of series was monthly revenue (in crore) of Bangabandhu Multipurpose Bridge. The seasonal forecasting with trend issue was considerable importance. The research work focus on the analysis of seasonal time series data using additive and multiplicative seasonal model of Holt–winters method and forecast the monthly revenue (in crore) using best model—additive Holt–Winters exponential smoothing up to January 2021.

Keywords: *Forecast, levels, seasonality, trend, root mean squared, Bangladesh*

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INTRODUCTION

Forecasting was most often part of a larger process of planning and managing, and a forecast was necessary to provide accurate estimates of the future for the larger process. The all of forecasting methods were classified under the types—time series univariate methods, causal or multivariate methods, qualitative or technological, and other quantitative methods.

The Holt–Winters method is a statistical forecasting method in time series univariate methods. The forecasting engages make projection about future performance on the basis of past and recent data. Dasgupta SS, Mahanta P, Roy R, Subramanian G. (2014) [1] forecast industry big data with Holt–Winters method. Exponential smoothing models and auto regressive moving average model compared to comprehend which method is more adapted to model the temperature behavior in Caserta, Italy (Guizzi G, Silvestri C, Romano E, Revetria R, 2015) [2].

Taylor JW (2003) [3] applied the Holt–Winters method to forecast short-term electricity demand. Holt (2004) [4] extended the exponential weighed moving averages to allow trend and seasonal variation. Holt–Winters exponential smoothing is a popular approach to forecasting seasonal time series. Winters method and Fourier series analysis are versatile methods because the methods model the level, trend, and seasonality of a time series (DeLurgio SA, 1998) [5]. Puah YJ, Huang YF, Chua KC, Lee TS. (2016) [6] were modeled the Rainfall series using additive Holt–Winters method to examine the rainfall pattern in Langat River Basin, Malaysia. The Holt–Winters exponential smoothing is one of the most popular methods to forecast the different time series. The main objective of the research work was fit the additive Holt–Winters exponential smoothing model, check the accuracy measures, and forecast the monthly revenue (in crore) of the Bangabandhu multipurpose bridge.

DATA DESCRIPTION

The monthly revenue (in crore) from July 1998 to July 2016 was plotted and shown in Figure 1. The mean revenue found to be 16.917 with standard deviation 8.9375. The quantile—quantile plot and box plot was presented in the Figure 2. The monthly revenue data normally distributed. From the above time series plot, it was evident that the seasonality and trend was present in the series. The seasonal length of the series was twelve (Table 1).

METHODOLOGY

The monthly revenue (in crore) of Bangabandhu Multipurpose Bridge collected from Bangladesh Bridge Authority, Bridges Division, Ministry of Road Transport and Bridges for July 1998 to July 2016. The toll was collected at Bangabandhu Multipurpose Bridge which stands on the river *Jamuna* between the district *Sirajgonj* and *Tangail.*

Exponential smoothing methods give larger weights to more recent observations, and the weights decrease exponentially as the observations become more distant. These methods are most effective when the parameters describing the time series are changing slowly over time. Holt–Winters Methods are applicable when both trend and seasonal pattern present. Two Holt–Winters methods were designed for time series that exhibit linear trend. Additive Holt–Winters method used for time series with constant (additive) seasonal variations and Multiplicative Holt–Winters method used for time series with increasing (multiplicative) seasonal variations.

Table 1: *Summary statistics.*

Monthly Revenue (in crore)	
Mean (in crore)	16.9170
Standard deviation (in crore)	8.9375
Kurtosis	0.9395
Skewness	0.4038
Coefficient of variation	0.5283
Minimum (in crore)	3.7900
Maximum (in crore)	37.7200
Count	217.0000

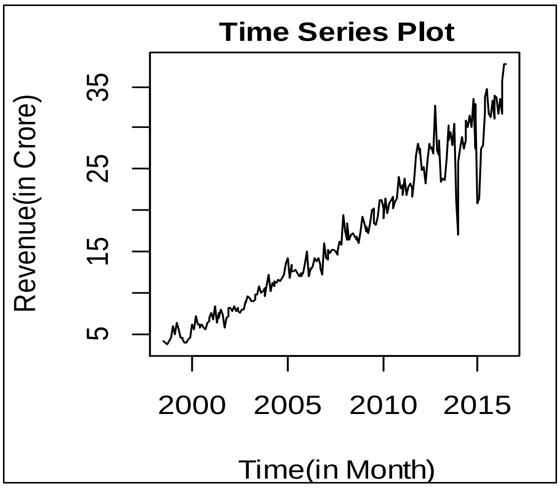

Fig. 1: *Time Series Plot of Monthly Revenue (in crore).*

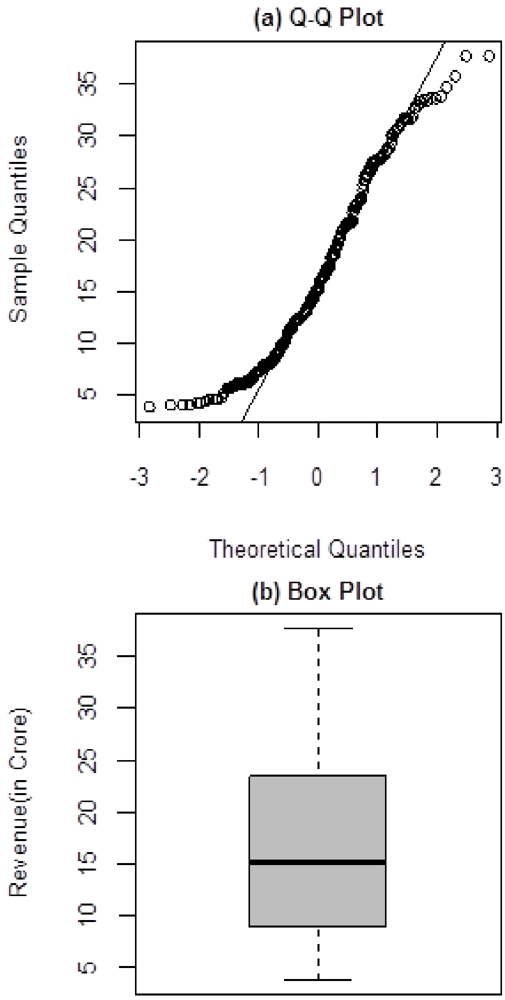

Fig. 2 *(a) Q-Q Plot and (b) Box Plot for Monthly Revenue (in crore).*

Holt–Winters Method

The Holt–Winters method was originally developed by Winters and involves estimating three smoothing parameters associated with the level, trend and seasonal factors (Bermudez JD, Segura JV, Vercher E, 2006) [7]. Holt–Winters method is an exponential smoothing approach for handling seasonal data.

Additive Holt–Winters Method

The additive Holt–Winters method is presented in the following equations.

$$y_T = \beta_0 + \beta_1 t + sn_T + \varepsilon_T$$

Estimate of the level at time T

$$l_T = \alpha\left(y_T - sn_{T-L}\right) + \left(1-\alpha\right)\left(l_{T-1} + b_{T-1}\right)$$

Estimate of the growth rate (or trend) at time T

$$b_T = \gamma\left(l_T - l_{T-1}\right) + \left(1-\gamma\right)b_{T-1},\ 0 \le \alpha, \gamma \le 1$$

Estimate of the seasonal factor at time T

$sn_T = \delta(y_T - l_T) + (1-\delta) sn_{T-L}$ where, $0 \le \delta \le 1$

p- Step ahead forecast made at time *T*

$\hat{y}_{T+p}(T) = l_T + pb_T + sn_{T+p-L}$ where, $p = 1, 2, \ldots$

Where,

Initial level$= l_0 = \beta_0 =$ intercept

Initial growth rate$= b_0 = \beta_1 =$ Slope

$S_T = y_T - \hat{y}_T$, and $\bar{S}_{[i]} = \frac{1}{L}\sum_{k=i-1}^{\le n} S_{2k+1}$

$L =$ No. of seasons in a year

Initial seasonal factors $= sn_{i-L} = \bar{S}_{[i]}$, $i = 1, 2, \ldots, L$

Multiplicative Holt–Winters Method

The multiplicative Holt–Winters method is presented in the following equations.

$y_T = (\beta_0 + \beta_1 t) \times sn_T \times \varepsilon_T$

Estimate of the level at time *T*

$l_T = \alpha(y_T / sn_{T-L}) + (1-\alpha)(l_{T-1} + b_{T-1})$

Estimate of the growth rate (or trend) at time *T*

$b_T = \gamma(l_T - l_{T-1}) + (1-\gamma) b_{T-1}, 0 \le \alpha, \gamma \le 1$

Estimate of the seasonal factor at time *T*

$sn_T = \delta(y_T / l_T) + (1-\delta) sn_{T-L}$ where, $0 \le \delta \le 1$

p- Step ahead forecast made at time *T*

$\hat{y}_{T+p}(T) = (l_T + pb_T) \times sn_{T+p-L}$, $p = 1, 2, \ldots$

Where,

Initial level$= l_0 = \beta_0 =$ intercept

Initial growth rate$= b_0 = \beta_1 =$ Slope

$S_T = y_T / \hat{y}_T$ and $\bar{S}_{[i]} = \frac{1}{L}\sum_{k=i-1}^{\le n} S_{2k+1}$

$L =$ No. of seasons in a year

Normalized Constant $= CF = L \Big/ \sum_{i=1}^{L} \bar{S}_{[i]}$

Initial seasonal factors $= sn_{i-L} = \bar{S}_{[i]}[CF]$

where, $i = 1, 2, \ldots\ldots, L$

Model Performance

Goodness of fit is the measure of the accuracy of the forecasted model to actual value. The model forecast accuracy was measured by the following criteria.

Mean Absolute Deviation : $MAD = \frac{\sum |e_t|}{n}$

Sum Squared Error : $SSE = \sum e_t^2$

Mean Squared Error : $MSE = \frac{\sum e_t^2}{n}$

Root Mean Squared : $RMS = \sqrt{\frac{\sum e_t^2}{n}}$

Mean Absolute Scaled Error:

$$MASE = \frac{1}{n}\sum_{t=1}^{n} \frac{\sum_{t=1}^{n} |y_t - \hat{y}_t|}{\frac{1}{n-1}\sum_{t=2}^{n} |y_t - y_{t-1}|}$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

Percentage Error $: PE_t = \frac{(y_t - \hat{y}_t)}{y_t} \times 100$

Adjusted $R^2 : \bar{R}^2 = 1 - \frac{(n-1)RSS}{(n-k)TSS} = \frac{(n-1)}{(n-k)} R^2$

Mean Percentage Error : $MPE = \frac{\sum PE_t}{n}$

Mean Absolute Percentage Error : $MAPE = \frac{\sum |PE_t|}{n}$

The TSS, ESS, and RSS mean total sum of square, explained sum of square and residual sum of square respectively.

RESULT AND DISCUSSIONS

The trend and the seasonal pattern were present in the time series (monthly revenue in crore). The Holt–winters method is applicable to forecast the future state. The Holt–Winters was fitted to forecast monthly revenue (in crore) for the Bangabandhu Multipurpose Bridge of Bangladesh. Finally, the equations take the forms and the fitted models with estimated parameters are shown in following.

Additive Holt–Winters Model

$y_T = \beta_0 + \beta_1 t + sn_T + \varepsilon_T$

$l_0 = \beta_0 = 35.2931090$ and $b_0 = \beta_1 = 0.1192485$

$S_T = y_T - \hat{y}_T$ where, $t = 1, 2, \ldots, n$ and $n = 217$

$\bar{S}_{[i]} = \frac{1}{L}\sum_{k=i-1}^{\le n} S_{2k+1}$

where, $L =$ No. of seasons in a year=12

$\bar{S}_{[1]} = 0.3680168$ $\quad \bar{S}_{[7]} = -1.2345571$

$\bar{S}_{[2]} = -0.3639068$ $\quad \bar{S}_{[8]} = 1.0573878$

$\bar{S}_{[3]} = 1.7726623$ $\quad \bar{S}_{[9]} = -0.1873891$

$\bar{S}_{[4]} = -1.652559$ $\quad \bar{S}_{[10]} = 2.0314614$

$\bar{S}_{[5]} = -0.1115694$ $\bar{S}_{[11]} = 3.0176877$

$\bar{S}_{[6]} = -0.6515678$ $\bar{S}_{[12]} = 1.9456332$

$sn_{i-L} = \bar{S}_{[i]}$ where, $i = 1, 2, ..., L$

$l_T = \alpha(y_T - sn_{T-L}) + (1-\alpha)(l_{T-1} + b_{T-1})$

where, $\alpha = 0.5370757$

$b_T = \gamma(l_T - l_{T-1}) + (1-\gamma)b_{T-1}$

where, $\gamma = 0.005237878$

$sn_T = \delta(y_T - l_T) + (1-\delta)sn_{T-L}$

where, $\delta = 0.4206121$

$p-$step ahead forecast made at time T

$\hat{y}_{T+p}(T) = l_T + pb_T + sn_{T+p-L}$ where, $p = 1, 2, ...$

$$SSE = \sum_{t=1}^{n}\left[y_T - \hat{y}_T(T-1)\right] = 691.3605$$

Multiplicative Holt–Winters Model

$y_T = (\beta_0 + \beta_1 t) \times sn_T \times \varepsilon_T$

$l_0 = \beta_0 = 32.5523721$ and $b_0 = \beta_1 = 0.1640977$

$S_T = y_T / \hat{y}_T$ where, $t = 1, 2, ..., n$ and $n = 217$

$$\bar{S}_{[i]} = \frac{1}{L}\sum_{k=i-1}^{\leq n} S_{2k+1}$$

where, $L =$ No. of seasons in a year=12

$\bar{S}_{[1]} = 1.0962413$ $\bar{S}_{[2]} = 1.0634873$

$\bar{S}_{[3]} = 1.1492499$ $\bar{S}_{[4]} = 1.0092294$

$\bar{S}_{[5]} = 1.0478964$ $\bar{S}_{[6]} = 1.0121130$

$\bar{S}_{[7]} = 0.9686139$ $\bar{S}_{[8]} = 1.0575681$

$\bar{S}_{[9]} = 1.0117472$ $\bar{S}_{[10]} = 1.1068369$

$\bar{S}_{[11]} = 1.1666387$ $\bar{S}_{[12]} = 1.1467173$

$sn_{i-L} = \bar{S}_{[i]}$ where, $i = 1, 2, ..., L$

$l_T = \alpha(y_T / sn_{T-L}) + (1-\alpha)(l_{T-1} + b_{T-1})$

where, $\alpha = 0.1766912$

$b_T = \gamma(l_T - l_{T-1}) + (1-\gamma)b_{T-1}$

where, $\gamma = 0.01577167$

$sn_T = \delta(y_T / l_T) + (1-\delta)sn_{T-L}$

where, $\delta = 0.3461822$

$p-$step ahead forecast made at time T

$\hat{y}_{T+p}(T) = (l_T + pb_T) \times sn_{T+p-L}$ where, $p = 1, 2, ...$

$$SSE = \sum_{t=1}^{n}\left[y_T - \hat{y}_T(T-1)\right] = 762.77962$$

The accuracy measures—Sum Square Error, Mean Sum Square Error, Root Mean Squared, Mean Percentage Error, Mean Absolute Percentage Error, Mean Absolute Deviation, Mean Absolute Scaled Error of the additive Holt–Winters method are lower than R^2 and Adjusted R^2 higher than the multiplicative Holt–Winters method (Table 2).

The fitted seasonal factor, growth rate (or trend), level and forecasted revenue were plotted against time (month) presented in Figure 3. The different types of forecast accuracy measures of additive Holt–Winters methods such as sum square error, root mean squared, mean absolute percentage error, mean absolute deviation, and R^2 are 691.3605056, 1.836434118, 5.899982004, 1.075149359, 0.972069439 respectively (Table 3).

Table 2: *Forecast Accuracy Measures.*

Accuracy Measures	Holt–Winters Additive Model	Holt–Winters Multiplicative Model
SSE	691.3605056	762.7796266
MSE	3.372490271	3.720876227
RMS	1.836434118	1.928957290
R^2	0.972069439	0.968063769
Adjusted R^2	0.981693889	0.955882545
MPE	0.542534439	0.967230691
MAPE	5.899982004	6.771234123
MAD	1.075149359	1.148890118
MASE	0.520340000	0.556028300

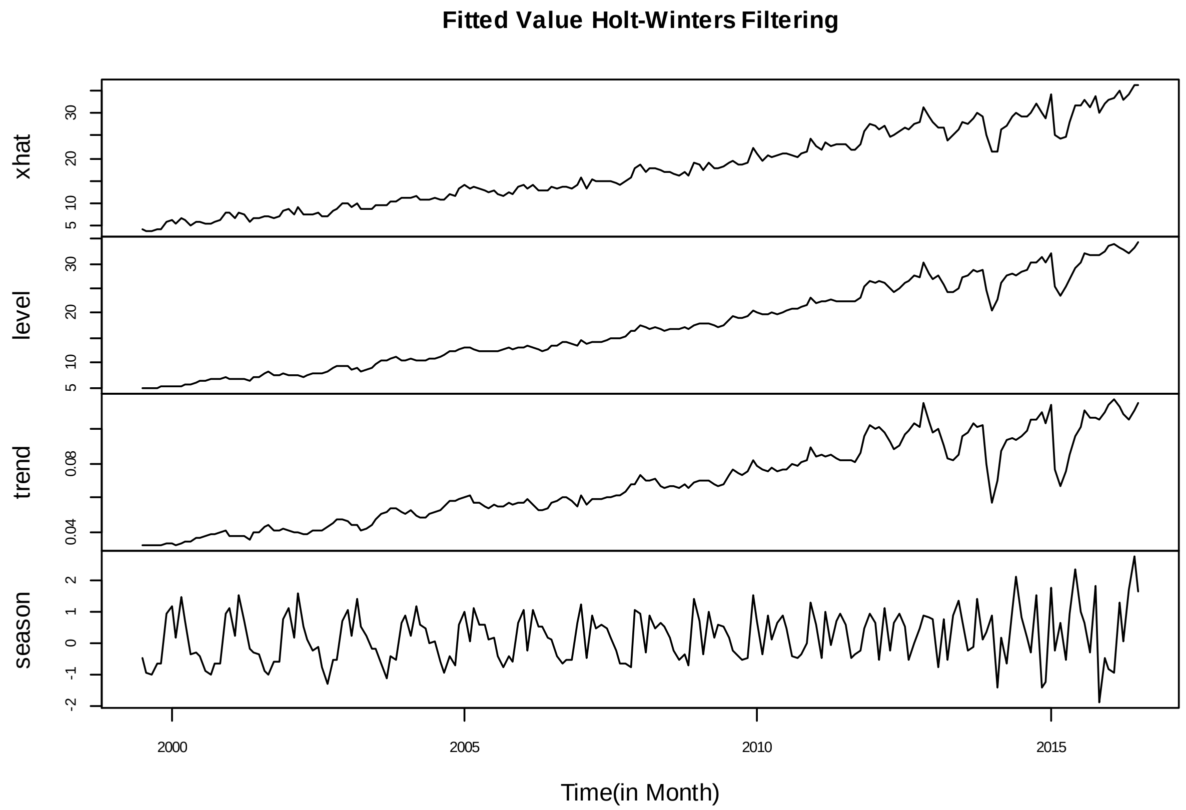

Fig. 3: *Fitted Seasonal Factor, Growth Rate (or trend), Level and Forecasted Revenue Plot against Time (Month) for Additive Holt–Winters Model.*

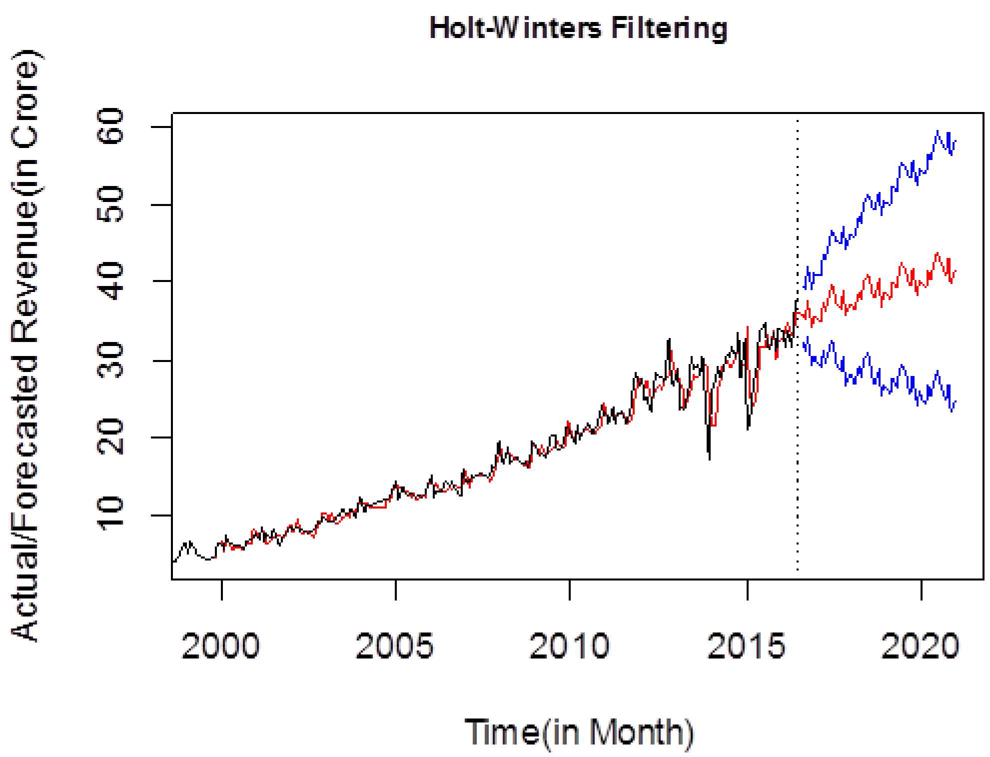

Fig. 4: *Actual revenue (Black colour) and Forecasted Revenue (in crore) with Confidence Limit Plot against Time (month) using Holt–Winters Additive Model.*

Table 3: *Monthly Forecasted Revenue (in crore) using Additive Holt–Winters Method.*

Year	Month	Forecasted Revenue (in Crore)
2016	Aug	35.78037
2016	Sep	35.16770
2016	Oct	37.42352
2016	Nov	34.11754
2016	Dec	35.77778
2017	Jan	35.35703
2017	Feb	34.89329
2017	Mar	37.30448
2017	Apr	36.17896
2017	May	38.51706
2017	Jun	39.62253
2017	Jul	38.66972
2017	Aug	37.21136
2017	Sep	36.59868
2017	Oct	38.85450
2017	Nov	35.54853
2017	Dec	37.20876
2018	Jan	36.78801
2018	Feb	36.32427
2018	Mar	38.73547
2018	Apr	37.60994
2018	May	39.94804
2018	Jun	41.05351
2018	Jul	40.10071
2018	Aug	38.64234
2018	Sep	38.02966
2018	Oct	40.28548
2018	Nov	36.97951
2019	Dec	38.63975
2019	Jan	38.21900
2019	Feb	37.75526
2019	Mar	40.16645
2019	Apr	39.04092
2019	May	41.37902
2019	Jun	42.48449
2019	Jul	41.53169
2019	Aug	40.07332
2019	Sep	39.46065
2019	Oct	41.71646
2019	Nov	38.41049
2019	Dec	40.07073
2020	Jan	39.64998
2020	Feb	39.18624
2020	Mar	41.59743
2020	Apr	40.47190
2020	May	42.81000
2020	Jun	43.91548
2020	Jul	42.96267
2020	Aug	41.50430
2020	Sep	40.89163
2020	Oct	43.14745
2020	Nov	39.84147
2020	Dec	41.50171
2021	Jan	41.08096

SUMMARY AND CONCLUSION

The forecasting accuracy measures of the multiplicative Holt–Winters method were higher than the additive Holt–Winters model. On the basis of the accuracy measures the monthly revenue (in crore) was forecasted by the additive Holt–Winters method.

The forecasted revenue (in crore) were presented where the forecasted value in December 2016, June 2017, January 2018, December 2019, and October 2020 were 35.77778, 39.62253, 36.78801, 40.07073, 43.14745 respectively. The forecasted revenue was also upward trend.

The coefficient of variation of the monthly revenue (in crore) and monthly forecasted revenue (in crore) using additive Holt–Winters method was 0.5283 and 0.06048151 respectively. The forecasting method—additive Holt–Winters method be the superior than the multiplicative Holt–Winters method (Figure 4).

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