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# Fitting the Probability Distribution of Monthly Maximum Temperature of Some Selected Stations from the Northern Part of Bangladesh

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**ABSTRACT**

*The weather of Bangladesh is varying as well as it is becoming more unpredictable in every year. However, the impacts of extreme temperatures, more fluctuated rainfall, sea level rise and other weather events have already occurred in Bangladesh and will continue to upsurge. So, this paper attempts to identify the most suitable probability distribution of the monthly maximum temperature of some selected stations from the northern part of Bangladesh. The Goodness-of-fit tests and the graphical comparison shows that the Generalized Skewed Logistic (GSL) provided the best fit for the observed monthly maximum temperatures for all the stations considered in this study.*

**Keywords:** Probability distribution, maximum temperature, Bangladesh.

**Mathematics Subject Classification:** 62G32, 62H10, 97K80

**Journal of Economic Literature (JEL) Classification :** C46, C51

## 1. INTRODUCTION

Nowadays, climate change is one of the great environmental issues. Climate change is global but its effect is not homogeneous all over the world. It is also expected that the earth will considerably worm in the coming decades. Temperatures chance can lead to changes in many activities in our daily life. Also, rising temperature causes sea level rise and as a result, the low-lying coastal areas and deltas of the world will be affected in future. That’s why we often imagine that some part of Bangladesh will be under water in future due to sea level rise. Moreover, ecosystems and wildlife are severely affected by the occurrence of extreme weather events. IPCC (2014) projected that the global mean surface temperature increases in 2100 from 3.7 to 4.8°C compared to pre-industrial levels. They also mentioned that the global climate change can have different temperature impacts on different areas, or other differential effects (e. g., on coastal areas via the rise in sea level). Mearns et al. (1984) and Hansen et al. (1988) make a conclusion that a significant change occurs in the frequency of temperature extremes due to the comparatively small changes in the mean temperature. Alexander et al. (2006) showed that annual trends in the lowest and highest daily minimum and maximum temperatures in the latter half of the twentieth century increased at many locations all over the world. Ahmad et al. (1996) shown that the temperature is increased by $0.5^{0}$C in Bangladesh during past 100 years. Mondal and Wasimi (2004) observed an increasing trend of $0.5^{0}$C and $1.1^{0}$C per century in day-time maximum and night-time minimum temperatures, respectively.

Donat and Alexander (2012) revealed that universally the distribution of daily temperatures has undeniably become “more extreme” since the middle of the 20th century. The probability distributions have been employed to categorized the extreme-value data. Several researchers applied the two-parameter Gumbel distribution and the three-parameter generalized extreme value (GEV) distribution in order to estimates the probability of annual temperature extremes (Faragó and Katz, 1990; Brown and Katz, 1995; Zwiers and Kharin, 1998; Kharin and Zwiers, 2000). Hasan et al., (2013) shown that the Generalized Extreme Value (GEV) distribution is the best-fitting model of the monthly maximum temperature of several stations in Malaysia. Hasan et al., (2012) shows that only weekly, biweekly, and monthly maximum temperatures in Penang significantly fitted the GEV model. Hossain et al., (2016) shown that the Generalized Skew Logistic distribution (GSL) provided the best fit for the observed monthly maximum temperature data of Dhaka in Bangladesh. Now, the researcher all over the world has a great interest in the issue of extreme weather events occurring. Thus, it is essential to identify the probability distribution of monthly maximum temperature in Bangladesh. So, the main aim of this paper is to identify the most suitable probability distribution of the monthly maximum temperature of the selected stations in Bangladesh.

## 2. MATERIALS AND METHODS

This paper considers a secondary data set collected from Bangladesh Meteorological Department.

### 2.1. Study Period and Area

This study selects five stations which cover almost all part of the northern section of Bangladesh. The selected stations are Bogra, Dinajpur, Rajshahi, Rangpur and Syedpur. The secondary data were collected over the period from January 1971 to December 2015 for Bogra, Dinajpur, Rajshahi and Rangpur. However, in case of Syedpur, the study period is January 1991 to December 2015 because of not available of the previous data.

### 2.2. Probability distributions

Many PDFs have been proposed in recent past, but in present study Weibull, Lognormal, Gamma, GEV etc are used to describe the characteristics of maximum temperature. This paper considers the following probability distributions.

**Weibull (W) Distribution**

The probability density function (PDF) of the Weibull distribution with two parameters is given by (Weibull, W., 1951):

$$f(v;k,c) \propto \frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[-\left(\frac{v}{c}\right)^{k}\right] \ ; \ 0 < v < \infty, k > 0, c > 0$$

where, $'k'$ and $'c'$ are the shape and scale parameters respectively.

**Lognormal (LN) Distribution**

The Lognormal distribution is a probability distribution of a random variable whose logarithm is normally distributed. The probability density function (PDF) of the Lognormal distribution is given by (Johnson, *et al.*, 1994):

$$f(v;\mu,\delta) \propto \frac{1}{v\sqrt{2\pi\delta^2}}\exp\left[\frac{-(\ln(v)-\mu)^2}{2\delta^2}\right] \ ; \ 0 < v, < \infty, \mu \in R, \delta > 0$$

where, $'\mu'$ and $'\delta'$ are the mean and standard deviation of the normal random variable $\ln(v)$.

**Gamma (G) Distribution**

Lancaster (1966) quotes from Laplace (1836) in which the latter obtains a Gamma distribution. The probability density function (PDF) of the gamma distribution is given by:

$$f(v;a,b) \propto \frac{v^{a-1}}{b^a\Gamma(a)}\exp\left(-\frac{v}{b}\right); \ v > 0, a > 0, b > 0$$

The parameters $'a'$ and $'b'$ are the shape and scale parameters respectively.

**Generalized Extreme Value (GEV) Distribution**

GEV distribution is a flexible model that combines the Gumbel, Frechet and Weibull maximum extreme value distributions (Ying and Pandey, 2007). For, GEV the probability density function is given by:

$$f(v;\mu,\sigma,\xi) \propto \frac{1}{\sigma}\left[1+\xi\left(\frac{v-\mu}{\sigma}\right)\right]^{(-1/\xi)-1}\exp\left\{-\left[1+\xi\left(\frac{v-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}; \ \xi \neq 0$$

again, for $v > \left(\mu - \frac{\sigma}{\xi}\right)$ in the case $\xi > 0$, and for $v < \left(\mu - \frac{\sigma}{\xi}\right)$ in the case $\xi < 0$ where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ is the shape parameter.

**Three Parameters Lognormal (3-P LN) Distribution**

The lognormal distribution derives its name from the relationship that exists between random variables $V$ and $Y = \ln(V-a)$. If $Y$ is distributed normally $(b,c)$, then $V$ is lognormal $(a,b,c)$. Accordingly, the probability density function of $V$ may be written as (Cohen and Whitten, 1980):

$$f(v;a,b,c) \propto \frac{1}{(v-a)c\sqrt{2\pi}} \exp\left[\frac{-\left[\ln(v-a)-b\right]^2}{2c^2}\right]; c^2 > 0,\ a < v < \infty .$$

**Skewed $t$ (ST) Distribution**

There are different parameterizations for the skewed generalized $t$ distribution and it was first introduced by Panayiotis Theodossiouin in 1998 (Theodossiou, 1998). Also, Indhumathy, *et al.*, (2014) and Davis (2015) used the following probability density function (pdf) for Skewed $t$ distribution:

$$f_{STD}(v) \propto \frac{2}{2\theta_1\theta_2^{1/2}\beta\left(1/2,\theta_2\right)\left[1+\frac{|v-\theta_0|^2}{\theta_2\left\{1+sign(v-\theta_0)\theta_3\right\}^2\theta_1^2}\right]^{1/2+\theta_2}}; -\infty < v < \infty; \theta_1 > 0, \theta_2 > 0, -\infty < (\theta_0,\theta_3) < \infty$$

where, $\theta_0$ and $\theta_1$ are the location and scale parameters respectively, $\theta_2$ and $\theta_3$ are the degrees of freedom and shape parameters respectively and $\beta$ is the beta function. The parameters of the skewed t distribution are calculated using the maximum likelihood method which maximizes the logarithm of the likelihood function.

**Generalized Skewed Logistic (GSL) Distribution**

Perks (1932) first introduced the generalized logistic distribution. Zeileis, A. and Windberger, T., (2014) wrote an R Package ‘glogis’ for Fitting and Testing Generalized Logistic Distributions and they used the following probability density function (pdf) of the generalized skew logistic distribution:

$$f_{GSLD}(v) \propto \frac{\gamma}{\sigma}\frac{e^{-\left(\frac{v-\mu}{\sigma}\right)}}{\left(1+e^{-\left(\frac{v-\mu}{\sigma}\right)}\right)^{\gamma+1}} \;;\; -\infty < v < \infty$$

where, $\mu$ , $\sigma$ and $\gamma$ are the location, scale and shape parameters respectively.

### 2.3. Goodness-of-fit tests

Goodness-of-fit tests are used to check the accuracy of the predicted data using theoretical probability function. To evaluate the goodness-of-fit of the PDFs to the monthly maximum temperature data, the KS, the $R^2$ , the $\chi^2$ , the RMSE, AIC and BIC were used.

**Kolmogorov-Smirnov (KS) Error Test**

The KS test computes the largest difference between the cumulative distribution function of the model and the empirical distribution function. The KS test statistic is defined as $D = \max_{1\le i\le n}\left|F_i - \hat{F}_i\right|$ where, $\hat{F}_i$ is the predicted cumulative probability of the $i^{th}$ observation obtained with the theoretical cdf and $F_i$ is the empirical probability of the $i^{th}$ observation are obtained with the Cunnane (1978)formula: $F_i = \frac{i-0.4}{n+0.2}$ where, $i = 1,\cdots,n$ is the rank for ascending ordered observations.

**$R^2$ Test**

The $R^2$ test is used widely for goodness-of-fit comparisons and hypothesis testing because it quantifies the correlation between the observed cumulative probabilities and the predicted cumulative probabilities of a monthly maximum temperature distribution. A larger value of $R^2$ indicates a better fit of the model cumulative probabilities $\hat{F}$ to the observed cumulative probabilities $F$. The $R^2$ is defined as: $R^2 = \frac{\sum_{i=1}^{n}\left(\hat{F}_i - \overline{F}\right)^2}{\sum_{i=1}^{n}\left(\hat{F}_i - \overline{F}\right)^2 + \sum_{i=1}^{n}\left(F_i - \hat{F}_i\right)^2}$ ; $\overline{F} = \frac{\sum_{i=1}^{n}\hat{F}_i}{n}$ .

**Chi-Square Error Test**

Chi-Square test is used to assess whether the observed probability differs from the predicted probability. Chi-Square test statistic is defined as $\chi^2 = \sum_{i=1}^{n}\frac{\left(F_i - \hat{F}_i\right)^2}{\hat{F}_i}$ .

**Root Mean Squared Error (RMSE) Test**

Root mean square error (RMSE) provides a term-by-term comparison of the actual deviation between observed probabilities and predicted probabilities. A lower value of RMSE indicates a better distribution function model. Root mean square error (RMSE) is defined as $RMSE = \left[\frac{\sum_{i=1}^{n}\left(F_i - \hat{F}_i\right)^2}{n}\right]^{1/2}$ .

**Akaike Information Criterion (AIC) Test**

The Akaike Information Criterion (AIC) is defined as $AIC = e^{2k/n}\frac{\sum \hat{u}_i^2}{n}$, $\sum \hat{u}_i^2 = \sum\left(F_i - \hat{F}_i\right)^2$ where, $k$ is the number of estimated parameters and $n$ is the number of observations.

**Bayesian Information Criterion (BIC)**

The Bayesian Information Criterion (BIC) is defined as $BIC = n^{k/n}\frac{\sum \hat{u}_i^2}{n}$, $\sum \hat{u}_i^2 = \sum\left(F_i - \hat{F}_i\right)^2$ , where, $k$ is the number of estimated parameters and $n$ is the number of observations.

## 3. RESULTS

The descriptive statistics of monthly maximum temperature for the selected stations in this study are presented in Table 1.

*Table 1:* Descriptive statistics of monthly maximum temperature for the selected stations.

| Months | | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bogra | Mean | 24.45 | 27.5 | 31.76 | 33.98 | 33.31 | 32.74 | 32.01 | 32.29 | 32.15 | 31.83 | 30.02 | 26.26 |
| | SD | 1.25 | 1.23 | 1.2 | 1.7 | 1.65 | 0.92 | 0.85 | 0.75 | 0.83 | 0.9 | 0.79 | 1.09 |
| | Min. | 21.3 | 24.6 | 29 | 30.6 | 30.3 | 30.7 | 29.9 | 30 | 30.5 | 30 | 28 | 23.1 |
| | Max. | 26.8 | 30.1 | 33.9 | 37.3 | 38.2 | 34.6 | 33.6 | 33.5 | 33.6 | 33.8 | 31.7 | 28.5 |
| Dinajpur | Mean | 23.24 | 26.66 | 31.24 | 33.14 | 32.76 | 32.62 | 32.04 | 32.3 | 31.88 | 31.14 | 28.98 | 25.27 |
| | SD | 1.34 | 1.16 | 1.09 | 1.48 | 1.3 | 0.74 | 0.8 | 0.73 | 0.85 | 0.86 | 0.65 | 1.11 |
| | Min. | 19.6 | 24.2 | 28.4 | 29.8 | 30.2 | 31 | 30 | 30.4 | 29.9 | 28.7 | 27.5 | 21.9 |
| | Max. | 25.6 | 29.3 | 33 | 36.2 | 36.4 | 34.8 | 33.6 | 33.9 | 33.8 | 33.5 | 31.2 | 28.4 |
| Rajshahi | Mean | 24.17 | 27.85 | 33.29 | 35.81 | 34.84 | 33.8 | 32.49 | 32.64 | 32.6 | 31.7 | 29.36 | 25.59 |
| | SD | 1.08 | 1.33 | 1.28 | 1.53 | 1.43 | 1.27 | 0.79 | 0.8 | 0.91 | 0.76 | 0.64 | 0.84 |
| | Min. | 21.3 | 25.2 | 30 | 32.3 | 32.4 | 31.4 | 30.9 | 30.8 | 30.9 | 29.8 | 27.8 | 23.1 |
| | Max. | 26 | 31.3 | 35.9 | 39.2 | 38.3 | 36.7 | 34.3 | 34.2 | 34.4 | 32.9 | 30.7 | 27 |
| Rangpur | Mean | 23.58 | 26.04 | 30.34 | 32.27 | 31.63 | 32.11 | 31.69 | 32.03 | 31.33 | 30.73 | 28.49 | 25.07 |
| | SD | 1.9 | 1.65 | 1.69 | 1.35 | 1.77 | 0.96 | 0.8 | 0.53 | 0.77 | 1.0 | 1.04 | 1.16 |
| | Min. | 17 | 19.7 | 23.8 | 29.4 | 29.2 | 31 | 30.4 | 30.7 | 30.1 | 28.8 | 27.2 | 23.3 |
| | Max. | 27.2 | 28.1 | 32.3 | 34.6 | 38 | 35.8 | 34.2 | 32.7 | 33.6 | 33 | 32.2 | 28.6 |
| Syedpur | Mean | 22.74 | 26.57 | 30.83 | 32.36 | 32.6 | 32.37 | 32.2 | 32.53 | 32.17 | 31.3 | 28.97 | 25.13 |
| | SD | 1.21 | 1.44 | 1.15 | 1.6 | 1.38 | 0.7 | 0.85 | 0.78 | 0.85 | 0.91 | 0.58 | 1.06 |
| | Min. | 20.3 | 24.1 | 28.4 | 29.8 | 29.5 | 31.2 | 30.7 | 31 | 30.5 | 29.4 | 28 | 22.7 |
| | Max. | 24.5 | 29.7 | 32.9 | 35.3 | 35.1 | 33.9 | 33.6 | 33.6 | 33.8 | 33.2 | 30.1 | 26.8 |

It is observed that January was the coldest month and April was the hottest month for all stations considered in this study. In case of Bogra, more fluctuations among maximum temperature was observed in the month April whereas it was more stable in the month August. Thise scenario is different for other stations considered in this study. The estimation of parameters of all the PDFs considered in this study was carried out using maximum likelihood method and computed parameter values of different PDFs used for all the two stations are presented in Table 2.

*Table 2:* Computed parameter values of different PDFs considered in this study.

| PDF | Parameters | Station | | | | |
|---|---|---|---|---|---|---|
| | | Bogra | Dinajpur | Rajshahi | Rangpur | Syedpur |
| W | Shape $(k)$ | 12.6035 (28.44) | 12.2686 (27.95) | 10.5180 (28.95) | 12.3777 (27.64) | 12.2245 (19.78) |
| | Scale $(c)$ | 32.0043 (277.04) | 31.4892 (272.20) | 32.7417 (231.93) | 30.9763 (275.28) | 31.3813 (194.77) |
| LN | Mean $(\mu)$ | 3.4185 (747.74) | 3.3981 (668.99) | 3.4324 (646.38) | 3.3825 (672.66) | 3.3935 (463.04) |
| | Standard Deviation $(\delta)$ | 0.1062 (32.86) | 0.1180 (32.86) | 0.1234 (32.86) | 0.1168 (32.86) | 0.1218 (23.49) |
| G | Shape $(a)$ | 91.9660 (18.63) | 75.2152 (16.45) | 68.0945 (17.11) | 76.9089 (16.82) | 70.9157 (15.81) |

| PDF | Parameters | Station | | | | |
|---|---|---|---|---|---|---|
| | | Bogra | Dinajpur | Rajshahi | Rangpur | Syedpur |
| | Scale $(b)$ | 0.3337 (18.57) | 0.4003 (14.89) | 0.4579 (15.73) | 0.3853 (16.35) | 0.4228 (16.92) |
| GEV | Shape $(\alpha)$ | -0.3834 (-28.80) | -0.4926 (-32.65) | -0.4181 (-24.34) | -0.3859 (-27.64) | -0.6000 (-20.91) |
| | Scale $(\beta)$ | 3.3004 (32.14) | 3.5474 (31.48) | 3.8852 (30.76) | 3.2366 (29.87) | 3.6335 (20.23) |
| | Location $(\mu)$ | 29.7248 (197.14) | 29.2604 (182.04) | 30.1247 (169.22) | 29.4898 (174.38) | 29.3056 (126.96) |
| 3-P LN | Shape $(c)$ | 0.0132 (33.13) | 0.0120 (32.70) | 0.0148 (29.84) | 0.0116 (30.89) | 0.0125 (23.23) |
| | Scale $(a)$ | 5.4639 (201.43) | 5.6312 (181.60) | 5.5152 (187.25) | 5.6366 (183.26) | 5.6141 (867.80) |
| | Threshold $(b)$ | -205.344 (-31.03) | -248.8605 (-29.0) | -217.2884 (-34.32) | -250.8805 (-38.31) | -244.3162 (-138.64) |
| ST | Location $(\theta_0)$ | 33.8161 (27.83) | 33.6840 (25.62) | 35.4711 (38.45) | 33.0505 (32.092) | 33.7086 (218.24) |
| | Scale $(\theta_1)$ | 3.567 (11.16) | 4.0059 (18.75) | 5.6424 (16.14) | 3.5340 (12.10) | 4.214 (12.71) |
| | Degrees of Freedom $(\theta_2)$ | 5 (30.30) | 5 (28.61) | 8 (24.20) | 4 (4.52) | 6 (3.99) |
| | Shape $(\theta_3)$ | -2.863 (-14.32) | -4.5001 (-18.34) | -3.3804 (-17.89) | -5.3108 (-6.57) | -5.6037 (-5.49) |
| GSL | Location $(\mu)$ | 33.3431 (245.02) | 33.2729 (302.57) | 34.3006 (165.15) | 32.7778 (352.94) | 33.3237 (233.08) |
| | Scale $(\sigma)$ | 0.7494 (12.47) | 0.5594 (11.31) | 1.0337 (11.79) | 0.4611 (10.65) | 0.4789 (7.18) |
| | Shape $(\gamma)$ | 0.2556 (9.29) | 0.1689 (9.10) | 0.2922 (8.34) | 0.1420 (8.87) | 0.1389 (5.97) |

The statistical parameters for fitness evaluation of PDFs currently analyzed are presented in Table 3. Considering K-S error, $\chi^2$ error, RMSE, AIC and BIC we may conclude that the Weibull, lognormal, Gamma, GEV, three parameters lognormal and skewed $t$ have large errors indicating their inadequacy in modelling monthly maximum temperatures for Bogra station whereas the higher value of $R^2$ and the lower values of K-S error, RMSE, chi-square error, AIC and BIC indicate that Generalized Skewed Logistic (GSL) distribution is more accurate than other PDFs in modelling monthly maximum temperatures for Bogra station. This scenario is same for all other stations considered in this study.

*Table 3:* Values of Statistical tests for different distributions of Stations considered in this study.

| PDF / Error | W | LN | G | GEV | 3-P LN | ST | GSL |
|---|---|---|---|---|---|---|---|
| **Bogra Station** | | | | | | | |
| $K-S$ | 0.09592 | 0.16150 | 0.15550 | 0.33263 | 0.14570 | 0.07357 | 0.05900 |
| $R^2$ | 0.96591 | 0.90708 | 0.91379 | 0.50231 | 0.92440 | 0.98066 | 0.98654 |
| $\chi^2$ | 5.07758 | 10.81601 | 10.20680 | 1381362.0 | 9.26423 | 3.26608 | 2.73007 |

| PDF / Error | W | LN | G | GEV | 3-P LN | ST | GSL |
|---|---|---|---|---|---|---|---|
| $RMSE$ | 0.05429 | 0.09036 | 0.08712 | 0.24393 | 0.08166 | 0.04106 | 0.03421 |
| AIC | 0.00297 | 0.00823 | 0.00765 | 0.06017 | 0.00674 | 0.00171 | 0.00118 |
| BIC | 0.00302 | 0.00836 | 0.00777 | 0.06162 | 0.00691 | 0.00177 | 0.00121 |
| **Dinajpur Station** | | | | | | | |
| $K-S$ | 0.13921 | 0.20115 | 0.19621 | 0.14492 | 0.18235 | 0.10733 | 0.08186 |
| $R^2$ | 0.94901 | 0.88942 | 0.89631 | 0.94121 | 0.91100 | 0.97053 | 0.98157 |
| $\chi^2$ | 9.02180 | 12.24868 | 11.68875 | 5.89231 | 10.80346 | 4.26225 | 3.32672 |
| $RMSE$ | 0.07006 | 0.10030 | 0.09740 | 0.07055 | 0.09020 | 0.05199 | 0.04080 |
| AIC | 0.00494 | 0.01013 | 0.00956 | 0.00503 | 0.00823 | 0.00274 | 0.00168 |
| BIC | 0.00502 | 0.01030 | 0.00971 | 0.00515 | 0.00843 | 0.00283 | 0.00172 |
| **Rajshahi Station** | | | | | | | |
| $K-S$ | 0.09302 | 0.15895 | 0.15277 | 0.10929 | 0.14190 | 0.10733 | 0.06345 |
| $R^2$ | 0.97954 | 0.93299 | 0.93954 | 0.96929 | 0.94962 | 0.98259 | 0.99111 |
| $\chi^2$ | 3.86916 | 8.89460 | 8.24178 | 4.05093 | 7.23395 | 2.87067 | 2.41018 |
| $RMSE$ | 0.04245 | 0.07748 | 0.07364 | 0.05077 | 0.06727 | 0.03873 | 0.02789 |
| AIC | 0.00182 | 0.00605 | 0.00546 | 0.00261 | 0.00458 | 0.00152 | 0.00079 |
| BIC | 0.00184 | 0.00614 | 0.00555 | 0.00267 | 0.00469 | 0.00157 | 0.00081 |
| **Rangpur Station** | | | | | | | |
| $K-S$ | 0.14324 | 0.19404 | 0.18923 | 0.18345 | 0.18181 | 0.09625 | 0.07833 |
| $R^2$ | 0.94689 | 0.88829 | 0.89512 | 0.90172 | 0.90619 | 0.97640 | 0.98324 |
| $\chi^2$ | 8.67396 | 12.68014 | 11.51624 | 10.53855 | 10.54994 | 3.82442 | 3.15904 |
| $RMSE$ | 0.07160 | 0.10068 | 0.09791 | 0.09466 | 0.09307 | 0.04691 | 0.03918 |
| AIC | 0.00517 | 0.01021 | 0.00966 | 0.00906 | 0.00876 | 0.00223 | 0.00155 |
| BIC | 0.00525 | 0.01037 | 0.00981 | 0.00928 | 0.00897 | 0.00231 | 0.00159 |
| **Syedpur Station** | | | | | | | |
| $K-S$ | 0.13696 | 0.18644 | 0.18266 | 0.11835 | 0.18181 | 0.10396 | 0.06922 |
| $R^2$ | 0.95131 | 0.88682 | 0.89450 | 0.95889 | 0.90699 | 0.97454 | 0.98796 |
| $\chi^2$ | 5.14276 | 6.74517 | 6.39328 | 2.34975 | 5.85220 | 1.97933 | 1.31820 |
| $RMSE$ | 0.06896 | 0.10099 | 0.09779 | 0.05887 | 0.09224 | 0.04838 | 0.03290 |
| AIC | 0.00483 | 0.01035 | 0.00970 | 0.00354 | 0.00869 | 0.00241 | 0.00111 |
| BIC | 0.00495 | 0.01062 | 0.00996 | 0.00368 | 0.00904 | 0.00254 | 0.00115 |

The graphical comparisons of different probability distribution considered in this study and the histogram of the observed monthly maximum temperature of different station are presented in the following figures.

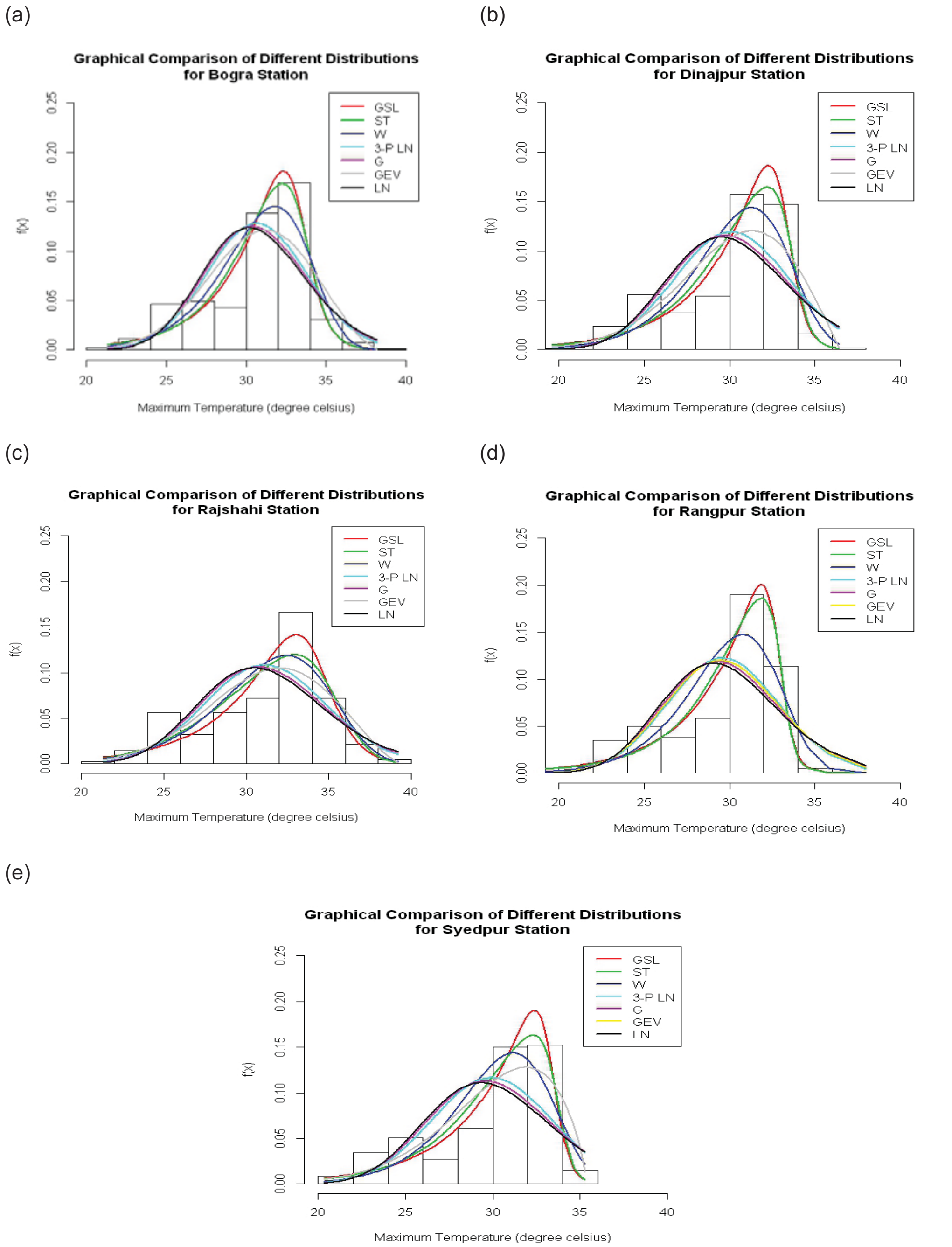


**Figure 1.** Graphical Comparison for different distributions of (a) Bogra (b) Dinajpur (c) Rajshahi (d) Rangpur (e) Syedpur station.

As seen from Figure 1 (a) to Figure 1 (e), Generalized Skewed Logistic (GSL) provided the best fit for the observed monthly maximum temperatures for all the stations considered in this study.

## 4. DISCUSSION AND CONCLUSION

The weather in Bangladesh is fluctuating and it is becoming more erratic in every year. The impacts of extreme temperatures, draughts, more inconstant rainfall, sea level rise and other extreme weather events are already observed in Bangladesh and will continue to exaggerate. It is observed that in Bangladesh, January is the coldest month whereas April is the hottest month for all stations considered in this study. From the results, we may conclude that the monthly maximum temperature fluctuated over the period of study. This paper firstly, estimate the parameters of all the probability distributions considered in this study with the help of maximum likelihood method. However, the Newton-Raphson method is used for iteration for convergence. In addition, considering K-S error, $\chi^2$ error, RMSE, AIC and BIC, we may conclude that the Weibull, Lognormal, Gamma, GEV, three parameters Lognormal and Skewed $t$ have large errors indicating their inadequacy in modelling monthly maximum temperatures for all stations whereas the higher value of $R^2$ and the lower values of K-S error, RMSE, chi-square error, AIC and BIC indicate that Generalized Skewed Logistic (GSL) distribution is more accurate than other PDFs in modelling monthly maximum temperatures for all stations considered in this study. Also, the graphical comparison of different distributions shows that the Generalized Skewed Logistic (GSL) provided the best fit for the observed monthly maximum temperatures for all the stations considered in this study. That is graphical comparison and most frequently used model selection criterion provide the same result. Therefore, Generalized Skewed Logistic distribution is the most appropriate to characterize the maximum temperature in the northern part of Bangladesh.

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