National College of Business Administration and Economics.

Sampling with Unequal Probabilities

BY

NADEEM SHAFIQUE BUTT

MASTER OF PHILOSOPHY

IN

STATISTICS

May, 2003

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A Dissertation Submitted to the
National College of Business Administration & Economics

In Partial Fulfillment of the
Requirements for the Degree of

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Director, Institute of Advanced Studies

Dissertation Committee:

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Chairman

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Member

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Member

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

IN THE NAME OF ALLAH, THE BENEFICIENT, THE MERCIFUL

Declaration

This is to certify that the research work I am submitting has not already been submitted and shall not in future be submitted for obtaining similar degree of any other university.

Nadeem Shafique Butt

20 – 05– 2003.

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Contents

Abbreviations

PPS	Probability Proportional to size sampling with replacement.
π PS	Probability Proportional to size sampling without replacement.
YG	Yates Grundy
d – b – d	Draw by Draw
proc	procedure
C.V	Co-efficient of Variation
ρ_{xy}	Correlation of x and y
$t_{symm\ (\mathrm{mod}\ )}$	Modified Murthy Estimator
$\mathrm{var}\left(t_{symm\ (\mathrm{mod}\ )}\right)$	Variance of Modified Murthy Estimator

CHAPTER 1

BASIC THEORY AND LITREATURE OF SURVEY

1.1 Introduction

Information is the key issue in society. Knowledge about the opinions in politics, consuming habits, preferences in sports and the arts, etc., are crucial for decision making and development in those areas. The theory of survey sampling offers tools and effective methods to obtain such information with a reliability that can be expressed and is based on data collection. In the frame of mathematical laws, a partial investigation of the finite population as whole. A firm basis for a kind of procedure may be achieved by probability sampling, which introduces an element of randomness into the sampling procedure. There are many probability sampling methods. But a good sampling scheme is one that is simply implemented, that leads to good estimation precision and that provides good variance estimation properties.

Sampling is the science and art of controlling and measuring reliability of useful statistical information through the theory of probability. It provides various types of statistical information of a quantitative or qualitative nature about the whole by examining a few units. The totality of aggregate, which favors the factors or characteristics under study, is called the *population. Sample* is a portion of population, which is selected using certain statistical tools with the view that it will exhibit the general characteristic of the population. Our knowledge, attitudes and actions are based to a very large extent on samples. The complete list of units from which we can draw our sample is called the *sampling frame*.

1.2 Probability Proportional to Size Sampling (PPS)

Probability proportional to size (PPS) sampling is an unequal probability sampling technique and is widely used in survey sampling. It is used to improve the efficiency of estimators by taking advantages of some known correlated auxiliary or supplementary information. Ideally, PPS sampling is done without replacement and numerous such procedures exists, Brewer and Hanif (1983) has listed some fifty PPS sampling methods. However, many of the procedures do not even extend beyond a sample of size two, and for those that do the calculation of the second order inclusion probabilities becomes steadily more complex and impracticable as the sample size increases. This creates a problem when estimating the efficiency of estimators because the variance involves second order inclusion probabilities, which differ depending on the selection method implemented.

1.3 Probability Proportional to Size Sampling With and Without Replacement

Probability proportional to size sampling with replacement was developed by Hansan and Hurwitz (1943). Prior to that, there had been a substantial development in sampling theory and practice but all these had been used on the assumption that the probability of selection within each stratum would be equal. Hansan and Hurwitz (1943) demonstrate the technique of unequal probability sampling via two-stage sampling. They selected the first stage units in usual way. The second stage units were selected with probabilities proportional to a size of measure.

This first suggestion for use of unequal probability sampling thus can be associated with the technique of multi-stage sampling with probability proportional to size. Unequal probability sampling can, however, be used in a single stage design and need not necessarily be with probability exactly proportional to size, though some sort of size of measure is almost always used as a starting point for assigning selection probabilities.

1.4 Hansen – Hurwitz Estimator

Hansen and Hurwitz (1943) proposed the idea of probability proportional to size sampling with replacement (ppswr). One unit was selected at each of n draws. They allocate the selection probability to ith unit of the population given by $p_i = \frac{Z_i}{Z}$ where Z_i is the measure of size for ith population unit and $Z = \sum_{i=1}^{N} Z_i$.

Hansen and Hurwitz (1943) proposed the following estimator for population total Y, denoted by y'_{HH}, for use with unequal probability sampling with replacement:

$$y'_{HH} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad (1.4.1)$$

Where y_i is measurement for the ith unit

The variance of this estimator is given by

$$Var(y'_{HH}) = \frac{1}{n}\left[\sum_{i=1}^{N}\frac{Y_i}{P_i} - Y^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{N} P_i\left(\frac{Y_i}{P_i} - Y\right)^2 \qquad (1.4.2)$$

$$= \frac{1}{2n}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} P_i P_j\left(\frac{Y_i}{P_i} - \frac{Y_j}{P_j}\right)^2 \qquad (1.4.3)$$

$$= \sum_{i=1}^{N}\frac{1}{P_i}\left(Y_i - P_i Y_i\right)^2$$

Formula (1.4.3) is suitable for algebraic manipulations while formula (1.4.2) is used for numerical comparison.

In probability Proportional to size sampling without replacement the units are selected with unequal probabilities at each draw and the units already selected are removed from the population before the next draw. A lot of work has been done in the field of sampling with unequal probabilities without replacement but the search for an optimum selection method and estimator is still continued. Brewer and Hanif (1983) and Chaudhry and Vos (1986) and Shahbaz (2002) have given a comprehensive bibliography of selection procedures and estimators used in unequal probability sampling.

1.5 Horvitz–Thompson Estimator in Unequal Probability Sampling without Replacement

Horvitz and Thompson (1952) were the first to provide a complete theoretical framework for unequal probability sampling without replacement. They suggested the following estimator of population total for use with unequal probability sampling without replacement:

$$y'_{HT} = \sum_{i \in S} \frac{Y_i}{\pi_i} \qquad (1.5.1)$$

where Y_i is the measurement for the ith unit

π_i is the probability that the ith unit is in the sample

π_{ij} is the probability that the ith and jth units are both in the sample

Horvitz and Thompson (1952) developed the following variance formula for y'_{HT}:

$$\mathrm{Var}_{HT}(y'_{HT}) = \sum_{i=1}^{N} \frac{1-\pi_i}{\pi_i} Y_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_{ij}} Y_i Y_j \qquad (1.5.2)$$

An alternative expression for the variance of y'_{HT}, developed independently by Sen (1953) and by Yates and Grundy (1953), is given by

$$\mathrm{Var}_{SYG}\left(y'_{HT}\right)=\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\pi_i\pi-\pi_{ij}\right)\left(\frac{Y_i}{\pi_i}-\frac{Y_j}{\pi_j}\right)^2 \qquad (1.5.3)$$

Horvitz and Thompson derived following unbiased estimator of

$$\hat{V}ar_{HT}\left(y'_{HT}\right)=\sum_{i=1}^{n}\frac{1-\pi_i}{\pi_i}Y_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}Y_iY_j \qquad (1.5.4)$$

This variance estimator suffers from the drawback that it may assume negative value for some samples. Therefore this estimator is not admissible for true variance of y'_{HT}.

Another estimator of variance of y'_{HT} proposed by Sen (1953) and independently by Yates and Grundy (1953) is given by

$$\mathrm{var}_{SYG}\left(y'_{HT}\right)=\frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\right)\left(\frac{Y_i}{\pi_i}-\frac{Y_j}{\pi_j}\right)^2 \qquad (1.5.5)$$

Hanif et al (1993) has given some other estimators for variance of Horvitz-Thompson estimator.

1.6 Murthy Estimator

In this method the successive units are drawn with probabilities $p_i, \frac{p_j}{(1-p_i)}, \frac{p_k}{(1-p_i-p_j)}$, and so on. Murthy (1957), estimator following the work of Des Raj (1956), produces ingenious unbiased estimates based on the specific order in which the n units in the sample were drawn. He showed that corresponding to any ordered estimate of this class we can construct an unordered estimate that is also unbiased and has smaller variance.

His proposed estimator is

$$t_{symm} = \frac{\sum_{i=1}^{n} P(s|\ i)y_i}{P(s)} \qquad (1.6.1)$$

Where

$P(s|i)$ = conditional probability of getting the set of units that was drawn given that the ith unit was drawn first

P(s) = unconditional probability of getting the set of units that was drawn

He has shown that the estimate t_{symm} is unbiased. For any unit i in the population, $\sum P(s|i) = 1$, where sum is taken over all samples having unit idrawn first. For n=2 the estimator is given as

$$t_{symm} = \frac{1}{2-p_i-p_j}\left[(1-p_j)\frac{y_i}{p_i}+(1-p_i)\frac{y_i}{p_j}\right]$$

(1.6.2)

with variance

$$V(t_{symm}) = \sum_{i=1}^{N}\sum_{j>i}^{N}\frac{p_i p_j (1-p_i-p_j)}{2-p_i-p_j}\left(\frac{y_i}{p_i}-\frac{y_j}{p_j}\right)^2$$

(1.6.3)

$V(t_{symm})$ its average is Σ P (S) $V(t_{symm})$, an unbiased sample estimate of variance for n=2 is

$$V(t_{symm}) = \frac{(1-p_i)(1-p_j)(1-p_i-p_j)}{(2-p_i-p_j)^2}\left(\frac{y_i}{p_i}-\frac{y_j}{p_j}\right)^2 \quad (1.6.4)$$

Pathak (1967) has given following formula for $V(t_{symm})$, for sample of size n:

$$Var(t_{symm}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} P_i P_j\left[1-\sum_{S}\frac{P(s|i)P(s|j)}{P(s)}\right]\left(\frac{Y_i}{P_i}-\frac{Y_j}{P_j}\right)^2 \quad (1.6.5)$$

1.7 Modification of Murthy Estimator

Following the procedure of Murthy under brewer selection procedure Shahbaz and Hanif (2003) develop new estimator for the estimation of population total in unequal probability sampling without replacement. Murthy used the idea of unordered estimators and proposed the estimator:

$$t_{symm} = \frac{\sum_{i=1}^{n} P(s|\ i) y_i}{P(s)} \qquad (1.7.1)$$

Shahbaz and Hanif (2002) has obtain following estimator

$$t_{symm} = \frac{b(1-2p_i)(1-2p_j)\left[\frac{y_i}{p_i}(1-p_j)+\frac{y_i}{p_i}(1-p_i)\right]}{4(1-p_i-p_j)(1-p_i)(1-p_j)} \qquad (1.7.2)$$

where

$$b = 1+\sum_{i=1}^{N}\frac{p_i}{1-2p_i}$$

Further, they have also obtained the design based variance of above estimator given as:

$$Var\left(t_{MM(1)}\right)=\frac{1}{2}\sum\sum\frac{p_i p_j}{4\left(1-p_i-p_j\right)}\left[\frac{y_i^{\ 2}}{p_i^{\ 2}}Aij+\frac{y_j^{\ 2}}{p_j^{\ 2}}B_{ij}-2\frac{y_i y_j}{p_i p_j}C_{ij}\right]$$

(1.7.3)

$$A_{ij}=\frac{b\left(1-2p_i\right)\left(1-2p_j\right)}{\left(1-p_i\right)^2}-\frac{4p_i\left(1-p_i-p_j\right)}{\left(1-p_i\right)} \qquad (1.7.4)$$

$$B_{ij}=\frac{b\left(1-2p_i\right)\left(1-2p_j\right)}{\left(1-p_j\right)^2}-\frac{4p_j\left(1-p_i-p_j\right)}{\left(1-p_j\right)} \qquad (1.7.5)$$

$$C_{ij}=4\left(1-p_i-p_j\right)-\frac{b\left(1-2p_i\right)\left(1-2p_j\right)}{\left(1-p_i\right)\left(1-p_j\right)} \qquad (1.7.5)$$

and have compared this estimator with a number of other estimator of population toaal in Unequal Probability Sampling Without Replacement.

1.8 Literature Survey

A lot of work on unequal probability sampling estimator has been done since early forties. Hansen and Horwitz (1943) were the first to introduce the idea of unequal probability sampling estimators.

Hansen and Horwitz (1943) showed that, for a finite population the use of varying probabilities for selecting the sample element is generally more efficient than selection with equal probability. Their sampling scheme was, however, confined to the selection of a single primary sampling unit (psu) from a stratum.

Midzuno (1952), Sen (1952a, 1952b) generalized the Hansen and Hurwitz (1943) scheme to sampling a combination of n elements from a stratum with probability proportional to size (pps) of the combination. Sen (1952c) further generalized the scheme for obtaining an unbiased estimate of the population total when the first r units are selected with pps and the remaining n-r units are selected with equal probability and without replacement. He also derived expression for an estimate of the variance of the estimate. Horvitz and Thomson (1952) presented another technique for dealing with the problem of selecting n p.s.u.'s without replacement and with varying probabilities from a finite population. They also presented the formula for obtaining unbiased estimate of the population total as well as of the variance of the estimate.

Sen (1954) derived expression for the unbiased estimate of the variance referred by Sen (1953) and showed that this estimate was always positive. He also gave a biased estimator for the sampling variance, which was more efficient than Horvitz and Thompson's (1952) unbiased estimate of the variance. Hartley and Rao (1962) derived expression for the variance of the estimates of the population total together with variance estimates for the selection procedure. The N units in the population are listed in a random order and their x_i are cumulated and a systematic selection of n elements from a random start is then made on the cumulation. Their formulae were applicable for moderate *N*. They also demonstrated that variance of the estimate was smaller as compared to sampling with pps with replacement. Raj (1964) showed that for g=1 pps sampling was superior to equal probability sampling and only slightly better than pps sampling. Pathak and Shukla (1966) investigated the non-negativity of the estimate of variance of Murthy (1957) estimator for n ≥2 and proved that in fact it is non-negative for n ≥2.

Rao (1972) showed that the generalized π ps sampling strategy consisting of the design with π_i, the probability of inclusion of the ith unit in the sample, proportional to the modified size together with the corresponding Horvitz-Thompson estimator, is superior to the symmeterized Des Raj strategy under a general super- population set up for all values of the super-population parameter, when the samples are of size two.

Hanif and Brewer (1979) presented a general theory of sampling with unequal probability, which allowed population units to appear more than once in sample. They presented two estimators for use in both single stage and multi-stage sample design.

Hanif and Brewer (1980) reviewed sampling with unequal probabilities without replacement and listed a number of selection procedures along with their properties. They also presented an approximate formula for the estimation of variance, which does not involve π_{ij}. Brewer, Hanif and Tam (1988) compared two general linear model based predicators, one of the expectations of a finite total and one of hat total itself, with the design-based generalized regression estimator (GRE). They showed that the choice of inclusion probabilities is more important asymptotically than the choice of estimator for the regression parameters. Further, if there is only one explanatory variable and no constant term in the model, the predictor of the expectation of the finite population total obtained by first procedure is identical to the Horvitz-Thompson (1952) ratio estimator. If the finite population total itself is to be predicated, the estimator is that suggested by Brewer (1979). Sahoo and Sahoo (1990) presented a new class of estimators for a finite population total with the aid of two auxiliary variables in a two-stage sampling. Samiuddin et al (1992) introduced three different classes of estimators and investigated their properties. They introduced the optimal class of estimators (given in design). Hanif et al (1993) proposed a new model-based variance estimator as special case of Hanif and Brewer (1980) and Kumar – Gupta - Agarwal (1985) model - based variance estimators. They showed empirically that the amount of bias was fairly small for all values of g and that it was more stable than all other estimators considered.

Hanif et al (1994) investigated the small sample performance of the revised ratio estimator introduced by Samiuddin et al (1992). They showed that this estimator was stable than the usual ratio estimator in small samples, when compared with other well-known estimators of the population total. Chaudhuri (1996) derived unbiased estimator of a population total along with a variance estimator formula.

Shahbaz, and Hanif (2003) has given some modification of Murthy Estimator by using Brewer (1963) selection procedure and by using another procedure developed by Shahbaz et al (2003)

CHAPTER 2

MODIFIED MURTHY ESTIMATOR

2.1 Introduction:

In this chapter modified Murthy estimator of the population total has been developed by using the estimator of Murthy (1957) defined as:

$$t_{symm} = \frac{1}{p(s)} \sum_{i=1}^{n} p(s|i)\, y_i \qquad (2.1.1)$$

Murthy (1957) used the Yates – Grundy (1953) d - b - d procedure to obtain following estimator of the population total for sample of size 2

$$t_{symm} = \frac{\left[\frac{y_i}{p_i}(1-p_j) + \frac{y_j}{p_j}(1-p_i)\right]}{(2-p_i-p_j)} \qquad (2.1.2)$$

Modified Murthy estimator has been developed by using the general selection procedure given by Shahbaz and Hanif (2003). The general selection procedure is stated as:

- Select first unit with probability proportional to $\frac{a\, p_i (1-p_i)}{1-2a\, p_i}$
- Select second unit with probability proportional to size of the remaining units.

Modified Murthy Estimator found to be unbiased and two special cases has be found under this estimator.

2.2 Modified Murthy Estimator:

Murthy Estimator for sample of size 2 is defined as:

$$t_{symm} = \frac{P(s|i)y_i + P(s|j)y_j}{P(s)} \qquad (2.2.1)$$

and the expression of p(s) given by Shahbaz and Hanif (2003) for their general selection procedure is:

$$p(s) = \frac{1}{\frac{k}{2}}\left[\frac{a\,p_i\,p_j}{1-2a\,p_i} + \frac{a\,p_i\,p_j}{1-2a\,p_j}\right] \qquad for \;\; a > 0.5 \; and \;\; p_i < \frac{1}{2a} \qquad (2.2.2)$$

with

$$k = 1 + (2a-1)\sum_{i=1}^{N}\frac{p_i}{1-2a\,p_i}$$

$$= \frac{2a\,p_i\,p_j}{k}\left[\frac{1}{1-2a\,p_i} + \frac{1}{1-2a\,p_j}\right]$$

$$= \frac{2a\,p_i\,p_j}{1+(2a-1)\sum_{i=1}^{N}\frac{p_i}{1-2a\,p_i}}\left[\frac{1}{1-2a\,p_i} + \frac{1}{1-2a\,p_j}\right]$$

By using (2.2.2) in (2.2.1)

$$t_{symm(\mathrm{mod})} = \frac{\frac{p_j}{1-p_i}y_i + \frac{p_i}{1-p_j}y_j}{\frac{2ap_i\,p_j}{k}\left[\frac{1}{1-2ap_i} + \frac{1}{1-2ap_j}\right]}$$

$$t_{symm(\mathrm{mod})} = \frac{k(1-2ap_i)(1-2ap_j)}{4a(1-ap_i-ap_j)}\left[\frac{y_i}{p_i(1-p_i)} + \frac{y_j}{p_j(1-p_j)}\right] \qquad (2.2.3)$$

2.3 Unbiasedness of the Modified Murthy Estimator:

In this section the Unbiasedness of the Modified Estimator has been proved.

Now consider

$$E\left(t_{symm}\ (\text{mod}\ )\right) = \sum_{\substack{i=1 \\ J>i}}^{N} \sum_{J=1}^{N} t_{symm}\ (\text{mod}\ )\, p\,(s)$$

$$= \frac{1}{2}\left[\sum_{j \neq i}^{N} \sum \frac{\left\{ \frac{P_j}{1-P_i} Y_i + \frac{P_i}{1-P_j} Y_j \right\}}{\frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right]} \frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right] \right]$$

$$= \frac{1}{2}\left[\sum_{j \neq i} \sum \frac{\left\{ \frac{P_j}{1-P_i} Y_i + \frac{P_i}{1-P_j} Y_j \right\}}{\frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right]} \frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right] \right]$$

$$= \frac{1}{2}\left[\sum_{j \neq i} \sum \frac{\left\{ \frac{P_j}{1-P_i} Y_i + \frac{P_i}{1-P_j} Y_j \right\}}{\frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right]} \frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right] \right]$$

$$= \frac{1}{2}\left[\sum_{j \neq i} \sum \frac{\left\{ \frac{P_j}{1-P_i} Y_i + \frac{P_i}{1-P_j} Y_j \right\}}{\frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right]} \frac{2aP_iP_j}{K}\left[\frac{1}{1-2aP_i} + \frac{1}{1-2aP_j}\right] \right]$$

$$= \frac{1}{2}\left[\sum_{i=1}^{N} \frac{Y_i}{1-P_i}\left(1-P_i\right) + \sum_{i=1}^{N} \frac{Y_j}{1-P_j}\left(1-P_j\right) \right]$$

$$= \frac{1}{2}\left[Y + Y\right]$$

$$= \frac{1}{2}[2Y]$$

$$E\left(t_{symm(\mathrm{mod})}\right) = Y \qquad (2.3.1)$$

From the equation (2.3.1) it can be seen that the Modified Murthy Estimator is an unbiased estimator of the population total.

2.4 Variance of the Modified Murthy Estimator:

In this section the variance of the Modified Murthy Estimator has been developed

$$\mathrm{var}\left(t_{symm\ (\mathrm{mod}\ )}\right) = E\left(t_{symm\ (\mathrm{mod}\ )}\right)^2 - \left[E\left(t_{symm\ (\mathrm{mod}\ )}\right)\right]^2$$

$$= \frac{1}{2}\sum_{j\neq i}^{N}\sum\left[\frac{\frac{P_j}{1-P_i}Y_i + \frac{P_i}{1-P_j}Y_j}{\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]}\right]^2\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right] - Y^2$$

$$= \frac{1}{2}\sum_{j\neq i}^{N}\sum\frac{\left(\frac{P_j}{1-P_i}Y_i + \frac{P_i}{1-P_j}Y_j\right)^2}{\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - \left[\sum_{i=1}^{N}Y_i\right]^2$$

$$= \frac{1}{2}\sum_{j\neq i}^{N}\sum\frac{\left(\frac{P_j}{1-P_i}Y_i + \frac{P_i}{1-P_j}Y_j\right)^2}{\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - \sum_{i=1}^{N}Y_i^{\ 2} - \sum_{j\neq i}^{N}\sum Y_iY_j$$

$$= \frac{1}{2}\left[\sum_{j\neq i}^{N}\sum\frac{\left(\frac{P_j}{1-P_i}Y_i + \frac{P_i}{1-P_j}Y_j\right)^2}{\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - 2\sum_{i=1}^{N}Y_i^{\ 2} - 2\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{1}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_i^{\,2} P_j^{\,2}\left(\frac{1}{P_i(1-P_i)}Y_i + \frac{1}{P_j(1-P_j)}Y_j\right)^2}{\frac{2aP_iP_j}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - 2\sum_{i=1}^{N} Y_i^{\;2} - 2\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{1}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_iP_j\left(\frac{Y_i^{\,2}}{P_i^{\,2}(1-P_i)_2} + \frac{Y_j^{\,2}}{P_j^{\,2}(1-P_i)^2} + 2\frac{Y_i}{Pi(1-Pi)}\frac{Y_j}{P_j(1-P_i)}\right)^2}{\frac{2a}{k}\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - 2\sum_{i=1}^{N} Y_i^{\;2} - 2\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{1}{2}\left[\sum_{j\neq i}^{N}\sum \frac{kP_iP_j\left(\frac{Y_i^{\,2}}{P_i^{\,2}(1-P_i)_2} + \frac{Y_j^{\,2}}{P_j^{\,2}(1-P_i)^2} + 2\frac{Y_i}{Pi(1-Pi)}\frac{Y_j}{P_j(1-P_i)}\right)^2}{2a\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - 2\sum_{i=1}^{N} Y_i^{\;2} - 2\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{k}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_iP_j\left(\frac{Y_i^{2}}{P_i^{2}(1-P_i)^2} + \frac{Y_j^{\,2}}{P_j^{\,2}(1-P_i)^2} + 2\frac{Y_i}{Pi(1-Pi)}\frac{Y_j}{P_j(1-P_i)}\right)^2}{2a\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - \frac{1}{k}\sum_{i=1}^{N} Y_i^{\;2} - \frac{1}{k}\sum_{i=1}^{N} Y_j^{\;2} - \frac{2}{k}\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{k}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_iP_j\left(\frac{Y_i^{2}}{P_i^{2}(1-P_i)_2} + \frac{Y_j^{\,2}}{P_j^{\,2}(1-P_i)^2} + 2\frac{Y_i}{Pi(1-Pi)}\frac{Y_j}{P_j(1-P_i)}\right)^2}{2a\left[\frac{1}{1-2aP_i}+\frac{1}{1-2aP_j}\right]} - \frac{1}{k}\sum_{j\neq i}^{N}\sum Y_i\frac{P_j}{1-P_i} - \frac{1}{k}\sum_{j\neq i}^{N}\sum Y_j\frac{P_i}{1-P_j} - \frac{2}{k}\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{k}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_iP_j\left(\frac{Y_i^{2}}{P_i^{2}(1-P_i)_2} + \frac{Y_j^{2}}{P_j^{2}(1-P_i)^2} + 2\frac{Y_i}{Pi(1-Pi)}\frac{Y_j}{P_j(1-P_i)}\right)^2}{2a\left[\frac{2(1-aP_i-aP_j)}{(1-2aP_i)(1-2aP_j)}\right]} - \frac{1}{k}\sum_{j\neq i}^{N}\sum Y_i\frac{P_j}{1-P_i} - \frac{1}{k}\sum_{j\neq i}^{N}\sum Y_j\frac{P_i}{1-P_j} - \frac{2}{k}\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{k}{2}\left[\sum_{j\neq i}^{N}\sum \frac{P_iP_j(1-2aP_i)(1-2aP_j)}{4a(1-aP_i-aP_j)}\left\{\frac{Y_i^2}{P_i^2(1-P_i^2)}+\frac{Y_j^2}{P_j^2(1-P_j^2)}+\frac{2Y_iY_j}{P_iP_j(1-P_i)(1-P_j)}\right\}\right.$$

$$\left. -\frac{1}{k}\sum_{j\neq i}^{N}\sum Y_i\frac{P_j}{1-P_i}-\frac{1}{k}\sum_{j\neq i}^{N}\sum Y_j\frac{P_i}{1-P_j}-\frac{2}{k}\sum_{j\neq i}^{N}\sum Y_iY_j\right]$$

$$= \frac{k}{8}\sum_{j\neq i}^{N}\sum \frac{P_iP_j(1-2aP_i)(1-2aP_j)}{a(1-aP_i-aP_j)}\left[\left\{\frac{Y_i^2}{P_i^2(1-P_i^2)}+\frac{Y_j^2}{P_j^2(1-P_j^2)}+\frac{2Y_iY_j}{P_iP_j(1-P_i)(1-P_j)}\right\}\right.$$

$$\left. -\frac{4a}{k}\frac{Y_i^2(1-aP_i-aP_j)}{P_i(1-P_i)(1-2aP_i)(1-2aP_j)}-\frac{4a}{k}\frac{Y_j^2(1-aP_i-aP_j)}{P_j(1-P_j)(1-2aP_i)(1-2aP_j)}-\frac{8a}{k}\frac{Y_iY_j(1-aP_i-aP_j)}{P_iP_j(1-2aP_i)(1-2aP_j)}\right]$$

$$= \frac{k}{8}\sum_{j\neq i}^{N}\sum \frac{P_iP_j(1-2aP_i)(1-2aP_j)}{a(1-aP_i-aP_j)}\left[\frac{Y_i^2}{P_i^2}\left\{\frac{1}{(1-P_i)^2}-\frac{4aP_i(1-aP_i-aP_j)}{k(1-P_i)(1-2aP_i)(1-2aP_j)}\right\}\right.$$

$$\left. +\frac{Y_j^2}{P_j^2}\left\{\frac{1}{(1-P_j)^2}-\frac{4aP_i(1-aP_i-aP_j)}{k(1-P_j)(1-2aP_i)(1-2aP_j)}\right\}-\frac{2Y_iY_j}{P_iP_j}\left\{-\frac{1}{(1-P_i)(1-P_j)}+\frac{4a(1-aP_i-aP_j)}{k(1-2aP_i)(1-2aP_j)}\right\}\right]$$

$$\text{var}(\ t_{symm}\ (\text{mod}\ )) = \frac{k}{8}\sum_{j\neq i}^{N}\sum D_{ij}\left[A_{ij}\frac{Y_i^2}{P_i^2}+B_{ij}\frac{Y_j^2}{P_j^2}-C_{ij}\frac{2Y_iY_j}{P_iP_j}\right] \qquad (2.4.1)$$

where

$$A_{ij} = \left\{\frac{1}{(1-P_i)^2}-\frac{4aP_i(1-aP_i-aP_j)}{k(1-P_i)(1-2aP_i)(1-2aP_j)}\right\} \qquad (2.4.2)$$

$$B_{ij} = \left\{\frac{1}{(1-P_j)^2}-\frac{4aP_j(1-aP_i-ap_j)}{k(1-P_j)(1-2aP_i)(1-2aP_j)}\right\} \qquad (2.4.3)$$

$$C_{ij} = \left\{ -\frac{1}{(1-P_i)(1-P_j)} + \frac{4a(1-aP_i-aP_j)}{k(1-2aP_i)(1-2aP_j)} \right\} \quad (2.4.4)$$

$$D_{ij} = \frac{P_i P_j (1-2aP_i)(1-2aP_j)}{a(1-aP_i-aP_j)} \quad (2.4.5)$$

2.5 Special Cases for $t_{symm(\mathrm{mod})}$:

In this section two special cases of $t_{symm(\mathrm{mod})}$ have been developed.

$$t_{symm} = \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2ap_i p_j}{1+(2a-1)\sum_{i=1}^{N} \frac{p_i}{1-2a\,p_i}} \left[\frac{1}{1-2ap_i} + \frac{1}{1-2ap_j} \right]}$$

2.5.1 Special Case - I

For a=0.5

$$t_{symm} = \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2\frac{1}{2} p_i p_j}{1+\left(2\frac{1}{2}-1\right)\sum_{i=1}^{N} \frac{p_i}{1-2\frac{1}{2}p_i}} \left[\frac{1}{1-2\frac{1}{2}p_i} + \frac{1}{1-2\frac{1}{2}p_j} \right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{p_i p_j \left[\frac{1}{1-p_i} + \frac{1}{1-p_j}\right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{p_i p_j [2-pi-pj]}{(1-p_i).(1-p_j)}}$$

$$= \frac{pipj\left[\frac{y_i}{p_i(1-p_i)} + \frac{y_j}{p_j(1-p_j)}\right]}{\frac{p_i p_j [2-pi-pj]}{(1-p_i).(1-p_j)}}$$

$$t_{symm} = \frac{\left[\frac{y_i}{p_i}(1-p_j) + \frac{y_j}{p_j}(1-p_i)\right]}{(2-p_i-p_j)}$$

From the above result it can be seen that at a=0.5 $t_{symm(\mathrm{mod})}$ reduces to the t_{symm} given by Murthy(1957).

2.5.2 Special Case - II

For a =1

$$t_{symm} = \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2ap_i p_j}{1+(2a-1)\sum_{i=1}^{N} \frac{p_i}{1-2a\,p_i}} \left[\frac{1}{1-2ap_i} + \frac{1}{1-2ap_j}\right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2p_i p_j}{1+(2-1)\sum_{i=1}^{N} \frac{p_i}{1-2\,p_i}} \left[\frac{1}{1-2p_i} + \frac{1}{1-2p_j}\right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2 p_i p_j}{1+\sum_{i=1}^{N} \frac{p_i}{1-2\ p_i}} \left[\frac{1}{1-2p_i} + \frac{1}{1-2p_j} \right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2 p_i p_j}{1+\sum_{i=1}^{N} \frac{p_i}{1-2\ p_i}} \left[\frac{2-2p_i-2p_j}{(1-2p_i)(1-2p_j)} \right]}$$

$$= \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{4 p_i p_j}{1+\sum_{i=1}^{N} \frac{p_i}{1-2\ p_i}} \left[\frac{1-p_i-p_j}{(1-2p_i)(1-2p_j)} \right]}$$

$$= \frac{p_i p_j [\frac{y_i}{p_i(1-p_i)} + \frac{y_j}{p_j(1-p_j)})}{\frac{4 p_i p_j}{1+\sum_{i=1}^{N} \frac{p_i}{1-2\ p_i}} \left[\frac{1-p_i-p_j}{(1-2p_i)(1-2p_j)} \right]}$$

consider

$$b = 1+\sum_{i=1}^{N} \frac{p_i}{1-2\ p_i}$$

$$t_{symm} = \frac{b(1-2p_i)(1-2p_j)\left[\frac{y_i}{p_i}(1-p_j) + \frac{y_i}{p_i}(1-p_i) \right]}{4(1-p_i-p_j)(1-p_i)(1-p_j)}$$

From the above result it can be seen that at a=1 $t_{symm(\text{mod})}$ reduces to the t_{symm} given by Shahbaz and Hanif (2003).

2.6 Verification of the results:

2.6.1 Verification of the Modified Murthy Estimator

The values of $t_{symm(\text{mod})}$ reduces to the standard results of simple random sampling for $p_i = p_j = \frac{1}{N}$.

Proof : Consider the expression of

$$t_{symm(\text{mod})} = \frac{\frac{p_j}{1-p_i} y_i + \frac{p_i}{1-p_j} y_j}{\frac{2ap_i p_j}{1+(2a-1)\sum_{i=1}^{N} \frac{p_i}{1-2a\,p_i}} \left[\frac{1}{1-2ap_i} + \frac{1}{1-2ap_j}\right]}$$

substituting $p_i = p_j = \frac{1}{N}$ in above equation:

$$= \frac{\frac{\frac{1}{N}}{1-\frac{1}{N}} y_i + \frac{\frac{1}{N}}{1-\frac{1}{N}} y_j}{\frac{2a\frac{1}{N}\frac{1}{N}}{1+(2a-1)\sum_{i=1}^{N} \frac{\frac{1}{N}}{1-2a\frac{1}{N}}} \left[\frac{1}{1-2a\frac{1}{N}} + \frac{1}{1-2a\frac{1}{N}}\right]}$$

consider

$$k = 1+(2a-1)\sum_{i=1}^{N} \frac{1/N}{1-2a/N}$$

$$= 1+(2a-1)\sum_{i=1}^{N}\frac{1}{N-2a}$$

$$= 1+\frac{(2a-1)N}{N-2a}$$

$$= \frac{2a(N-1)}{N-2a}$$

$$= \frac{\dfrac{\frac{1}{N}}{1-\frac{1}{N}}y_i + \dfrac{\frac{1}{N}}{1-\frac{1}{N}}y_j}{\dfrac{2a\frac{1}{N}\frac{1}{N}}{\frac{2a(N-1)}{N-2a}}\left[\dfrac{1}{1-2a\frac{1}{N}} + \dfrac{1}{1-2a\frac{1}{N}}\right]}$$

$$= \frac{\dfrac{y_i}{N-1}+\dfrac{y_j}{N-1}}{\dfrac{2\frac{a}{N^2}}{\frac{2a(N-1)}{N-2a}}\left[\dfrac{2N}{N-2a}\right]}$$

$$= \frac{\dfrac{y_i+y_j}{N-1}}{\dfrac{2a(N-2a)}{2aN^2(N-1)}\left[\dfrac{2N}{N-2a}\right]}$$

$$= N\left[\frac{y_i+y_j}{2}\right]$$

$$t_{symm(\mathrm{mod})} = N\,\bar{y} \qquad (2.6.1.1)$$

From the equation (2.6.1.1) it can be seen that the Modified Murthy Estimator reduce to the estimator for the population total of Simple Random Sampling for $p_i = p_j = \frac{1}{N}$.

2.6.2 Verification of the Variance Formula of Modified Murthy Estimator

The values of $\mathrm{var}\left(t_{symm(\mathrm{mod})}\right)$ reduces to the standard results of simple random sampling for $p_i = p_j = \frac{1}{N}$.

Proof :

By using equation (2.4.1)

$$\mathrm{var}(t_{symm(\mathrm{mod})}) = \frac{k}{8}\sum_{j \neq i}^{N}\sum D_{ij}\left[A_{ij}\frac{Y_i^2}{P_i^2} + B_{ij}\frac{Y_j^2}{P_j^2} - C_{ij}\frac{2Y_iY_j}{P_iP_j}\right]$$

$$put \;\; p_i = p_j = \frac{1}{N}$$

$$k = \sum_{i=1}^{N}\frac{ap_i(1-p_i)}{1-2p_i}$$

$$= \sum_{i=1}^{N}\frac{a\frac{1}{N}(1-\frac{1}{N})}{1-2\frac{1}{N}}$$

$$= \frac{2a(N-1)}{(N-2a)}$$

Consider equation (2.4.2)

$$A_{ij} = \left\{\frac{1}{\left(1-\frac{1}{N}\right)^2} - \frac{4a\frac{1}{N}\left(1-a\frac{1}{N}-a\frac{1}{N}\right)}{k\left(1-\frac{1}{N}\right)\left(1-2a\frac{1}{N}\right)\left(1-2a\frac{1}{N}\right)}\right\}$$

$$A_{ij} = \left\{\frac{N^2}{(N-1)^2} - \frac{4aN^2}{kN(N-1)(N-2a)}\right\}$$

$$= \left\{ \frac{N^2}{(N-1)^2} - \frac{4aN^2}{\frac{2a(N-1)}{(N-2a)} N(N-1)(N-2a)} \right\}$$

$$A_{ij} = \frac{N(N-2)}{(N-1)^2} = B_{ij}$$

Consider equation (2.4.4)

$$= \left\{ -\frac{1}{\left(1-\frac{1}{N}\right)\left(1-\frac{1}{N}\right)} + \frac{4a\left(1-a\frac{1}{N}-a\frac{1}{N}\right)}{k\left(1-2a\frac{1}{N}\right)\left(1-2a\frac{1}{N}\right)} \right\}$$

$$= \left\{ -\frac{1}{\left(1-\frac{1}{N}\right)\left(1-\frac{1}{N}\right)} + \frac{4a\left(1-a\frac{1}{N}-a\frac{1}{N}\right)}{\frac{2a(N-1)}{(N-2a)}\left(1-2a\frac{1}{N}\right)\left(1-2a\frac{1}{N}\right)} \right\}$$

$$= \frac{2N^2 - 2N - N^2}{(N-1)^2}$$

$$C_{ij} = \frac{N(N-2)}{(N-1)^2}$$

Consider equation (2.4.5)

$$D_{ij} = \frac{\frac{1}{N}\frac{1}{N}\left(1-2a\frac{1}{N}\right)\left(1-2a\frac{1}{N}\right)}{a\left(1-a\frac{1}{N}-a\frac{1}{N}\right)}$$

$$= \frac{\frac{1}{N}\frac{1}{N}\left(1-2a\frac{1}{N}\right)}{a}$$

$$D_{ij} = \frac{(N-2a)}{aN^3}$$

$$\text{var}(t_{symm(\text{mod})}) = \frac{k}{8}\sum_{j\neq i}^{N}\sum \frac{N-2a}{aN^3}\left[\frac{N(N-2)}{(N-1)^2}\frac{y_i^{\,2}}{\left(\frac{1}{N}\right)^2} + \frac{N(N-2)}{(N-1)^2}\frac{y_j^{\,2}}{\left(\frac{1}{N}\right)^2} - \frac{N(N-2)}{(N-1)^2}\frac{y_i^{\,2}}{\left(\frac{1}{N}\right)\left(\frac{1}{N}\right)}2y_i y_j\right]$$

$$= \frac{k}{8}\sum_{j\neq i}^{N}\sum \frac{N-2a}{aN^3}\frac{N^3(N-2)}{(N-1)^2}\left[y_i^{\,2} + y_j^{\,2} - 2y_i y_j\right]$$

$$= \frac{k}{8}\frac{N-2a}{aN^3}\frac{N^3(N-2)}{(N-1)^2}\sum_{j\neq i}^{N}\sum (y_i - y_j)^2$$

$$= \frac{\frac{2a(N-1)}{N-2a}}{8}\frac{N-2a}{aN^3}\frac{N^3(N-2)}{(N-1)^2}\sum_{j\neq i}^{N}\sum (y_i - y_j)^2$$

$$= \frac{N-2}{4(N-1)}\sum_{j\neq i}^{N}\sum (y_i - y_j)^2$$

$$= \frac{N(N-2)}{2\{2N^2(N-1)\}}\sum_{j\neq i}^{N}\sum (y_i - y_j)^2$$

$$Var(t_{symm(\text{mod})}) = \frac{N(N-2)}{2}s^2$$

where

$$s^2 = \frac{1}{2N^2(N-1)}\sum_{j\neq i}^{N}\sum (y_i - y_j)^2$$

It can be seen that the variance formula reduces to the expression of Simple Random Sampling for $p_i = p_j = \frac{1}{N}$

CHAPTER 3

EMPIRICAL STUDY

3.1 Introduction:

In this chapter sample variances of the Modified Murthy Estimator for various values of “a” are calculated. For this study 50 natural populations have taken form standard text on sampling techniques. Further ranks of sample variances are calculated to check the performance of Modified Murthy Estimator for selected values of “a”.

Sampling Variance of Modified Murthy estimator for different values of “a” using selected Natural Populations.

Table 3.1

“a”	Pop. – 1	Pop. – 2	Pop. – 3	Pop. – 4	Pop. – 5
0.1	13078.78	1469535.62	237035.52	370.63	6031.69
0.2	13301.97	1478038.62	241457.52	342.69	6092.88
0.3	13535.28	1488702.62	246516.02	317.22	6163.63
0.4	13779.59	1501843.62	252291.02	294.57	6245.56
0.5	14035.72	1517845.62	258877.52	275.09	6339.56
0.6	14304.25	1537189.62	266391.50	259.56	6447.63
0.7	14586.25	1560427.62	274960.00	248.66	6571.69
0.8	14882.28	1588279.62	284746.00	243.06	6713.38
0.9	15193.5	1621591.62	295930.50	243.94	6875.06
1.0	15520.56	1661468.62	308737.50	252.03	7060.00
1.1	15865.06	1709282.62	323433.50	269.03	7270.75
1.2	16227.97	1766837.62	340349.50	296.47	7511.12
1.3	16610.66	1836476.62	359881.50	336.10	7785.69
1.4	17014.69	1921342.62	382524.00	390.22	8099.69
1.5	17441.71	2025734.62	408896.00	461.69	8458.62

Table 3.1 (Continued)

“a”	Pop. – 6	Pop. – 7	Pop. – 8	Pop. – 9	Pop. – 10
0.1	57804.45	11579.88	467485.84	137782.12	51170916.00
0.2	53843.95	10478.25	466645.84	136517.12	52684388.00
0.3	50804.95	9956.12	465901.84	135233.12	54281892.00
0.4	48883.45	10248.25	465221.84	133937.12	55970276.00
0.5	48339.45	11703.00	464591.00	132623.88	57757092.00
0.6	49485.95	14854.50	464023.00	131295.38	59650532.00
0.7	52731.45	20551.62	463495.00	129951.62	61661156.00
0.8	58603.95	30205.62	463069.84	128591.38	63798948.00
0.9	67786.45	46300.62	462701.84	127218.12	66076836.00
1.0	81199.45	73559.38	462431.00	125832.12	68508512.00
1.1	100078.95	121955.12	462221.84	124430.62	71108704.00
1.2	126135.45	216165.62	462133.84	123016.12	73897120.00
1.3	161790.95	435925.88	462183.00	121589.38	76893216.00
1.4	210568.45	-	462333.84	120153.62	80121824.00
1.5	277778.47	-	462607.00	118704.94	83610144.00

Table 3.1(Continued)

"a"	Pop. – 11	Pop. – 12	Pop. – 13	Pop. – 14	Pop. – 15
0.1	25472680.00	9013.06	13521029.00	417374.44	185697.20
0.2	26491138.00	8892.62	14353941.00	413134.44	181827.20
0.3	27641576.00	8771.75	15371109.00	413288.00	178875.20
0.4	28941288.00	8650.12	16620709.00	418742.44	176939.20
0.5	30411816.00	8528.19	18169798.00	430494.44	176065.20
0.6	32078530.00	8405.75	20112934.00	449846.44	176394.66
0.7	33971648.00	8282.75	22588166.00	478394.44	178027.20
0.8	36128576.00	8159.41	25803782.00	518114.44	181105.20
0.9	38593600.00	8035.50	30087716.00	571550.44	185753.20
1.0	41422912.00	7911.50	35983364.00	641918.44	192141.20
1.1	44685376.00	7787.31	44450916.00	733490.44	200455.20
1.2	48467496.00	7663.06	57343268.00	851786.44	210891.20
1.3	52880616.00	7538.88	78697768.00	1004136.00	223703.20
1.4	58067008.00	7414.50	119125160.00	1200642.38	239142.66
1.5	64215848.00	7290.53	217522528.00	1455162.38	257552.66

Table 3.1 (Continued)

"a"	Pop. – 16	Pop. – 17	Pop. – 18	Pop. – 19	Pop. – 20
0.1	31.73	381.95	457155712.00	2024379648.00	1267.18
0.2	31.12	384.14	484448384.00	2083065088.00	1257.83
0.3	30.56	386.70	514557056.00	2145129728.00	1248.25
0.4	30.03	389.68	548018816.00	2210882816.00	1238.45
0.5	29.55	393.14	585524352.00	2280690944.00	1228.40
0.6	29.13	397.16	627990144.00	2354959104.00	1218.11
0.7	28.77	401.82	676654464.00	2434152704.00	1207.57
0.8	28.49	407.23	733237888.00	2518814976.00	1196.76
0.9	28.29	413.52	800205696.00	2609582848.00	1185.69
1.0	28.21	420.83	881215616.00	2707182848.00	1174.34
1.1	28.24	429.36	981939584.00	2812478720.00	1162.71
1.2	28.42	439.35	1111667072.00	2926531328.00	1150.80
1.3	28.76	451.10	1286695808.00	3050565888.00	1138.59
1.4	29.31	465.00	1538408320.00	3186075904.00	1126.08
1.5	30.09	481.53	1935675264.00	3334927616.00	1113.27

Table 3.1 (Continued)

"a"	Pop. – 21	Pop. – 22	Pop. – 23	Pop. – 24	Pop. – 25
0.1	4611.32	22086.83	1383.80	1136.29	6071356.00
0.2	4588.51	22230.30	1395.89	1176.96	6133036.00
0.3	4583.32	22384.86	1408.93	1223.89	6208028.00
0.4	4598.23	22551.33	1422.91	1278.92	6297724.00
0.5	4634.82	22730.93	1437.95	1344.69	6403532.00
0.6	4695.61	22924.58	1454.11	1425.10	6527260.00
0.7	4782.95	23133.64	1471.49	1526.17	6670652.00
0.8	4900.23	23359.46	1490.22	1657.64	6835804.00
0.9	5050.32	23603.89	1510.34	1836.25	7025484.00
1.0	5237.12	23868.52	1532.04	2093.34	7242332.00
1.1	5465.04	24155.52	1555.40	2494.69	7489676.00
1.2	5739.06	24467.08	1580.60	3204.60	7771292.00
1.3	6064.68	24805.93	1607.80	4776.30	8091436.00
1.4	6448.26	25174.96	1637.16	-	8455564.00
1.5	6897.48	25577.92	1668.93	-	8869436.00

Table 3.1 (Continued)

"a"	Pop. – 26	Pop. – 27	Pop. – 28	Pop. – 29	Pop. – 30
0.1	8807632.00	12928481.00	272721376.00	273816.00	379766.50
0.2	8775104.00	13007553.00	275497472.00	279866.00	377296.00
0.3	8759776.00	13093873.00	278379552.00	286378.25	374813.16
0.4	8764633.00	13188145.00	281376608.00	293356.00	372321.00
0.5	8793216.00	13290823.00	284498240.00	300846.00	369817.66
0.6	8849776.00	13402849.00	287755648.00	308890.25	367303.16
0.7	8939401.00	13524897.00	291161792.00	317512.00	364778.16
0.8	9068752.00	13657841.00	294731200.00	326766.00	362246.50
0.9	9245824.00	13802683.00	298480928.00	336692.00	359700.50
1.0	9480752.00	13960417.00	302430880.00	347346.00	357150.50
1.1	9786592.00	14132375.00	306604032.00	358772.25	354584.66
1.2	10180112.00	14319857.00	311027744.00	371022.25	352014.00
1.3	10683376.00	14524465.00	315734688.00	384176.00	349433.66
1.4	11326000.00	14747809.00	320764192.00	398306.00	346843.16
1.5	12148080.00	14991889.00	326163648.00	413478.00	344243.66

Table 3.1 (Continued)

"a"	Pop. – 31	Pop. – 32	Pop. – 33	Pop. – 34	Pop. – 35
0.1	97825.27	631145.00	43360652.00	273842528.00	1284.80
0.2	98284.77	630833.00	43160460.00	281690528.00	1294.33
0.3	98760.27	630530.62	42968204.00	289798496.00	1304.60
0.4	99251.14	630236.00	42784868.00	298213792.00	1315.66
0.5	99759.77	629950.00	42605964.00	306928032.00	1327.54
0.6	100285.52	629675.00	42439564.00	315959648.00	1340.35
0.7	100830.14	629418.62	42281060.00	325323104.00	1354.09
0.8	101392.89	629172.00	42132364.00	335059360.00	1368.86
0.9	101977.02	628939.62	41996900.00	345153952.00	1384.73
1.0	102582.52	628720.00	41873548.00	355643808.00	1401.77
1.1	103209.77	628529.00	41763724.00	366541152.00	1420.07
1.2	103861.14	628355.00	41667468.00	377862560.00	1439.73
1.3	104537.39	628200.00	41585252.00	389650848.00	1460.84
1.4	105238.52	628082.62	41518948.00	401916320.00	1483.54
1.5	105969.27	627986.62	41473164.00	414679392.00	1507.94

Table 3.1 (Continued)

"a"	Pop. – 36	Pop. – 37	Pop. – 38	Pop. – 39	Pop. – 40
0.1	35857.00	3941028.00	1353.98	23891.65	424055.09
0.2	34882.00	3963924.00	1385.98	23819.84	430786.53
0.3	34134.50	4002932.00	1401.98	23750.00	437935.09
0.4	33649.00	4060292.00	1329.98	23683.07	445522.53
0.5	33466.54	4138404.00	1353.98	23619.03	453585.53
0.6	33630.00	4240284.00	1393.98	23558.45	462158.09
0.7	34196.00	4369340.00	1401.98	23501.46	471276.53
0.8	35229.50	4529812.00	1417.98	23448.57	480987.53
0.9	36807.00	4726724.00	1473.98	23400.40	491340.53
1.0	39025.50	4965892.00	1465.98	23357.28	502383.53
1.1	41996.50	5254692.00	1433.98	23320.03	514176.53
1.2	45862.50	5602292.00	1409.98	23289.20	526790.56
1.3	50794.00	6019764.00	1473.98	23265.46	540292.06
1.4	57012.00	6520996.00	1497.98	23249.62	554769.56
1.5	64786.00	7124036.00	1529.98	23242.95	570311.06

Table 3.1 (Continued)

"a"	Pop. – 41	Pop. – 42	Pop. – 43	Pop. – 44	Pop. – 45
0.1	177.15	988.08	595.19	10.92	1242.83
0.2	177.13	889.51	542.78	11.10	1250.65
0.3	177.15	790.92	500.01	11.33	1261.00
0.4	177.21	693.15	474.57	11.60	1274.21
0.5	177.33	597.57	480.08	11.91	1290.55
0.6	177.50	506.53	542.07	12.28	1310.43
0.7	177.74	424.15	711.92	12.69	1334.21
0.8	178.06	357.97	1104.33	13.16	1362.38
0.9	178.47	322.72	2021.49	13.72	1395.39
1.0	178.98	349.49	4529.47	14.30	1433.90
1.1	179.61	512.14	16250.68	14.97	1478.53
1.2	180.37	1021.03	-	15.71	1530.13
1.3	181.29	2709.19	-	16.52	1589.54
1.4	182.38	-	-	17.43	1657.87
1.5	183.66	-	-	18.40	1736.35

Table 3.1 (Continued)

“a”	Pop. – 46	Pop. – 47	Pop. – 48	Pop. – 49	Pop. – 50
0.1	10236.94	11.20	96996.50	121651.27	156.00
0.2	10849.38	11.17	89969.50	131971.77	184.00
0.3	11517.62	11.26	83547.50	144692.27	156.00
0.4	12248.28	11.43	77795.50	160251.27	160.00
0.5	13048.94	11.73	72810.50	179193.77	164.00
0.6	13928.00	12.24	68683.50	202198.77	176.00
0.7	14896.00	12.86	65528.50	230132.27	144.00
0.8	15964.88	13.65	63487.50	264101.75	152.00
0.9	17148.56	14.65	62694.50	305552.75	148.00
1.0	18464.44	15.83	63346.50	356402.25	152.00
1.1	19933.06	17.27	65652.50	419243.25	164.00
1.2	21579.19	18.90	69854.50	497672.75	180.00
1.3	23433.50	20.89	76271.50	596817.75	172.00
1.4	25534.31	23.10	85253.50	724225.25	180.00
1.5	27929.44	25.63	97249.50	891499.25	184.00

Table: Ranks of Various values of “a” along with the ranks of C.V. (X) and ρ_{XY}

Table 3.2

Pop No	*CV*	ρ_{XY}	a=0.1	a=0.2	a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0	a=1.1	a=1.2	a=1.3	a=1.4	a=1.5
1.	20	7	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2.	46	49	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3.	38	35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4.	10	41	13	12	10	8	7	5	3	1	2	4	6	9	11	14	15
5.	9	10	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
6.	45	50	7	6	4	2	1	3	5	8	9	10	11	12	13	14	15
7.	44	44	4	3	1	2	5	6	7	8	9	10	11	12	13	-	-
8.	4	9	15	14	13	12	11	10	9	8	7	5	3	1	2	4	6
9.	12	21	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
10.	33	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11.	39	25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12.	15	28	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
13.	48	29	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14.	34	45	3	1	2	4	5	6	7	8	9	10	11	12	13	14	15
15.	37	40	8	7	5	3	1	2	4	6	9	10	11	12	13	14	15
16.	19	27	15	14	13	11	10	8	7	5	3	1	2	4	6	9	12

Table 3.2 (Continued)

Pop No	*CV*	ρ_{XY}	a=0.1	a=0.2	a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0	a=1.1	a=1.2	a=1.3	a=1.4	a=1.5
17.	25	12	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
18.	40	32	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
19.	43	13	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
20.	22	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
21.	29	33	4	2	1	3	5	6	7	8	9	10	11	12	13	14	15
22.	23	11	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
23.	18	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
24.	50	4	1	2	3	4	5	6	7	8	9	10	11	12	13	-	-
25.	31	30	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
26.	36	26	5	3	1	2	4	6	7	8	9	10	11	12	13	14	15
27.	21	8	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
28.	35	2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
29.	16	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
30.	17	22	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
31.	13	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
32.	5	3	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
33.	11	24	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Table 3.2 (Continued)

Pop No	***CV***	ρ_{XY}	**a=0.1**	**a=0.2**	**a=0.3**	**a=0.4**	**a=0.5**	**a=0.6**	**a=0.7**	**a=0.8**	**a=0.9**	**a=1.0**	**a=1.1**	**a=1.2**	**a=1.3**	**a=1.4**	**a=1.5**
34.	14	37	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
35.	27	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
36.	30	42	8	6	4	3	1	2	5	7	9	10	11	12	13	14	15
37.	41	38	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
38.	1	19	3	4	7	1	2	5	6	9	13	11	10	8	12	14	15
39.	24	15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
40.	28	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
41.	3	5	2	1	3	4	5	6	7	8	9	10	11	12	13	14	15
42.	47	48	11	10	9	8	7	5	4	3	1	2	6	12	13	-	-
43.	49	47	6	5	3	1	2	4	7	8	9	10	11	-	-	-	-
44.	7	39	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
45.	26	34	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
46.	8	6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
47.	6	46	2	1	3	4	5	6	7	8	9	10	11	12	13	14	15
48.	32	43	14	13	11	10	8	6	4	3	1	2	5	7	9	12	15
49.	42	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
50.	2	36	6	15	5	7	8	11	1	4	2	3	9	12	10	13	14

Frequency Table of Ranks of $\mathrm{var}(t_{symm\,(\mathrm{mod})})$ for various “a”

Table 3.3

Ranks	a=0.1	a=0.2	a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0	a=1.1	a=1.2	a=1.3	a=1.4	a=1.5
1	26	3	3	2	3	0	1	1	2	1	0	1	0	0	7
2	2	27	1	3	2	2	0	0	2	2	1	0	1	7	0
3	2	2	29	3	0	1	1	2	1	1	1	0	7	0	0
4	2	1	2	29	1	1	3	1	0	1	0	8	0	1	0
5	1	1	2	0	31	3	2	1	0	1	8	0	0	0	0
6	2	2	0	0	0	33	1	1	0	7	2	0	1	0	1
7	1	1	1	1	2	0	34	1	8	0	0	1	0	0	0
8	2	0	0	2	2	1	0	42	0	0	0	1	0	0	0
9	0	0	1	0	0	0	8	1	36	0	1	1	1	1	0
10	0	1	1	1	1	8	0	0	0	36	1	0	1	0	0
11	1	0	1	1	8	1	0	0	0	1	36	0	1	0	0
12	0	1	0	8	0	0	0	0	0	0	0	37	1	1	1
13	1	1	9	0	0	0	0	0	1	0	0	0	36	1	0
14	1	9	0	0	0	0	0	0	0	0	0	0	0	35	1
15	9	1	0	0	0	0	0	0	0	0	0	0	0	0	36
Average	5.14	5.34	5.28	5.46	5.88	6.46	6.84	7.48	8.04	8.60	9.44	10.22	11.00	11.78	12.59

Average Ranks of $\mathrm{var}(t_{symm\,(\mathrm{mod})})$ for various "a" with ranks of Coefficient of Variation

Table 3.4

CV	a=0.1	a=0.2	a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0	a=1.1	a=1.2	a=1.3	a=1.4	a=1.5
1 – 10	6.73	7.36	6.91	6.55	6.82	7.36	6.55	7.09	7.55	7.73	8.45	8.91	9.64	10.82	11.55
11 – 20	8.00	8.00	8.00	7.90	7.90	7.80	7.80	7.70	7.60	7.50	7.70	8.00	8.30	8.70	9.10
21 – 30	4.80	4.80	4.90	5.40	5.80	6.40	7.20	7.90	8.60	9.20	9.80	10.40	11.00	11.60	12.20
31 – 40	3.60	3.60	3.70	4.30	4.80	5.60	6.40	7.30	8.20	9.20	10.40	11.50	12.60	13.80	15.00
41 – 50	3.40	3.60	3.50	3.70	4.50	5.40	6.50	7.50	8.20	9.20	10.50	12.00	13.00	14.00	15.00

Average Ranks of $\text{var}(t_{symm\,(\text{mod})})$ for various "a" with ranks of with ranks of Correlation Coefficient.

Table 3.5

ρ_{XY}	a=0.1	a=0.2	a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0	a=1.1	a=1.2	a=1.3	a=1.4	a=1.5
1 – 10	3.64	4.09	4.82	5.45	6.09	6.73	7.36	8.00	8.64	9.18	9.73	10.27	11.09	11.80	12.70
11 – 20	4.00	4.60	5.40	5.30	5.90	6.70	7.30	8.10	9.00	9.30	9.70	10.00	10.90	11.60	12.20
21 – 30	8.40	8.10	7.80	7.70	7.80	7.80	7.80	7.70	7.60	7.50	7.70	8.00	8.30	8.70	9.10
31 – 40	2.50	3.80	3.20	4.10	4.90	6.10	6.10	7.40	8.30	9.30	10.80	12.00	12.70	13.90	14.90
41 – 50	6.90	5.90	5.00	4.60	4.60	4.90	5.60	6.20	6.70	7.80	9.40	11.11	12.33	13.71	15.00

Conclusions:

The empirical study has been carried out for the Modified Murthy Estimator for various values of “a”. Results of this study have been given in tables 3.1–3.5. Following conclusions have been drawn from analysis of these tables.

Table 3.1 constitute sampling variance of Modified Murthy Estimator for various values of “a” for selected 50 natural populations. From this table it can be seen that the value of a=0.1 performs better than others values in 27 populations, that is in 54% of the populations a=0.1 is better. Further from table 3.1 it can be seen that in most of populations the sampling variance increases with increase in the value of “a”.

Table 3.2 constitute ranks of variances for values of “a”. The ranking has been done following the justification of Jefres (1961). This table also contains the ranks of correlation coefficient and coefficient of variation.

Table 3.3 constitute the frequencies of ranks for various values of “a”. From this table it can be seen that a=0.1 clearly outperform other values involved in study.

Table 3.4 and 3.5 contains average rank of various values of "a" for various ranges of coefficient of variation and correlation coefficient. From these tables it can be clearly seen that the value a=0.1 dominate other values involved in study.

Appendix

Description of the Populations

This appendix contains fifty natural populations that have been studied in this thesis. Information about the source of all populations has also been given. The coefficient of variation of measure of size and coefficient of variation for basic variable of study is also given. The description of measure of size, basic variable of study along with the coefficient of correlation between two variables is also given. Complete listing of populations is given in the following pages

Table A.1

Description of Natural Populations

Sr. No	***Source***	***N***	***Y***	***X***	***c.v. (X)***	***c.v. (Y)***	ρ_{xy}
1.	Cochran, 1977 p. 34	20	Food Cost	Family Size	0.3832	0.3059	0.4852
2.	Cochran, 1977 p. 152, 1 –20	20	City Size 1930	City Size 1920	1.0328	1.0309	0.9906
3.	Cochran, 1977 p. 152, 21 – 40	20	City Size 1930	City Size 1920	0.7940	0.6746	0.9376
4.	Cochran, 1977 p. 203	10	Actual Weight	Eye Estimate	0.1708	0.1939	0.9736
5.	Cochran, 1977 p. 325	10	No. of Persons	No. of Rooms	0.1350	0.1527	0.6515
6.	Cochran, 1977 p. 152, 1 – 15	15	City Size 1930	City Size 1920	0.9623	0.9723	0.9932
7.	Cochran, 1977 p. 152, 40 – 49	10	City Size 1930	City Size 1920	0.9377	0.8674	0.9764
8.	Singh & Chaudhary 1986, p. 107, B – 1	16	Timber Volume	Strip Length	0.0761	0.1461	0.5635
9.	Singh & Chaudhary 1986, p. 107, B – 3	16	Timber Volume	Strip Length	0.1892	0.4503	0.8020
10.	Singh & Chaudhary 1986, p. 107, 1 - 20	20	Cultivated Area	Population	0.6925	0.3614	0.8160
11.	Singh & Chaudhary 1986, p. 108, 21-40	20	Cultivated Area	Population	0.7984	0.6164	0.8556
12.	Singh & Chaudhary 1986 p. 116	20	Yield of Crop	Area under Crop	0.2636	0.4249	0.8807

Table A.1 (Continued)

Sr. No	Source	N	Y	X	c.v. (X)	c.v. (Y)	ρxy
13.	Singh & Chaudhary 1986, p. 141, 1 - 14	14	No. of Lime Trees	Area under Lime	1.2316	1.0226	0.9011
14.	Singh & Chaudhary 1986, p. 166	16	Area under Wheat 1979 – 80	Area under Wheat 1978 – 79	0.7141	0.6950	0.9784
15.	Singh & Chaudhary 1986, p. 177	20	Area under Wheat 1974	Area under Wheat 1973	0.7519	0.7638	0.9714
16.	Mukhopadhyay 1998, p. 43, 1 – 12	12	Household Income	Household Size	0.3829	0.4910	0.8795
17.	Mukhopadhyay 1998, p. 43, 13-24	12	Household Income	Household Size	0.4300	0.5309	0.6726
18.	Mukhopadhyay 1998, p. 110	10	Output	No. of Workers	0.8715	0.3648	0.9228
19.	Mukhopadhyay 1998, p. 157	20	Output	No. of Workers	0.9248	0.3892	0.6903
20.	Mukhopadhyay 1998, p. 190, 1–20	20	Cultivated Area	Household Size	0.4085	1.0732	0.7401
21.	Mukhopadhyay 1998, p. 192, 1 - 15	15	Value Added	No. of Workers	0.5163	0.5350	0.9305
22.	Mukhopadhyay 1998, p. 192, 16-30	15	Value Added	No. of Workers	0.4113	0.4972	0.6619
23.	Mukhopadhyay 1998, p. 207, 1 - 15	15	Income	Household Size	0.3613	0.3942	0.7037
24.	Murthy, 1976 p. 91, 1 – 20	20	Cultivated Area	Holding Size	1.5417	0.4742	0.3918
25.	Murthy, 1976 p. 127, 1 – 15	15	No. of Cultivators	Cultivated Area	0.5693	0.5680	0.9060

Table A.1 (Continued)

Sr. No	*Source*	*N*	*Y*	*X*	*c.v. (X)*	*c.v. (Y)*	*ρxy*
26.	Murthy, 1976 p. 127, 21 – 35	15	No. of Cultivators	Cultivated Area	0.7363	0.8360	0.8754
27.	Murthy, 1976 p. 128, 51 – 65	15	No. of Cultivators	Cultivated Area	0.3887	0.5099	0.5438
28.	Murthy, 1976 p. 129, 86 – 100	15	No. of Cultivators	Cultivated Area	0.7276	1.0438	0.1799
29.	Murthy, 1976 p. 132, 108 – 123	16	Timber Volume	Length	0.3191	0.2737	0.8019
30.	Murthy, 1976 p. 132, 151 – 170	20	Timber Volume	Length	0.3227	0.5318	0.8080
31.	Murthy, 1976 p. 178, 1 – 20	20	Area under Paddy	Geograph-ical Area	0.1930	0.3776	-0.073
32.	Murthy, 1976 p. 178, 66 – 80	15	Area under Paddy	Geograph-ical Area	0.0911	0.3888	0.2541
33.	Murthy, 1976 p. 228, 1 – 20	20	Output	No. of Workers	0.1725	0.2519	0.8250
34.	Murthy, 1976 p. 228, 51 – 70	20	Output	No. of Workers	0.2601	0.0772	0.9464
35.	Murthy, 1976 p. 398, 1 – 20	20	No. of Absentees	No. of Workers	0.4892	0.5152	0.7657
36.	Murthy, 1976 p. 399, 21 – 34	14	Area under Wheat 1964	Area under Wheat 1963	0.5249	0.5421	0.9739
37.	Murthy, 1976 p. 422, 1 – 20	20	Cattle in Survey	Cattle in Census	0.9042	0.8870	0.9579
38.	Deming, 1950 p. 184, 1 – 20	20	Marked Weight	Weight by Bureau	0.0036	0.0030	0.7996

Table A.1 (Continued)

Sr. No	Source	N	Y	X	c.v. (X)	c.v. (Y)	ρxy
39.	Yates, 1981 p. 150, 21 – 40	20	No. of Absentees	No. of Workers	0.4213	0.6815	0.7342
40.	Yates, 1981 p. 153, 1 – 20	20	Measured Volume	Eye Estimate	0.5008	0.4391	0.7506
41.	Kish, 1965 p. 146	10	Sample Size	Class Size	0.0595	0.1368	0.4069
42.	Kish, 1965 p. 42, 1 – 10	10	Rented Du's	Total Du's	1.1525	1.4483	0.9888
43.	Kish, 1965 p. 42, 11 – 20	10	Rented Du's	Total Du's	1.2568	1.2457	0.9833
44.	Scheaffers, 1986 p. 129	20	Current Value	Value two years ago	0.1225	0.1162	0.9662
45.	Scheaffers, 1986 p. 142	15	Actual Value	Value from Computer	0.4692	0.4658	0.9320
46.	Scheaffers, 1986 p. 157	14	Amount on Food	Total Income	0.1275	0.1779	0.4414
47.	Scheaffers, 1986 p. 159	15	Present Salary	Past Salary	0.0941	0.0917	0.9809
48.	Som, 1976 p. 73, 1 – 15	15	Area under Wheat 1937	Area under Wheat 1936	0.6794	0.7182	0.9746
49.	Som, 1976 p. 73, 16 – 30	15	Area under Wheat 1937	Area under Wheat 1936	0.9244	0.7475	0.9071
50.	Deming, 1950 p. 184, 21 – 36	16	Marked Weight	Weight by Bureau	0.0074	0.0083	0.9462

Table A.2

Details of Natural Populations

Pop. – 1	***X :***	2, 3, 3, 5, 4, 7, 2, 4, 2, 5, 3, 6, 4, 4, 2, 5, 3, 4, 2, 4
	Y :	14.3, 20.8, 22.7, 30.5, 41.2, 28.2, 24.2, 30.0, 24.2, 44.4, 13.4, 19.8, 29.4, 27.1, 22.2, 37.7, 22.6, 36.0, 20.6, 27.7.
Pop. – 2	***X :***	76, 138, 67, 29, 381, 23, 37, 120, 61, 387, 93, 172, 78, 66, 60, 46, 2, 507, 179, 121
	Y :	80, 143, 67, 50, 464, 48, 63, 115, 69, 459, 104, 183, 106, 86, 57, 65, 50, 634, 260, 113
Pop. – 3	***X :***	50, 44, 77, 64, 64, 56, 40, 40, 38, 136, 116, 46, 243, 87, 30, 71, 256, 43, 25, 94
	Y :	64, 58, 89, 63, 77, 142, 60, 64, 52, 139, 130, 53, 291, 105, 111, 79, 288, 61, 57, 85
Pop. – 4	***X :***	59, 47, 52, 60, 67, 48, 44, 58, 76, 58
	Y :	61, 42, 50, 58, 67, 45, 39, 57, 71, 53
Pop. – 5	***X :***	60, 52, 58, 56, 62, 51, 72, 48, 71, 58
	Y :	115, 80, 82, 93, 105, 109, 130, 93, 109, 95
Pop. – 6	***X :***	76, 138, 67, 29, 381, 23, 37, 120, 61, 387, 93, 172, 78, 66, 60
	Y :	80, 143, 67, 50, 464, 48, 63, 115, 69, 459, 104, 183, 106, 86, 57
Pop. – 7	***X :***	94, 43, 298, 36, 161, 74, 45, 36, 50, 48
	Y :	85, 50, 317, 46, 232, 93, 53, 54, 58, 75
Pop. – 8	***X :***	12, 12, 12, 12, 12, 12, 12, 12, 11, 11, 11, 11, 10, 10, 10, 10
	Y :	762, 651, 461, 521, 653, 544, 542, 590, 533, 517, 520, 539, 509, 449, 492, 498

Table A.2 (Continued)

Pop. – 9	***X :***	6, 6, 6, 6, 6, 5, 5, 5, 4, 4, 4, 4, 4, 4, 4, 4
	Y :	165, 224, 192, 161, 104, 94, 102, 115, 110, 109, 83, 36, 61, 92, 75, 64
Pop. – 10	***X :***	226, 670, 4505, 1732, 2874, 2282, 793, 895, 1157, 3201, 1117, 1236, 5201, 848, 1238, 1917, 1800, 2335, 4396, 1607
	Y :	678, 663, 1290, 1170, 1390, 1110, 760, 730, 950, 1700, 909, 1169, 1840, 660, 1140, 1360, 1509, 1810, 2240, 1225
Pop. – 11	***X :***	2071, 2155, 7780, 2746, 2549, 1007, 1567, 5271, 659, 3209, 2902, 2955, 1746, 1045, 666, 904, 773, 1040, 760, 2084
	Y :	1250, 1690, 3200, 1744, 2400, 680, 970, 1850, 340, 2450, 1760, 2120, 1220, 860, 620, 760, 602, 532, 438, 633
Pop. – 12	***X :***	5.2, 5.9, 3.9, 4.2, 4.7, 4.8, 4.9, 6.8, 4.7, 5.7, 5.2, 5.2, 4.9, 4.0, 1.3, 7.4, 7.4, 4.8, 6.2, 6.2
	Y :	28, 29, 30, 22, 22, 25, 28, 37, 26, 32, 25, 38, 31, 16, 6, 61, 61, 29, 47, 47
Pop. – 13	***X :***	32.77, 7.97, 0.62, 15.61, 42.85, 40.03, 9.39, 6.33, 5.05, 94.55, 53.71, 0.67, 0.82, 2.15
	Y :	2328, 754, 105, 949, 3091, 1736, 840, 311, 0, 3044, 2483, 128, 102, 60
Pop. – 14	***X :***	100, 200, 423, 503, 258, 1275, 89, 699, 243, 672, 597, 455, 282, 421, 80, 465
	Y :	85, 239, 406, 503, 217, 1191, 69, 584, 294, 745, 611, 421, 361, 277, 96, 489
Pop. – 15	***X :***	70, 163, 320, 440, 250, 125, 558, 254, 101, 359, 109, 481, 125, 5, 427, 78, 75, 45, 564, 238
	Y :	50, 149, 284, 381, 278, 111, 634, 278, 112, 355, 99, 498, 111, 6, 339, 80, 105, 27, 515, 249

Table A.2 (Continued)

Pop. – 16	***X :***	7, 3, 5, 8, 12, 4, 6, 11, 8, 9, 5, 7
	Y :	3.2, 1.1, 2.4, 3.7, 6.2, 1.7, 2.1, 3.9, 4.7, 5.1, 2.3, 1.8
Pop. – 17	***X :***	6, 11, 5, 5, 7, 17, 12, 10, 10, 6, 5, 8
	Y :	1.6, 3.5, 4.1, 2.4, 6.8, 11.7, 8.3, 5.9, 6.1, 2.3, 7.5, 8.4
Pop. – 18	***X :***	57, 69, 81, 107, 253, 352, 425, 545, 750, 985
	Y :	2552, 3975, 3607, 3975, 5712, 6903, 6973, 7075, 7545, 8975
Pop. – 19	***X :***	68, 52, 54, 76, 107, 253, 452, 52, 71, 217, 82, 112, 544, 352, 75, 482, 644, 88, 125, 144
	Y :	3890, 1451, 2800, 3520, 4700, 5712, 6854, 1600, 3802, 6203, 8512, 5203, 7072, 6507, 3512, 6660, 7185, 3752, 4502, 4962
Pop. – 20	***X :***	7, 6, 3, 3, 8, 5, 5, 7, 9, 13, 4, 6, 4, 7, 6, 4, 3, 7, 5, 5
	Y :	0.82, 3.0, 3.32, 5.0, 7.92, 1.62, 2.66, 4.66, 6.64, 18.0, 0.0, 0.16, 0.50, 4.08, 2.65, 1.95, 2.0, 0.44, 1.98, 3.57
Pop. – 21	***X :***	95, 79, 30, 45, 28, 142, 125, 81, 43, 53, 148, 89, 57, 132, 47
	Y :	47.1, 45.2, 15.6, 23.2, 11.7, 98.6, 51.7, 42.2, 27.3, 31.8, 67.2, 50.9, 30.2, 70.7, 31.2
Pop. – 22	***X :***	43, 116, 65, 103, 52, 67, 22, 75, 69, 63, 83, 124, 31, 96, 60
	Y :	11.2, 61.7, 21.9, 15.7, 38.4, 32.2, 31.3, 36.5, 38.9, 41.2, 51.7, 70.2, 16.9, 51.3, 15.5
Pop. – 23	***X :***	8, 10, 7, 5, 12, 4, 10, 6, 7, 5, 11, 3, 7, 8, 11
	Y :	9.2, 15.7, 8.4, 7.5, 18.3, 7.6, 9.5, 9.3, 8.4, 16.5, 20.1, 7.2, 11.6, 9.2, 18.9

Table A.2 (Continued)

Pop. – 24	***X :***	21.04, 12.59, 20.30, 16.16, 23.82, 1.79, 26.91, 7.41, 7.68, 66.55, 141.80, 28.12, 8.29, 7.27, 1.47, 1.12, 10.67, 5.94, 3.15, 4.84
	Y :	2.70, 1.76, 1.47, 1.64, 1.56, 1.79, 5.44, 1.97, 2.45, 3.26, 3.20, 3.90, 1.95, 2.20, 0.48, 1.12, 1.98, 2.45, 2.60, 1.67
Pop. – 25	***X :***	2544, 428, 1177, 4567, 2618, 4113, 1869, 2713, 2237, 600, 3420, 4012, 1949, 695, 1569
	Y :	806, 193, 819, 1970, 1149, 1510, 611, 888, 1291, 257, 1855, 1741, 1345, 343, 586
Pop. – 26	***X :***	527, 2767, 2770, 719, 607, 482, 1527, 1367, 767, 1648, 2440, 2434, 1638, 61, 4505
	Y :	126, 1906, 1454, 191, 336, 419, 432, 393, 192, 1091, 1830, 1219, 543, 93, 1675
Pop. – 27	***X :***	811, 1453, 1665, 2350, 564, 2487, 904, 2040, 1314, 1506, 1657, 1053, 2071, 872, 1718
	Y :	446, 888, 277, 793, 421, 1085, 383, 1813, 394, 1116, 1145, 955, 1158, 826, 1565
Pop. – 28	***X :***	1031, 1930, 1333, 1509, 509, 4424, 1881, 4139, 4072, 612, 5507, 4634, 1667, 2013, 156
	Y :	910, 563, 506, 168, 217, 1409, 1, 109, 0, 1167, 690, 2659, 67, 591, 1506
Pop. – 29	***X :***	3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 8, 8
	Y :	144, 159, 209, 210, 227, 247, 277, 372, 304, 208, 305, 353, 369, 361, 333, 290
Pop. – 30	***X :***	1, 1, 2, 3, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5
	Y :	21, 23, 40, 43, 50, 96, 145, 156, 127, 161, 214, 143, 201, 198, 173, 220, 297, 151, 158, 163

Table A.2 (Continued)

Pop. – 31	***X :***	103, 106, 120, 120, 121, 121, 124, 128, 135, 137, 145, 147, 151, 153, 160, 166, 176, 178, 185, 206
	Y :	41, 33, 87, 78, 56, 62, 58, 19, 64, 61, 74, 13, 81, 41, 58, 44, 65, 69, 29, 46
Pop. – 32	***X :***	453, 460, 462, 467, 501, 503, 514, 515, 541, 542, 543, 562, 570, 586, 601
	Y :	194, 161, 222, 223, 96, 164, 318, 272, 155, 292, 214, 275, 100, 418, 189
Pop. – 33	***X :***	51, 51, 52, 52, 53, 54, 57, 60, 65, 67, 68, 70, 71, 73, 74, 76, 78, 80, 81, 85
	Y :	1350, 1176, 1841, 2606, 2656, 2546, 2911, 3280, 3425, 3416, 3390, 3395, 3417, 3290, 3481, 3520, 3570, 3740, 3520, 3601
Pop. – 34	***X :***	253, 285, 291, 314, 335, 352, 375, 387, 425, 443, 452, 466, 481, 495, 528, 544, 563, 585, 598, 644
	Y :	5684, 5790, 5839, 5920, 6315, 6540, 6567, 6719, 6752, 6660, 6854, 6760, 6825, 6940, 7295, 7070, 7152, 7186, 7215, 7288
Pop. – 35	***X :***	95, 79, 30, 45, 28, 142, 125, 81, 43, 53, 148, 89, 57, 132, 47, 43, 116, 65, 103, 52
	Y :	9, 7, 3, 2, 3, 8, 9, 10, 6, 2, 16, 4, 5, 13, 4, 9, 12, 8, 9, 8
Pop. – 36	***X :***	92, 247, 134, 131, 129, 190, 363, 235, 73, 62, 71, 137, 196, 255
	Y :	85, 221, 133, 144, 103, 175, 335, 219, 62, 79, 60, 100, 141, 263
Pop. – 37	***X :***	623, 690, 534, 293, 69, 842, 475, 371, 161, 298, 2045, 1069, 706, 1795, 1406, 118, 330, 218, 160, 210
	Y :	654, 696, 530, 315, 78, 640, 692, 292, 210, 555, 2110, 592, 707, 1890, 1123, 115, 375, 212, 147, 297

Table A.2 (Continued)

Pop. – 38	***X :***	980, 978, 975, 982, 986, 978, 981, 982, 978, 980, 988, 981, 983, 977, 979, 987, 979, 979, 979, 983
	Y :	447, 445, 456, 446, 449, 447, 447, 449, 445, 448, 449, 448, 449, 446, 447, 449, 447, 448, 447, 449
Pop. – 39	***X :***	67, 64, 75, 69, 63, 83, 124, 31, 96, 42, 85, 91, 78, 150, 54, 69, 64, 164, 132, 82
	Y :	27, 12, 12, 15, 9, 14, 25, 3, 45, 25, 35, 28, 13, 36, 26, 27, 2, 69, 41, 10
Pop. – 40	***X :***	102, 14, 57, 70, 95, 92, 110, 208, 208, 110, 110, 110, 110, 128, 79, 177, 65, 196, 167, 268
	Y :	170, 47, 64, 91, 126, 146, 87, 195, 255, 135, 146, 154, 110, 112, 153, 216, 125, 100, 287, 261
Pop. – 41	***X :***	94, 85, 85, 80, 82, 79, 86, 82, 93, 88
	Y :	16, 20, 18, 17, 14, 15, 19, 13, 19, 17
Pop. – 42	***X :***	5, 9, 18, 68, 32, 48, 11, 1, 1, 4
	Y :	3, 5, 5, 52, 21, 34, 3, 0, 0, 0
Pop. – 43	***X :***	29, 31, 5, 2, 4, 102, 20, 15, 1, 29
	Y :	17, 14, 0, 0, 2, 54, 11, 11, 0, 23
Pop. – 44	***X :***	6.7, 8.2, 7.9, 6.4, 8.3, 7.2, 6.0, 7.4, 8.1, 9.3, 8.2, 6.8, 7.4, 7.5, 8.3, 9.1, 8.6, 7.9, 6.3, 8.9
	Y :	7.1, 8.4, 8.2, 6.9, 8.4, 7.9, 6.5, 7.6, 8.9, 9.9, 9.1, 7.3, 7.8, 8.3, 8.9, 9.6, 8.7, 8.8, 7.0, 9.4
Pop. – 45	***X :***	15.0, 9.5, 14.2, 20.5, 6.7, 9.8, 25.7, 12.6, 15.1, 30.9, 7.3, 28.6, 14.7, 20.5, 10.9
	Y :	14.0, 9.0, 12.5, 22.0, 6.3, 8.4, 28.5, 10.0, 14.4, 28.2, 15.5, 26.3, 13.1, 19.5, 9.8
Pop. – 46	***X :***	39, 43, 21, 64, 57, 47, 28, 75, 34, 52
	Y :	65, 78, 52, 82, 92, 89, 73, 98, 56, 75

Table A.2 (Continued)

Pop. – 47	***X :***	15.4, 16.7, 17.7, 19.9, 16.4, 15.1, 17.9, 15.2, 14.4, 15.4, 18.2, 16.4, 19.2, 17.0, 17.3
	Y :	16.5, 17.6, 18.9, 21.4, 17.0, 16.3, 19.1, 16.3, 16.4, 16.6, 19.6, 17.7, 20.8, 18.3, 18.9
Pop. – 48	***X :***	75, 163, 326, 442, 254, 125, 559, 254, 101, 359, 109, 481, 125, 5, 427
	Y :	52, 149, 289, 381, 278, 111, 634, 278, 112, 355, 99, 498, 111, 6, 399
Pop. – 49	***X :***	78, 78, 45, 564, 238, 92, 247, 134, 131, 129, 192, 663, 236, 73, 62
	Y :	79, 105, 27, 515, 249, 85, 221, 133, 144, 103, 179, 330, 219, 62, 79
Pop. – 50	***X :***	983, 987, 987, 979, 980, 980, 979, 985, 980, 987, 981, 987, 987, 977, 970, 959
	Y :	447, 448, 448, 445, 448, 447, 447, 448, 447, 450, 449, 449, 449, 446, 441, 435

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