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The Trivariate Pseudo Inverse Weibull Distribution

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We have proposed the trivarite Pseudo Inverse Weibull distribution. Standard distributional properties of the proposed distribution has been obtained. We have also obtained the bivariate distribution of concomitants of records. The conditional distribution of one of the concomitant has also been obtained when information of other concomitant is available. Moments of the resulting distributions has been computed.

Keywords: Inverse Weibull Distribution; , Concomitants of Records; Conditional distributions.

1. Introduction

Order and Record Statistics has wide sprfead applications in many areas of life. The quantities have attracted number of statisticians in developing new probability distributions with many applications in extreems. The distribution of record statistics has been proposed by Chandler(1952). A comprehensive treaty on records is given in Ahsanullah(1995). Formally, the distribution of kth lower records based upon a random sample of size n from the distribution $F(x)$ is given by Ahsanullah(1995) as:

$$f_{k:n}(x_k)=\frac{1}{\Gamma(k)}f(x_k)\left[H(x_k)\right]^{k-1}; \qquad (1.1)$$

where $R(x_k)=-\ln\left[F(x)\right]$.

The concomitants of record statistics has also found many applications when random sample if available from a bivariate distribution with density function $f(x,y)$ and sample is arranged with respect to records of one of the random variable X .

The distribution of kth concomitant of record statistics is defined byAhsanullah(1995)as:

$$f(y_k)=\int_{-\infty}^{\infty}f(y_k\,|\,x_k)\,f_{k:n}(x_k)\,dx_k, \qquad (1.2)$$

where $f(y\,|\,x)$ is conditional distribution of Y given $X=x$ and $f_{k:n}(x_k)$ is defined in (1.1). The joint distribution of two concomitants of records is given as:

$$f(y_k,y_m)=\int_{-\infty}^{\infty}\int_{-\infty}^{x_m}f(y_k\,|\,x_k)\,f(y_m\,|\,x_m)\,f_{k,m:n}(x_k)\,dx_k\,dx_m. \qquad (1.3)$$

The distribution of concomitants of records has been studied by many authors; see for exampleKirmani & Beg(1984),M. Q. Shahbaz & Shahbaz(2009) among others. The distribution of the pair of concomitanst of records is defined byM. Q. Shahbaz, Shahbaz, Mohsin, & Rafiq(2010) as:

$$f(y_k,z_k)=\int_{-\infty}^{\infty}f(y_k,z_k\,|\,x_k)\,f_{k:n}(x_k)\,dx_k; \qquad (1.4)$$

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where $f_{k:n}(x_k)$ is defined in (1.1).

Recently a new class of distributions called the Pseudo distributions has attracted number of statisticians due to their wider applicability. The pseudo distributions have found many applications in stochastic processes, actuarial sciences and many other fields where standard probability distributions do not provide better fit. The pseudo distributions are introduced by Filus & Filus(2000, 2006). Filus & Filus(2000) introduced the pseudo Gaussian distribution as a linear combination of normal random variables. M. Q. Shahbaz & Shahbaz(2009) has introduced the pseudo Rayleigh distribution as the compound distribution of two random variables. M. Q. Shahbaz, et al.(2010) has studied the distribution of bivariate concomitants of record statistics for trivariate pseudo exponential distribution. The trivariate pseudo Weibull distribution has been studied by S. Shahbaz, Shahbaz, & Rafiq(2011). Mohsin, Ahmad, Shahbaz, & Shahbaz(2009) studied properties of lower records for bivariate pseudo inverse Rayleigh distribution.

In the following section we have introduced trivariate pseudo Inverse Weibull distribution with some of its basic properties. The distribution of bivariate concomitants of records for the said distribution has been studied in section 3.

2. The Trivariate Pseudo Inverse Weibull Distribution

In this section we have introduced the trivariate Pseudo Inverse Weibull distribution as a compound distribution of three random variables. The distribution is defined in the following:

Suppose that the random variable X has Inverse Weibull distribution with shape parameter β_1. The density function of X is:

$$f(x;\beta_1) = \beta_1 x^{-(\beta_1+1)} \exp\left(-x^{-\beta_1}\right); x > 0, \beta_1 > 0 \quad (2.1)$$

Suppose further that the random variable Y has Inverse Weibull distribution with shape parameter β_2 and scale parameter $\phi_1(x)$, where $\phi_1(x)$ is some function of random variable X. The density function of Y is:

$$f(y|x) = \beta_2\phi_1(x) y^{-(\beta_2+1)} \exp\left\{-\phi_1(x) y^{-\beta_2}\right\}; y, \phi_1(x), \beta_2 > 0$$

$$f(y|x) = \beta_2\phi_1(x)y^{-(\beta_2+1)}\exp\{-\phi_1(x)y^{-\beta_1}\}; y, \phi_1(x), \beta_2 > 0 \quad (2.2)$$

Finally, suppose that the random variable Z also has Inverse Weibull distribution with shape parameter β_3 and scale parameter $\phi_2(x,y)$, where $\phi_2(x,y)$ is some function of random variables X and Y. The density function of Z is therefore:

$$f(z|x,y) = \beta_3\phi_2(x,y)z^{-(\beta_3+1)}\exp\{-\phi_2(x,y)z^{-\beta_3}\}; z, \phi_2(x,y), \beta_3 > 0 \quad (2.3)$$

We define the trivariate pseudo Inverse Weibull distribution as compound distribution of (2.1), (2.2) and (2.3). The density function of trivariate pseudo Inverse Weibull distribution is given as:

$$f(x,y,z) = \beta_1\beta_2\beta_3\phi_1(x)\phi_2(x,y)x^{-(\beta_1+1)}y^{-(\beta_2+1)}z^{-(\beta_3+1)} \times \exp\left[-\{x^{-\beta_1} + \phi_1(x)y^{-\beta_2} + \phi_2(x,y)z^{-\beta_3}\}\right] \phi_1(x) > 0;\ \phi_2(x,y) > 0; x, y, z, \beta_1, \beta_2, \beta_3 > 0 \quad (2.4)$$

The density (2.4) can be used to defined several distributions based upon various choices of $\phi_1(x)$ and $\phi_2(x,y)$. We have defined the trivariate pseudo Inverse Weibull distributions by using $\phi_1(x) = x^{-\beta_1}$ and $\phi_2(x,y) = x^{-\beta_1}y^{-\beta_2}$. The densityt function of pseudo Inverse Weibull distribution is therefore:

$$f(x,y,z) = \beta_1\beta_2\beta_3 x^{-(3\beta_1+1)} y^{-(2\beta_2+1)} z^{-(\beta_3+1)} \times \exp\left[-x^{-\beta_1}\left\{1 + y^{-\beta_2} + y^{-\beta_2}z^{-\beta_3}\right\}\right]; \quad (2.5)$$
$$x, y, z, \beta_1, \beta_2, \beta_3 > 0.$$

The product moments for distribution (2.5) are given as:

$$\mu'_{r,s,t} = \int_0^\infty\int_0^\infty\int_0^\infty x^r y^s z^t f(x,y,z)\,dxdydz$$
$$= \int_0^\infty\int_0^\infty\int_0^\infty x^r y^s z^t \beta_1\beta_2\beta_3 x^{-(3\beta_1+1)} y^{-(2\beta_2+1)} z^{-(\beta_3+1)} \times \exp\left[-x^{-\beta_1}\left\{1 + y^{-\beta_2} + y^{-\beta_2}z^{-\beta_3}\right\}\right] dxdydz$$

Making the transformation $w = x^{-\beta_1}\left\{1 + y^{-\beta_2} + y^{-\beta_2}z^{-\beta_3}\right\}$ and after some calculus, the product moments turned out to be:

$$\mu'_{r,s,t} = \Gamma\left(1-\frac{r}{\beta_1}+\frac{s}{\beta_2}\right)\Gamma\left(1-\frac{s}{\beta_2}+\frac{t}{\beta_3}\right)\Gamma\left(1-\frac{t}{\beta_3}\right) \quad (2.6)$$

The product moments (2.6) exist if $s > \frac{\beta_2(r-\beta_1)}{\beta_1}$ and $t > \frac{\beta_3(s-\beta_2)}{\beta_2}$. Using (2.6), we can easily obtain the marginal moments. The conditional distribution of X given Y and Z is obtained as:

$$f(x\,|\,y,z) = \frac{f(x,y,z)}{f(y,z)}. \quad (2.7)$$

We first obtain the joint marginal distribution $f(y,z)$ as:

$$\begin{aligned} f(y,z) &= \int_0^\infty f(x,y,z)\,dx \\ &= \int_0^\infty \beta_1\beta_2\beta_3 x^{3\beta_1-1} y^{2\beta_2-1} z^{\beta_3-1} \\ &\quad \times \exp\left[-x^{\beta_1}\left\{1+y^{\beta_2}+y^{\beta_2}z^{\beta_3}\right\}\right]dx \\ &= \frac{2\beta_2\beta_3 y^{\beta_2-1} z^{\beta_3-1}}{\left(1+z^{\beta_3}+y^{\beta_2}z^{\beta_3}\right)^3}; y,z,\beta_2,\beta_3 > 0 \end{aligned} \quad (2.8)$$

Graph of the density given in (2.8) is:

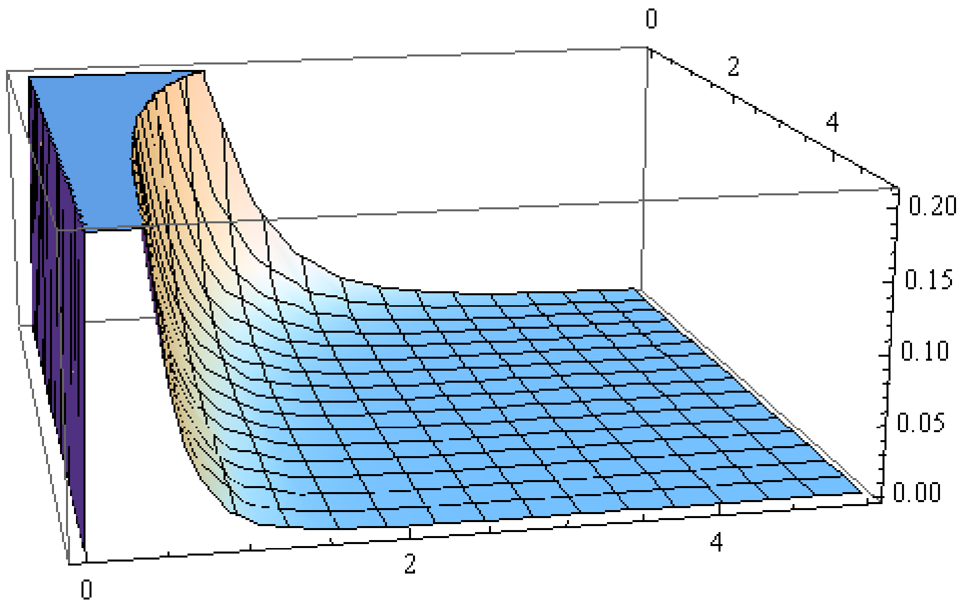

Figure 2.1: Joint marginal distribution $f(y,z)$

Using (2.4) and (2.8) in (2.7), the conditional distribution of X given Y and Z is given as:

$$\begin{aligned} f(x\,|\,y,z) &= \frac{1}{2}\beta_1\left(1+z^{\beta_2}+y^{\beta_2}z^{\beta_3}\right)^3 x^{-(3\beta_1+1)} y^{-3\beta_2} z^{-3\beta_3} \\ &\times \exp\left[-x^{-\beta_1}\left\{1+y^{-\beta_2}+y^{-\beta_2}z^{-\beta_3}\right\}\right]. \end{aligned}$$

The *hth* conditional moment of X is:

$$\begin{aligned} E\left(X^h\,|\,y,z\right) &= \frac{1}{2}\int_0^\infty x^h\beta_1\left(1+z^{\beta_2}+y^{\beta_2}z^{\beta_3}\right)^3 x^{-(3\beta_1+1)} y^{-3\beta_2} z^{-3\beta_3} \\ &\times \exp\left[-x^{\beta_1}\left\{1+y^{\beta_2}+y^{\beta_2}z^{\beta_3}\right\}\right]dx \\ &= \frac{1}{2}\left(1+y^{-\beta_2}+y^{-\beta_2}z^{-\beta_3}\right)^{h/\beta_2}\Gamma\left(3-\frac{h}{\beta_1}\right). \end{aligned} \quad (2.9)$$

The conditional moments exist if $3\beta_1 > h$. The conditional mean and variance can be readily obtained from (2.9). We now find the conditional distribution of Y and Z given X by using:

$$f(y,z\,|\,x) = \frac{f(x,y,z)}{f(x)} \quad (2.10)$$

Using (2.1) and (2.4) in (2.10), the conditional distribution of Y and Z given X is:

$$\begin{aligned} f(y,z\,|\,x) &= \beta_2\beta_3 x^{-2\beta_1} y^{-(2\beta_2+1)} z^{-(\beta_3+1)} \\ &\times \exp\left\{-x^{-\beta_1} y^{-\beta_2}\left(1+z^{-\beta_3}\right)\right\} \end{aligned} \quad (2.11)$$

The joint conditional moments of Y and Z given X can be obtained from (2.11) as:

$$E\left(Y^h Z^q \mid x\right) = \int_0^\infty\int_0^\infty y^h z^q f\left(y,z \mid x\right)dydz$$
$$= \int_0^\infty\int_0^\infty y^h z^q \beta_2\beta_3 x^{-2\beta_1} y^{-(2\beta_2+1)} z^{-(\beta_3+1)} \times \exp\left\{-x^{-\beta_1} y^{-\beta_2}\left(1+z^{-\beta_3}\right)\right\}dydz \quad (2.12)$$
$$= x^{-s\beta_1/\beta_2}\Gamma\left(1-\frac{h}{\beta_2}+\frac{q}{\beta_3}\right)\Gamma\left(1-\frac{q}{\beta_3}\right).$$

The conditional means, variances and covariances can be readily obtained from (2.12). In the following section we have obtained the distribution of the concomitants of record statistics for (3.4).

3. Bivariate Concomitants of Lower Records

The trivariate pseudo Inverse Weibull distribution has been defined in (2.3). We have also defiened the specific distribution in (2.4) for certain choices of $\phi_1(x)$ and $\phi_2(x,y)$. We now obtain the distribution of bivariate concomitant of records defined in (1.4). The joint conditional distribution of Y and Z given X is given in (2.11). Using marginal distribution of X; given in (2.1); in (1.1), the distribution of k-th lower record for random variable $X_k = X$ is:

$$f_{k:n}\left(x_k\right) = \frac{\beta_1}{\Gamma(k)} x^{-(k\beta_1+1)} \exp\left(-x^{-\beta_1}\right); x,\beta_1 > 0 \quad (3.1)$$

Using (2.11) and (3.1) in (1.4), the joint distribution of the pair of concomitants; $Y_k = Y$ and $Z_k = Z$; is obtained below:

$$f\left(y_k,z_k\right) = \int_0^\infty \beta_2\beta_3 x^{-2\beta_1} y^{-(2\beta_2+1)} z^{-(\beta_3+1)} \times \exp\left\{-x^{-\beta_1} y^{-\beta_2}\left(1+z^{-\beta_3}\right)\right\} \times \frac{\beta_1}{\Gamma(k)} x^{-(k\beta_1+1)} \exp\left(-x^{-\beta_1}\right)dx$$
$$= \frac{\beta_1\beta_2\beta_3}{\Gamma(k)} y^{-(2\beta_1+1)} z^{-(\beta_3+1)} \int_0^\infty x^{-\beta_1(k+2)-1} \times \exp\left\{-x^{-\beta_1}\left(1+y^{-\beta_2}+y^{-\beta_2}z^{-\beta_3}\right)\right\}dx$$

Making the transformation $x^{-\beta_1}\left(1+y^{-\beta_2}+y^{-\beta_2}z^{-\beta_3}\right) = w$ and simplifying, the joint distribution of two concomitants of lower record; $Y_k = Y$ and $Z_k = Z$; is obtained as:

$$f\left(y_k,z_k\right) = \frac{\beta_2\beta_3 k(k+1) y^{2k\beta_2-1} z^{\beta_3(k+1)-1}}{\left(1+z^{\beta_3}+y^{\beta_2}z^{\beta_3}\right)^{k+2}}; y,z > 0. \quad (3.2)$$

Graph of the density given in (3.2) is:

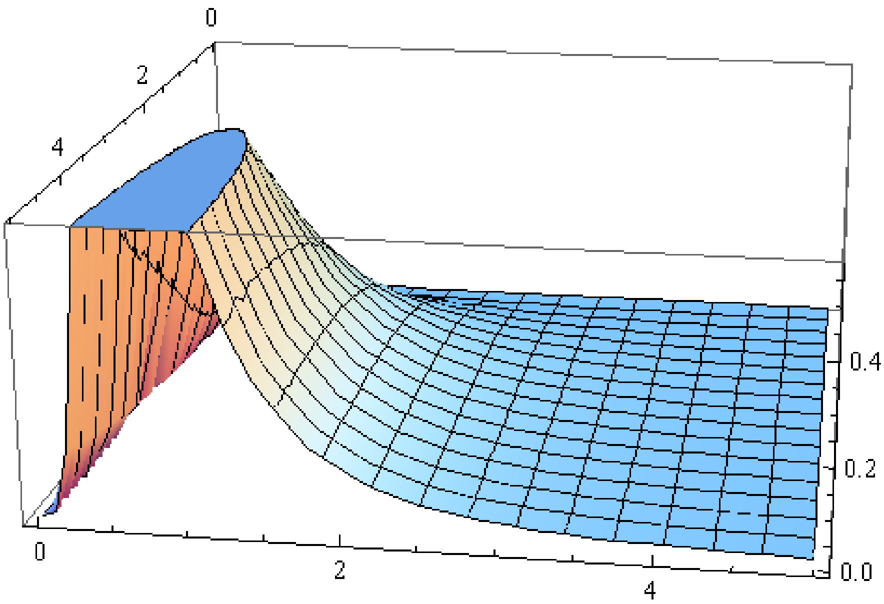

Figure 3.1: Joint distribution of two concomitants of lower record

The joint moments of two concomitants are given by:

$$\mu_{s,t}^{/} = E\left(Y_k^s, Z_k^t\right) = \int_0^\infty\int_0^\infty y^s z^t \frac{\beta_2\beta_3 k(k+1) y^{2\beta_2 - 1} z^{\beta_3 - 1}}{\left(1 + y^{\beta_2} + y^{\beta_2} z^{\beta_3}\right)^{k+2}} dydz$$

UsingAbramowitz(1964) and some simplification, the product moments of two concomitants are given as:

$$\mu_{s,t}^{/} = \frac{1}{\Gamma(k)}\Gamma\left(k + \frac{s}{\beta_2}\right)\Gamma\left(1 - \frac{s}{\beta_2} + \frac{t}{\beta_3}\right)\Gamma\left(1 - \frac{t}{\beta_3}\right). \quad (3.3)$$

We can easily compute the means, variances and covariance from (3.3). We now find the marginal distribution of $Z_k = z$ from (3.2) as:

$$f(z_k) = \int_0^\infty \frac{\beta_2\beta_3 k(k+1) y^{2\beta_2 - 1} z^{\beta_3 - 1}}{\left(1 + y^{\beta_2} + y^{\beta_2} z^{\beta_3}\right)^{k+2}} dy$$
$$= \frac{\beta_3 z^{\beta_3 - 1}}{\left(1 + z^{\beta_3}\right)^2}; z, \beta_3 > 0 \cdot \quad (3.4)$$

Using (3.2) and (3.4), the conditional distribution of $Y_k = Y$ given $Z_k = Z$ is:

$$f(y_k \mid z_k) = \frac{\beta_2 k(k+1) y^{k\beta_2 - 1} z^{k\beta_3}\left(1 + z^{\beta_3}\right)^2}{\left(1 + y^{\beta_2} + y^{\beta_2} z^{\beta_3}\right)^{k+2}}; y, z > 0 \quad (3.5)$$

The conditional moments of Y_k given Z_k are given as:

$$E\left(Y_k^s \mid z_k\right) = \int_0^\infty y_k^s \frac{\beta_2 k(k+1) y^{2\beta_2 - 1}\left(1 + z^{\beta_3}\right)^2}{\left(1 + y^{\beta_2} + y^{\beta_2} z^{\beta_3}\right)^{k+2}} dy$$

Using Abramowitz(1964), the conditional moments are given as:

$$E\left(Y_k^s \mid Z_k = z\right) = \frac{z^{-s\beta_3/\beta_2}\Gamma\left(k + \frac{s}{\beta_2}\right)\Gamma\left(2 - \frac{s}{\beta_2}\right)}{\left(1 + z^{\beta_3}\right)^{-s/\beta_2}\Gamma(k)}; s < k\beta_2 \quad (3.6)$$

The conditional mean and variance of Y_k given z_k can be readily obtained from (3.6).

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