MANOVA with Summary Statistics: A STATA Program

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Abstract

Almost all available statistical packages are capable of performing Multivariate Analysis of Variance (MANOVA) from raw data. Some of statistical packages have capability to perform independent sample t-test, ANOVA and some other tests of significance on summary data, but you come across not single software that has the capability to perform MANOVA directly on summary data. A STATA programme has been written to perform Multivariate ANOVA on summary data. The programme computes available statistics for Multivariate ANOVA (i.e Willk's Lembda, Lawley's-Hotelling trace, Pillae's trace and Roy's largest root). The programme is also capable to perform Box–M test for testing equality of covariance matrices on summary data. Example has been given by using the programme on summary data to perform Multivariate ANOVA.

Keywords: ANOVA, MANOVA, Summary data.

1. Introduction

Multivariate ANOVA (MANOVA) has been widely used for comparison of several factors when information on several variables has been collected. The simplest case is One Way MANOVA where levels of single factors are compared on the basis of information of several variables. One simple use of One Way MANOVA is to test the equality of mean vectors of several Multivariate Normal populations having common covariance matrix.

Most of the statistical packages are designed to conduct several test of significance when raw data is available, but in many practical situations only summary information is available (e.g. $n's$, $\bar{X}'s$, $S.D's$, . . .) from some report/research paper etc. Now some statistical package providers start realizing this situation and they have included some procedures in their software to perform some test of significance from summary data (e.g. Chi-Square, t-tests, ANOVA etc). David A. Larson (1992) describes a method to generate surrogate data from the summary statistics that can be used to perform one way ANOVA.

According to Larson one has to generate two new columns namely $X_j's$ and $X_n's$ as

$$X's = \bar{X}'s_j + \sqrt{\frac{S_j^2}{n_j}} \text{ and } X_n's = n_j\bar{X}_j - \left(n_j - 1\right)X_j's$$

And after some data manipulation data is ready to perform ANOVA in usual way.

Butt et al (2006) used an alternative method that can be used to perform one way, two way and higher way ANOVA by using summary measures $n_j, \bar{x}_j$ and s_j for $j = 1, 2, ..., k,$ where $n_j, \bar{x}_j$ and s_j are, respectively, the size, mean and standard deviation of j–th treatment. In this paper we extend this idea for the Multivariate Analysis of Variance case and have developed a STATA programme that can be used to perform one way MANOVA on summary data.

2. Methodology

The Multivariate ANOVA is a natural extension of the univariate ANOVA. In this technique the observation vectors are available from the multivariate normal populations having common covariance matrices. The task is to compare the mean vectors of these multinormal populations. Extending the idea of univariate study, the multivariate ANOVA can be presented as the Multivariate Linear Model. Specifically, the one way multivariate ANOVA model is given as:

$$\mathbf{x}_{ij} = \boldsymbol{\mu} + \boldsymbol{\tau}_j + \boldsymbol{\varepsilon}_{ij} \;;\; \boldsymbol{\varepsilon}_{ij} \;\; \textit{has} \;\; N_p\left(\mathbf{0};\boldsymbol{\Sigma}\right) \;\; \textit{and} \;\; \begin{cases} j = 1, 2, ..., k \\ i = 1, 2, ..., n_j \end{cases} \tag{2.1}$$

The hypothesis of interest in the fixed effect one way MANOVA model is generally given as:

$$H_0 : \boldsymbol{\tau}_j = 0 \;\; , \;\; j = 1,2,...,k \tag{2.2}$$

Another simple use of the one way MANOVA model is to test the hypothesis:

$$H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \cdots\cdots = \boldsymbol{\mu}_k \tag{2.3}$$

when independent random samples are available from $N_p\left(\boldsymbol{\mu}_j;\boldsymbol{\Sigma}\right)$.

Several test statistics are available in literature that enables us to test the hypothesis given in (2.2) and (2.3). The most popular of these statistics is the one given by Willks (1932). The statistic given by Willks (1932) is the ratio of two generalized variances and is given as:

$$F_0 = \left[\frac{ms - p\left(k-1\right)/2 + 1}{p\left(k-1\right)}\right] \cdot \frac{1 - \Lambda^{1/S}}{\Lambda^{1/S}} \tag{2.4}$$

with $$\Lambda = \frac{|\mathbf{W}|}{|\mathbf{B}+\mathbf{W}|}\ ;\ \mathbf{W} = \sum_{j=1}^{k}\left(n_j - 1\right)\mathbf{S}_j\ ;\ \mathbf{B} = \sum_{j=1}^{k} n_j \bar{\mathbf{x}}_j \bar{\mathbf{x}}_j^{/} - n\bar{\mathbf{x}}\bar{\mathbf{x}}^{/} \qquad (2.5)$$

The quantities $\bar{\mathbf{x}}_j$ and $\mathbf{S}_j$ are the mean vector and covariance matrix for *j* – th group and $\bar{\mathbf{x}}$ is the combined mean. The statistic given in (2.4) has an exact $F_{p(k-1);ms-p(k-1)/2+1}$ where:

$$m = (n-1) - (p+k)/2\ ;\ S^2 = \frac{p^2(k-1)^2 - 4}{p^2 + (k-1)^2 - 5} \qquad (2.6)$$

Another statistic proposed by Hotelling (1951) is based upon the trace of $\mathbf{T}^{-1}\mathbf{B}$ and is given as:

$$V = tr\left(\mathbf{T}^{-1}\mathbf{B}\right)\ ;\ F_1 = \frac{\left[(n-k-p-1)+a+1\right]V}{\left[\left(|k-1-p|-1\right)+a+1\right](a-V)}\ ; a = \min(k-1,p) \qquad (2.7)$$

The expression given in (2.7) has: $F_{\left[\left(|k-1-p|-1\right)+a+1\right];\left[(n-k-p-1)+a+1\right]}$

Pallie's trace (1955) has proposed yet another statistic based upon the trace of $\mathbf{W}^{-1}\mathbf{B}$. This statistic is given as:

$$U = tr\left(\mathbf{W}^{-1}\mathbf{B}\right);\ \mathrm{F}_2 = \frac{2U\left[a(n-k-p-1)/2+1\right]}{a^2\left[\left(2|k-p-1|-1\right)/2+a+1\right]} \qquad (2.8)$$

The expression given in (2.8) has: $F_{a\left[|k-1-p|+a\right];a(n-k-p-1)+2}$

Roy (1953) built his statistic on the largest root of $\mathbf{W}^{-1}\mathbf{B}$. His statistic is given as:

$$R = \max(\lambda_i);\quad \mathrm{F}_3 = \frac{R(n-b-1)}{b};\ b = \max(k-1,p) \qquad (2.9)$$

λ_i are eigen values of $\mathbf{W}^{-1}\mathbf{B}$;

The expression given in (2.9) has : $F_{b;n-b-1}$

The programme also performs the Box (1949) test for equality of covariance matrices by using the statistic:

$$B_0 = MC\ ;\ M = (n-k)\ln|\mathbf{S}| - \sum_{j=1}^{k}\left(n_j-1\right)\mathbf{S}_j\ ;\ \mathbf{S} = (n-k)^{-1}\mathbf{W} \qquad (2.10)$$

$$\text{and}\qquad C = 1 - \frac{2p^2+3p-1}{6(k-1)(p+1)}\left[\sum_{j=1}^{k}\frac{1}{n_j-1} - \frac{1}{n-k}\right]$$

In the following section we have given the programme and a live example is given in section 4 of the paper.

3. STATA Program

In this section we have given a Stata Mata program named “manovai.ado” that can be used to perform the MANOVA on summary data. The program “manovai.ado” requires four arguments as input, variable of group identification, variable of sample sizes, variable of mean vectors and variables of variance-covariance matrices respectively.

```
-------------------------------------------------- manovai.ado
program manovai, rclass
                version 9
                syntax varlist
                marksample touse
                mata: manovai("`varlist'", "`touse'")
                                                      end
        version 9
        mata:
        function manovai(string scalar varnames, string scalar touse)
            {
                string rowvector        vars, rhsvars
                string scalar           lhsvar, samp, gr
                real matrix             v, b,t, sum, sumb, sumw
                real scalar             count, gn, gm, dt
                real colvector          m, g, W
                vars = tokens(varnames)
                gr = vars[1]
                samp = vars[2]
                lhsvar  = vars[3]
                rhsvars = vars[|4\.|]
                st_view(v, ., rhsvars, touse)
                st_view(m, ., lhsvar, touse)
                st_view(s, ., samp, touse)
                st_view(g, ., gr, touse)
                info = panelsetup(g,1)
                  stats=panelstats(info)
                  count = stats[4]
                  k = stats[1]
                  sumw = J(count,count,0)
                  sumb = J(count,count,0)
                  gn = 0
                  dt = 0
                  nj = 0
                  gm = J(count,1,0)
  for (i=1; i<=rows(info); i++)
                              {
```

```
                        v_i = panelsubmatrix(v, i, info)
                        m_i = panelsubmatrix(m,i,info)
                        n_i = panelsubmatrix(s,i,info)
                        n = mean(n_i)
                        det = (log(det(v_i)))*(n-1)
                              w_i=(n-1)*v_i
                              b_i=n*m_i*(m_i)'
                              gn = gn + n
                              gm = ((gm + m_i*n))
                              sumw = sumw + w_i
                              sumb = sumb + b_i
                              dt = dt + det
                              nj = nj + 1/(n-1)
                                                }
summ = summ
sumb = sumb
gm = gm/gn
sumb=sumb - gn*gm*gm'
t = sumb + sumw

mm = (gn-1)-(count+k)/2
s = ((count^2*(k-1)^2 - 4)/(count^2+ (k-1)^2 - 5))^0.5
df1 = count*(k-1)
df2 = mm*s - ((count*(k-1))/2)+1
gdf = (k-1)
df = (gn -1)
edf= df - gdf
W = (k-1,count)
w = min(W)
d = max(W)

m = (gn-k)*log(det(sumw/(gn-k)))-dt
c = 1- (((2*count^2 + 3*count + 1)/(6*(count+1)*(k-1)))*(nj-1/(gn-k)))
q = m*c
df_q= (count*(count+1)*(k-1))/2
PQ = 1- chi2(df_q,q)

Lam = det(sumw)/det(t)
FW = (df2/df1)*(1-Lam^(1/s))/(Lam^(1/s))
PW=1- F(df1,df2,FW)

V= trace(invsym(t)*sumb)
FP = (((gn-k-count-1)+ w + 1)*V)/(((abs((k-1)-count)-1)+ w +1)*(w-V))
PP = 1- F(df1,df2,FP)
```

```
df1_p= w*((abs((k-1)-count)-1)+ w +1)
df2_p= w*((gn-k-count-1)+ w + 1)

U = trace(invsym(sumw)*sumb)
FL = (2*(w*(gn-k-count-1)/2+1)*U)/(w^2*((2*(abs(k-1-count)-1)/2)+w+1))
PL = 1- F(df1,df2,FL)
df1_l= w*((abs((k-1)-count)-1)+ w +1)
df2_l= w*(gn-k-count-1)+2

r = eigenvalues(invsym(sumw)*sumb)
r = Re(r)
R = max(r)
FR = R*((gn-k-d+k-1)/d)
PR = 1- F(df1,df2,FR)
df1_r= d
df2_r= (gn-k-d+k-1)

                        }
                              end

------------------------------------------------- manovai.ado
```

4. Numerical Example

In this section we have given a hypothetical example to demonstrate the usefulness of the program that can be used to perform MANOVA on summary statistics.

Example

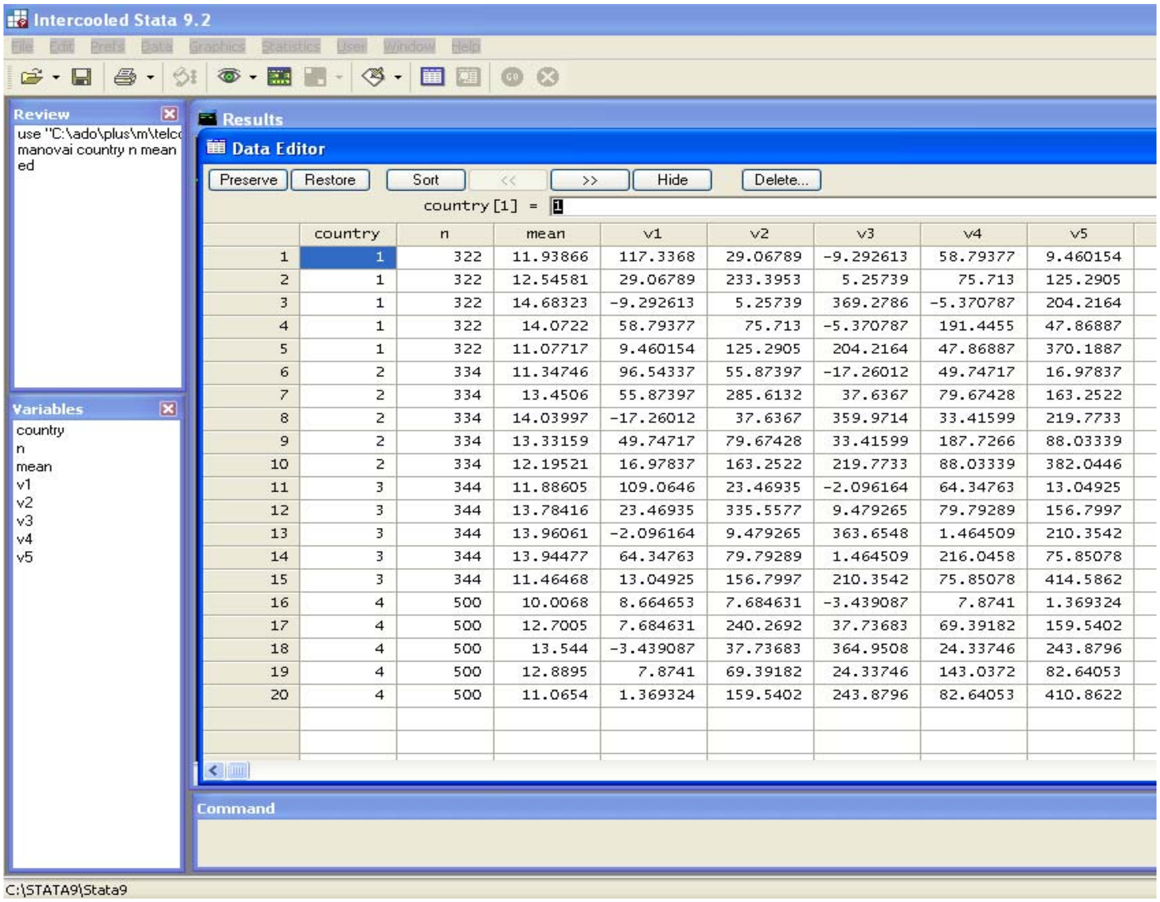
In particular, the telecommunication company has separated the monthly bill into amounts spent on Long distance, Toll free, Equipment, Calling card, and Wireless services, and categorized the customers based upon countries (India, China, Malaysia) and apply MANOVA test to see is there significant difference in monthly billings of these three countries. Suppose the results are published in a report along with summary statistics. Now suppose you wish to compare how monthly billings in Pakistan are different from these three countries.

You can have raw data for Pakistan but may not have raw data for other three countries. In such situation our program manovai can be used to perform MANOVA.

Table 4.1: Summary data that obtained from published report and calculated from Pakistani sample

				Variance - Covariance					
Country		**N**	**Mean**	**V1**	**V2**	**V3**	**V4**	**V5**	
India	Long Distance last month	322	11.9	117.3	29.1	-9.3	58.8	9.5	**Summary Information form published report**
	Toll free last month	322	12.5	29.1	233.4	5.3	75.7	125.3	
	Equipment last month	322	14.7	-9.3	5.3	369.3	-5.4	204.2	
	Calling card last month	322	14.1	58.8	75.7	-5.4	191.4	47.9	
	Wireless last month	322	11.1	9.5	125.3	204.2	47.9	370.2	
China	Long Distance last month	334	11.3	96.5	55.9	-17.3	49.7	17.0	
	Toll free last month	334	13.5	55.9	285.6	37.6	79.7	163.3	
	Equipment last month	334	14.0	-17.3	37.6	360.0	33.4	219.8	
	Calling card last month	334	13.3	49.7	79.7	33.4	187.7	88.0	
	Wireless last month	334	12.2	17.0	163.3	219.8	88.0	382.0	
Malaysia	Long Distance last month	344	11.9	109.1	23.5	-2.1	64.4	13.0	
	Toll free last month	344	13.8	23.5	335.6	9.5	79.8	156.8	
	Equipment last month	344	14.0	-2.5	9.5	363.7	1.5	210.4	
	Calling card last month	344	13.9	64.3	79.8	1.5	216.0	75.9	
	Wireless last month	344	11.5	13.0	156.8	210.4	75.9	414.6	
Pakistan	Long Distance last month	500	10.0	8.7	7.7	-3.4	7.9	1.4	**Summary calculated from Pakistani sample**
	Toll free last month	500	12.7	7.7	240.3	37.7	69.4	159.5	
	Equipment last month	500	13.9	-3.4	37.7	365.0	24.3	243.9	
	Calling card last month	500	12.9	7.9	69.4	24.3	143.0	82.6	
	Wireless last month	500	11.1	1.4	159.5	243.9	82.6	410.9	

Table given in table 4.1 can be entered directed into Stata data editor in the following manner



use "C:\ado\plus\m\telco
manovai country n mean
ed

country
n
mean
v1
v2
v3
v4
v5

country [1] = 1

	country	n	mean	v1	v2	v3	v4	v5
1	1	322	11.93866	117.3368	29.06789	-9.292613	58.79377	9.460154
2	1	322	12.54581	29.06789	233.3953	5.25739	75.713	125.2905
3	1	322	14.68323	-9.292613	5.25739	369.2786	-5.370787	204.2164
4	1	322	14.0722	58.79377	75.713	-5.370787	191.4455	47.86887
5	1	322	11.07717	9.460154	125.2905	204.2164	47.86887	370.1887
6	2	334	11.34746	96.54337	55.87397	-17.26012	49.74717	16.97837
7	2	334	13.4506	55.87397	285.6132	37.6367	79.67428	163.2522
8	2	334	14.03997	-17.26012	37.6367	359.9714	33.41599	219.7733
9	2	334	13.33159	49.74717	79.67428	33.41599	187.7266	88.03339
10	2	334	12.19521	16.97837	163.2522	219.7733	88.03339	382.0446
11	3	344	11.88605	109.0646	23.46935	-2.096164	64.34763	13.04925
12	3	344	13.78416	23.46935	335.5577	9.479265	79.79289	156.7997
13	3	344	13.96061	-2.096164	9.479265	363.6548	1.464509	210.3542
14	3	344	13.94477	64.34763	79.79289	1.464509	216.0458	75.85078
15	3	344	11.46468	13.04925	156.7997	210.3542	75.85078	414.5862
16	4	500	10.0068	8.664653	7.684631	-3.439087	7.8741	1.369324
17	4	500	12.7005	7.684631	240.2692	37.73683	69.39182	159.5402
18	4	500	13.544	-3.439087	37.73683	364.9508	24.33746	243.8796
19	4	500	12.8895	7.8741	69.39182	24.33746	143.0372	82.64053
20	4	500	11.0654	1.369324	159.5402	243.8796	82.64053	410.8622

Following out can be obtained after using “manovai” command

```
. manovai country n mean v1-v5

 Number of obs = 1500

 W = Wilks' lambda                  L = Lawley-Hotelling trace
 P = Pillai's trace                 R = Roy's largest root

-------------------------------------------------------------
 Source      Statistic     df  F(df1      df2)        F    prob>F
-------------------------------------------------------------
 Group     W    0.9879      3   15.0   4119.2      1.21   0.2540
           P    0.0121          15.0   4482.0      1.21   0.2552
           L    0.0122          15.0   4472.0      1.21   0.2530
           R    0.0102           5.0   1494.0      3.05   0.0001
-------------------------------------------------------------
 Residual                 1496
-------------------------------------------------------------
 Total                    1499
-------------------------------------------------------------
 Test of Homogeneity of Covariance Matrices
 Chi-Sq = 794.097     df = 45       P = 0.0000
```

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