Analytical Modeling of Magnetic DW Motion

S. A. Nasseri[1], G. Durin[1,2], C. Serpico[1,3]

1. ISI Foundation, Torino, Italy
2. Istituto Nazionale di Ricerca Metrologica (INRIM), Torino, Italy
3. Università degli Studi di Napoli Federico II, Napoli, Italy

1. Introduction

- Manipulating magnetic domain walls (DWs) within nanostructures has been linked with spintronic logic, storage and sensing devices.
- The 1-D model is the prime analytical model for describing magnetic DW motion [1-3]. While qualitatively successful, this model may at times fail to quantitatively match experimental and numerical results.

2. The Landau-Lifshitz Gilbert Equation

$$\frac{d\vec{m}}{dt} = -\gamma_0 \vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{d\vec{m}}{dt} + (\vec{u}.\nabla)\vec{m} - \beta \vec{m} \times [(\vec{u}.\nabla)\vec{m}] + \gamma_0 H_{FL} \vec{m} \times \hat{u}_{SOT} - \gamma_0 H_{SL} \vec{m} \times (\vec{m} \times \hat{u}_{SOT})$$

$$H_{eff} = -\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \vec{m}} \qquad e_D = \frac{dE}{dV} = A\sum |\nabla \vec{m}_i|^2 + K_U \sin^2\theta - \mu_0 M_s \vec{m}\cdot\vec{H}_a - \frac{1}{2}\mu_0 M_s \vec{m}\cdot\vec{H}_d + D[m_z \nabla\cdot\vec{m} - (\vec{m}\cdot\nabla)m_z]$$

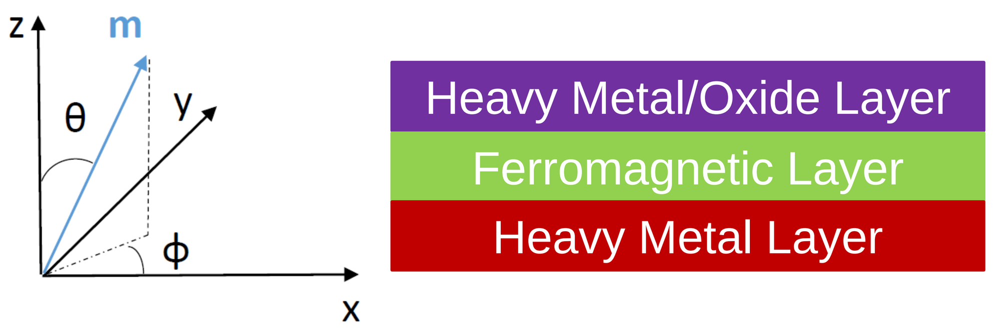

Fig. 1. Material and coordinate systems under study.

3. A Collective Coordinates Approach

Magnetic DWs in perpendicularly magnetized heterostructures may be described using a set of collective coordinates. We have selected the following 4 time-dependent coordinates which can lead to a generalized 1-D model:

- The position of the center of the DW (q)
- The tilt angle of the magnetization in the plane of the sample (ϕ)
- The domain wall width (Δ)
- The tilt angle of the wall (χ)

$$\phi(x,y,t) = \phi(t)$$

$$\theta(x,y,t) = 2\text{atan}\left(\exp\left(\frac{(x-q)\cos\chi + y\sin\chi}{\Delta}\right)\right)$$

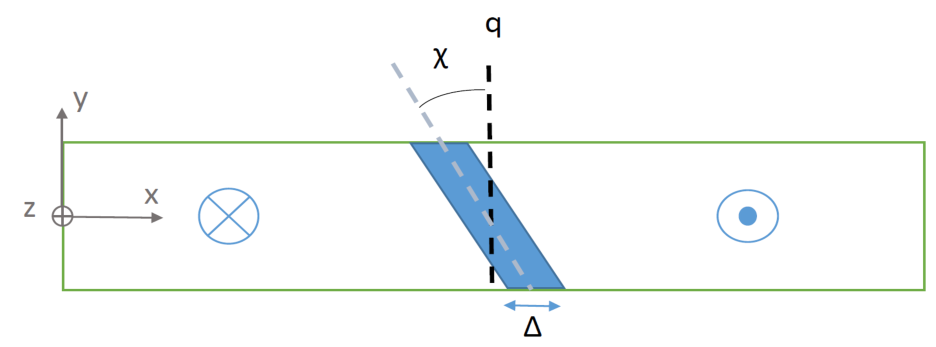

Fig. 2. Model of a tilted DW illustrating the collective coordinates.

$$(1+\alpha^2)\frac{\dot{q}}{\Delta}\cos\chi = \alpha\mu_0\gamma\left[H_z - \frac{\pi}{2}H_{sl}\left[\sin\phi\, u_{SOT_x} - \cos\phi\, u_{SOT_y}\right]\right] + (1+\alpha\beta)\frac{u}{\Delta}\cos\chi + \mu_0\gamma H_{sl}u_{SOT_z} + \frac{1}{2}\gamma M_s(N_y - N_x)\sin 2(\phi-\chi) + \frac{\pi}{2}\left[\mu_0\gamma[H_x\sin\phi - H_y\cos\phi] - \frac{\gamma D}{M_s\Delta}\sin(\phi-\chi)\right]$$

$$(1+\alpha^2)\dot{\phi} = \mu_0\gamma\left[H_z - \frac{\pi}{2}H_{sl}\left[\sin\phi\, u_{SOT_x} - \cos\phi\, u_{SOT_y}\right]\right] + (\beta-\alpha)\frac{u}{\Delta}\cos\chi - \alpha\mu_0\gamma\tau_{sl}u_{SOT_z} - \frac{\alpha}{2}\gamma M_s(N_y - N_x)\sin 2(\phi-\chi) - \alpha\frac{\pi}{2}\left[\mu_0\gamma[H_x\sin\phi - H_y\cos\phi] - \frac{\gamma D}{M_s\Delta}\sin(\phi-\chi)\right]$$

$$\frac{\alpha}{2}\frac{1}{6}\left(\frac{w}{\Delta}\right)^2\frac{\dot{\Delta}}{\Delta} = \frac{\gamma}{M_s}\frac{A}{\Delta^2}\left(\left(\frac{w}{\pi\Delta}\right)^2 + \sin^2\chi\right) + \frac{\gamma}{M_s}\left(\frac{w}{\pi\Delta}\right)^2\left[K_u + \frac{1}{2}\mu_0 M_s^2\left(N_x\cos^2(\phi-\chi) + N_y\sin^2(\phi-\chi) - N_z\right)\right] - \frac{1}{2}\gamma c M_s(N_y - N_x)\cos\chi\sin 2(\phi-\chi) + \frac{\pi}{2}\frac{\gamma D}{M_s\Delta}\sin\phi + \frac{\pi}{2}\left(\frac{w}{\pi\Delta}\right)^2\mu_0\gamma[H_x\cos\phi + H_y\sin\phi]$$

$$-\frac{\alpha}{2}\frac{1}{6}\left(\frac{w}{\Delta}\right)^2\frac{\dot{\chi}}{\cos\chi} = 2\frac{\gamma}{M_s}\frac{A}{\Delta^2}\sin\chi - \frac{1}{2}\gamma M_s(N_y - N_x)\cos\chi\sin 2(\phi-\chi) + \frac{\pi}{2}\frac{\gamma D}{M_s\Delta}\sin\phi$$

4. Controlling DW Motion with In-plane Fields

In-plane fields could be used to control the direction of DW motion in PMA heterostructures, due to the Dzyloshinskii-Moriya Interaction (DMI).

$$(1+\alpha^2)\frac{\dot{q}}{\Delta}\cos\chi = \frac{\pi}{2}H_{sl}\alpha\mu_0\gamma\cos\phi + \frac{1}{2}\gamma M_s(N_y - N_x)\sin 2(\phi-\chi) + \frac{\pi}{2}\left[\mu_0\gamma H_x\sin\phi - \frac{\gamma D}{M_s\Delta}\sin(\phi-\chi)\right]$$

$$H_{SL} = \frac{\hbar\theta_{SHE}J}{2\,e\,M_s t_f}$$

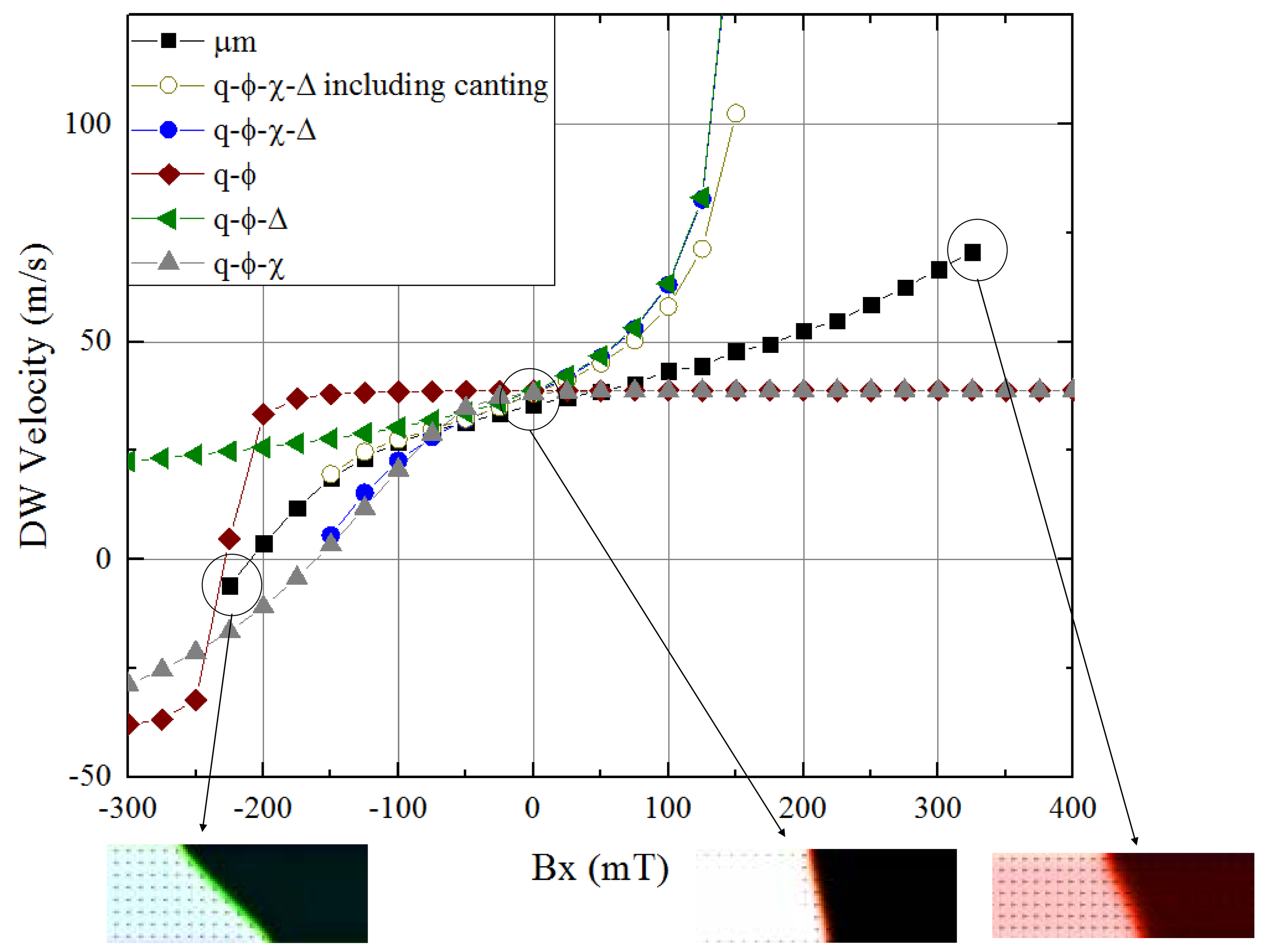

Fig 2. SHE-driven DW motion under in-plane fields in a Pt-Co-Mgo heterostructure with high DMI. Material properties [4]: saturation magnetization M_s =700 kA/m, exchange constant A = 0.1 pJ/m, uniaxial perpendicular anisotropy constant K_u = 480 kJ/m^3, Gilbert damping α = 0.3, SHE angle θ_{SHE}=0.07, Interfacial DMI D = 1.2 mJ/m^2.

5. Comparison to a Model based on Statistical Averages

- A model based on statistical averages has been proposed for field and spin-transfer torque driven DW motion [5].
- Provides a means of performing system identification (or as they call it "fingerprinting") on the system.
- Comparison to the analytical model could shed some light on the limitations and means of improving the analytical models.
- In a system with perpendicular magnetic anisotropy but no DMI:

1-D Model

$$(1+\alpha^2)\dot{\phi} = \mu_0\gamma H_z + (\beta-\alpha)\frac{u}{\Delta} - \alpha\gamma M_s(N_y - N_x)\frac{\sin 2\phi}{2}$$

Ghent Model

$$(1+\alpha^2)\left\langle\delta\frac{\partial\Phi}{\partial t}\right\rangle = \mu_0\gamma H_z - (\beta-\alpha)\frac{u}{\left(\frac{L_x}{2}\right)} - \alpha\gamma M_s(N_y - N_x)\frac{\langle m_j\rangle\langle m_k\rangle}{\langle\delta\rangle}$$

$$\Phi = \text{atan}\left(\frac{m_y}{m_x}\right) \qquad \delta = m_j^2 + m_k^2$$

6. References

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