Modeling and Simulation of an Induction Drive with Application to a Small Wind Turbine Generator

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***Abstract*- Among renewable energies the solution of utilizing wind energy conversion systems is now in a growing trend. A valid choice for operation of such systems may be the use of the induction machine. This study presents modeling and simulation of a stand-alone induction drive with application to a small wind turbine generator system. The model of the induction machine is written in terms of fluxes as state variables and is used to design an estimator model that can provide the correct feedback signals for the vector control of a wind energy conversion system. The estimation of magnetic flux and electromagnetic torque is based on stator currents and stator voltages. Simulation results are provided indicating good estimator design and operation.**

***Index Terms* — induction generator, wind turbine system model, estimator simulation, Kalman Filtering**

I. INTRODUCTION

In recent years the rapid development of digital signal processors (DSP) based systems and the decreasing cost of power electronics allowed the complex models and control algorithms of AC machines to become popular for a large range of applications. Among renewable energies the solution of utilizing induction generators for wind energy conversion systems is now in a growing trend. Currently, a wide spread control concept is that of a variable speed rotor with pitch regulation, and this concept is combined with both direct drive and geared drive trains the latter dominating the wind market.

A valid choice for operation at variable speed may be the use of induction machine. An induction motor can operate as a generator in super synchronous speed raised by an overhauling type of load, or by lowering the inverter frequency below the machine speed, when there is a converter-fed machine drive. Continuous regenerative operation of a drive is possible if the load machine is a source of power, such as in a wind generation system.

In this paper the model of the induction machine is written in terms of fluxes as state variables. This model is then used to design an Extended Kalman Filter (EKF) based estimator that can provide the correct feedback signals for the vector control of a wind energy conversion system.

The sensors employed to obtain state feedback information for the field- oriented control (FOC) are usually expensive, hard to mount and provide usually inadequate measurements because of the defective and aggressive environment where they act.

For the elimination of these errors several methods where proposed. One of them is the EKF algorithm which is presented in this work.

II. WIND TURBINE SYSTEM DESCRIPTION

In Figure 1 is shown a basic wind turbine system that converts the wind energy into mechanical energy which at its turn is used by an electric machine to generate power. The basic configuration consists of the wind turbine blades, the drive train, the induction generator and the *AC-AC* converter (not shown in Figure 2 to connect the system to the grid.

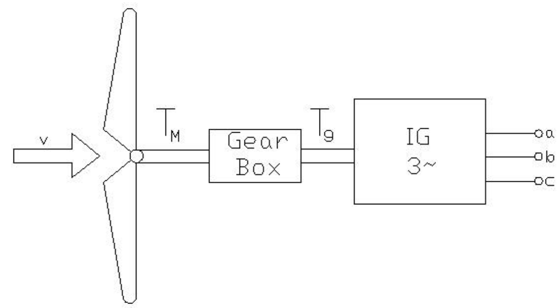

Figure 1. Basic wind turbine model

A. Wind turbine Model

The models of wind turbines take into account several characteristics such as the size, blade radius, nominal power, shafts stiffness, losses, gear box ratio, etc. The mechanical power produced by the wind turbine is:

$P_M = \frac{1}{2}\rho_{air} c_p \pi R^2 v^3$ where ρ is the air density, πR^2 is the turbine swept area, v is the speed of the wind, and $c_p(\lambda)$ is the power efficiency coefficient of the wind turbine which depends on the tip-speed ratio, λ. This is defined as:

$$\lambda = \frac{\omega_M R}{v} \tag{2}$$

where, $\omega_M = \frac{d\theta_M}{dt}$ is the angular speed of the turbine blades, and R is the length of the turbine blade.

There is an optimal value of the tip speed ratio, λ_{opt} , which allows a maximum capture of power from the wind. This value is found from the typical characteristic of the wind turbine power coefficient $c_p = f(\lambda)$ provided by the manufacturers.

When the wind turbine is operating with λ_{opt}, the nonlinear power expression from (1) may be recalculated as:

$$P_{M opt} = c_{opt}\omega_M^3 \quad (3)$$

and the corresponding mechanical torque produced by the blades is:

$$T_{M opt} = c_{opt}\omega_M^2 \quad (4)$$

where, C_{opt} is a constant which depends on the turbine characteristics and air density.

B. Drive train model

The drive train dynamics consists of the dynamics of the wind turbine rotor, low-speed shaft, gear box, high-speed shaft, and the induction generator. Figure 2 shows the basic diagram of the drive train that was used for the derivation of the mathematical model, [1].

Thus, for the wind turbine rotor the following torques equation can be written:

$$J_M \frac{d^2\theta_M}{dt^2} + T_G = T_M(t) \quad (5)$$

where, $T_G = K_1 \Delta\theta_{K1}$ is the reaction torque exerted by the flexible low-speed shaft on the rotor of the wind turbine.

Similarly, for the rotor of the induction generator the following torques equation is valid:

$$J_g \frac{d^2\theta_r}{dt^2} - T_g + T_e = 0 \quad (6)$$

where $T_g = K_2 \Delta\theta_{K2}$ is the reaction torque exerted by the high-speed shaft on the induction generator rotor, and T_e is the electromagnetic torque of the induction machine in generator operation mode.

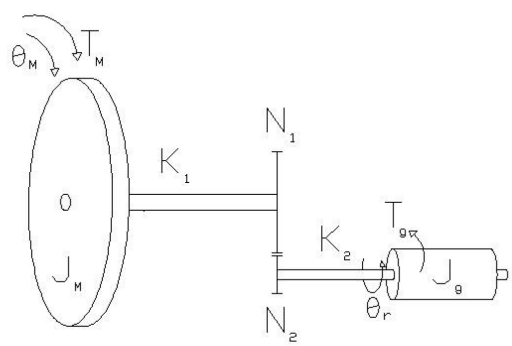

Figure 2. Drive train model

The reference values for the above mechanical parameters where reported in [2].

C. Induction generator model

In this paper, the model of the induction generator is derived using as state parameters the stator and rotor flux d-q axis components and the angular speed of the induction generator. The model is a fifth order dynamic model. It is also assumed that the induction drive is a stand alone generator so that the excitation capacitor and load impedance are added to the model.

The mathematical model of the induction machine having fluxes as state variables, and written in terms of inductances, is represented by the set of (7) – (11):

$$\frac{d\psi_{sd}}{dt} = -\frac{R_s}{\sigma L_s}(\psi_{sd} - \frac{L_m}{L_r}\psi_{rd}) + \omega_\lambda \psi_{sq} + u_{sd} \quad (7)$$

$$\frac{d\psi_{sq}}{dt} = -\frac{R_s}{\sigma L_s}(\psi_{sq} - \frac{L_m}{L_r}\omega_{rq}) - \omega_\lambda \psi_{sd} + u_{sq} \quad (8)$$

$$\frac{d\psi_{rd}}{dt} = -\frac{R_r}{\sigma L_s}(\psi_{sd} - \frac{L_m}{L_r}\psi_{sd}) + (\omega_\lambda - \omega)\psi_{rq} + u_{rd} \quad (9)$$

$$\frac{d\psi_{rq}}{dt} = -\frac{R_r}{\sigma L_r}(\psi_{rq} - \frac{L_m}{L_s}\psi_{sq}) - (\omega_\lambda - \omega)\psi_{rd} + u_{rq} \quad (10)$$

$$\frac{d\omega}{dt} = \frac{3}{2}\frac{z_p^2}{J}\frac{L_m}{L_s}(\psi_{sq} i_{rd} - \psi_{sd} i_{rq}) - \frac{z_p}{J} T_g \quad (11)$$

An induction generator may be self-excited by providing the magnetizing reactive power by a capacitor bank [3]. In Figure 3 a stand-alone induction generator under an *R-L* resistive-inductive load is shown.

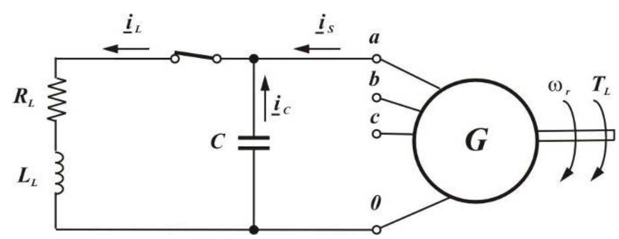

Figure 3. Single-phase R-L load circuit

The stator voltages u_{sd} and u_{sq} are computed as state variables in the generator's load impedance model as follows:

$$\frac{du_{sd}}{dt} = -\frac{1}{C} i_{sd} + \frac{1}{C} i_{Ld} \quad (12)$$

$$\frac{du_{sq}}{dt} = -\frac{1}{C} i_{sq} + \frac{1}{C} i_{Lq} \quad (13)$$

$$\frac{di_{Ld}}{dt} = \frac{1}{L_L}(u_{sd} - R_L i_{Ld}) + \omega_\lambda i_{Lq} \quad (14)$$

$$\frac{di_{Lq}}{dt} = \frac{1}{L_L}(u_{sq} - R_L i_{Lq}) - \omega_\lambda i_{Ld} \quad (15)$$

The electromagnetic torque expression can be written in terms of the state variables:

$$T_e = \frac{3}{2} z_p \frac{L_m}{\sigma L_s L_r} \left(\psi_{sq} \psi_{rd} - \psi_{sd} \psi_{rq} \right) \qquad (16)$$

Figure 4 shows the complete Simulink model of the wind conversion system described by (1)-(16).

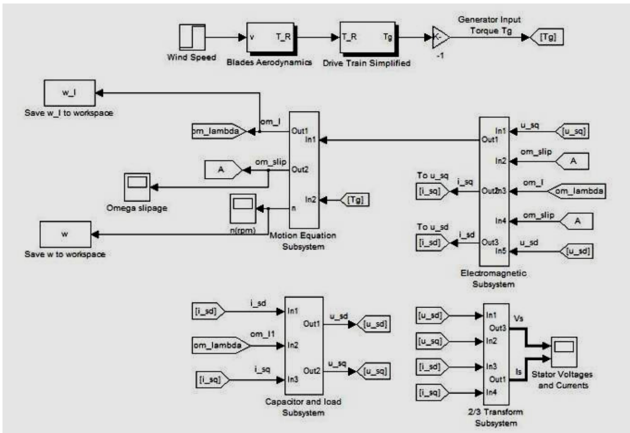

Figure 4. Simulink block diagram of the dynamic system

D. State space model of the induction machine

The state space equations (7)-(11) of the induction machine can be rewritten in matrix form as in (17):

$$\frac{dx}{dt} = A \cdot x + B \cdot u$$

$$y = C \cdot x \qquad (17)$$

In (17) the state variables input and output vectors are as follows:

$$x = \begin{bmatrix} \psi_{sd} & \psi_{sq} & \psi_{rd} & \psi_{rq} & 0 \end{bmatrix}^T \qquad (18)$$

$$u = \begin{bmatrix} u_{sd} & u_{sq} & 0 & 0 & 0 \end{bmatrix}^T \qquad (19)$$

$$y = \begin{bmatrix} i_{sd} & i_{sq} & i_{rd} & i_{rq} & 0 \end{bmatrix}^T \qquad (20)$$

The system matrix A, the input matrix B and the output matrix C are:

$$A = \begin{bmatrix} -\frac{R_s}{\sigma L_s} & \omega_\lambda & \frac{R_s L_m}{\sigma L_s L_r} & 0 & 0 \\ -\omega_\lambda & -\frac{R_s}{\sigma L_s} & 0 & \frac{R_s L_m}{\sigma L_s L_r} & 0 \\ \frac{R_r L_m}{\sigma L_s L_r} & 0 & -\frac{R_r}{\sigma L_r} & \omega_\lambda - \omega & 0 \\ 0 & \frac{R_r L_m}{\sigma L_s L_r} & \omega - \omega_\lambda & -\frac{R_r}{\sigma L_r} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (21)$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (22)$$

$$C = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & -\frac{L_m}{\sigma L_s L_r} & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & -\frac{L_m}{\sigma L_s L_r} & 0 \\ -\frac{L_m}{\sigma L_s L_r} & 0 & \frac{1}{\sigma L_r} & 0 & 0 \\ 0 & -\frac{L_m}{\sigma L_s L_r} & 0 & \frac{1}{\sigma L_r} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (23)$$

III. EXTENDED KALMAN FILTER STATE ESTIMATOR

The filtering problem involved in this paper is to find the best estimate of the state vector x_k of the induction machine which evolves according to the discrete- time nonlinear state transition equation:

$$x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \qquad (24)$$

where, f(.,.) is the machine dynamics, x_{k-1} is the state observation of the machine at sampling time k-1, u_{k-1} is the known input at time step k-1 and w_{k-1} is the system noise, which is white noise with a Gaussian distribution with mean zero and covariance Q.

Also we admit that we possess a set of noisy measurements, noted as z:

$$z_k = h(x_k) + v_k \qquad (25)$$

where, h(.) is a function of the state parameters and v_k is the measurement noise, with a Gaussian distribution with mean zero and covariance R.

First the estimated states are calculated (prediction step) then using the measurements these states are updated in function of the Kalman gain (K).

The estimation (prediction) equations are:

$$\hat{x}_k^- = f(\hat{x}_{k-1}^+, \hat{u}_{k-1}) + \hat{w}_{k-1} \qquad (26)$$

$$P_k^- = A_k P_{k-1} A_k^T + B_k U_{k-1} B_k^T + Q_{k-1} \qquad (27)$$

where, $\hat{x}_{k-1}^+$ is the previous state estimate, with covariance matrix P_{k-1}, $\hat{u}_{k-1}$ is the control input with covariance matrix U_{k-1}, $f(\cdot,\cdot)$ is the system dynamics function and $\hat{w}_{k-1}$ is the system noise with covariance Q_{k-1}.

The update equations following a measurement:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-)) \qquad (28)$$

$$P_K^+ = (I - K_k H_k) P_k^- \qquad (29)$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \qquad (30)$$

where, K_k is the Kalman gain, z_k is the measurement at time step k, h(.) is a function of the state vector, H is the observation matrix, R_k is the covariance of the measurement noise v_k.

At the first step of the algorithm the values of *x* and *P* is initialized with the prior knowledge about the system. It is not a trivial task to tune the values of the covariance *R* and *Q*. These values influence the performance of the filter although there is no direct method of choosing them in real time applications and it is often the case that they are selected in a trial and error method.

IV. Simulation Results

The induction machine model using fluxes as state variables has been used with numerical data shown in the Appendix.

The Matlab-Simulink software package was used for the modeling and simulation of the system.

The transient selected to validate the proposed model is the start-up regime of a loaded induction generator.

A successful build-up process, is predicted as Figure 5 shows for a slip frequency of -0.5ω, an excitation capacitance value of 700 μF, and a resistive inductive load with R=7kΩ and L=10 H.

The simulation was carried out for a speed of 400 rpm of the induction generator as shown in Figure 6.

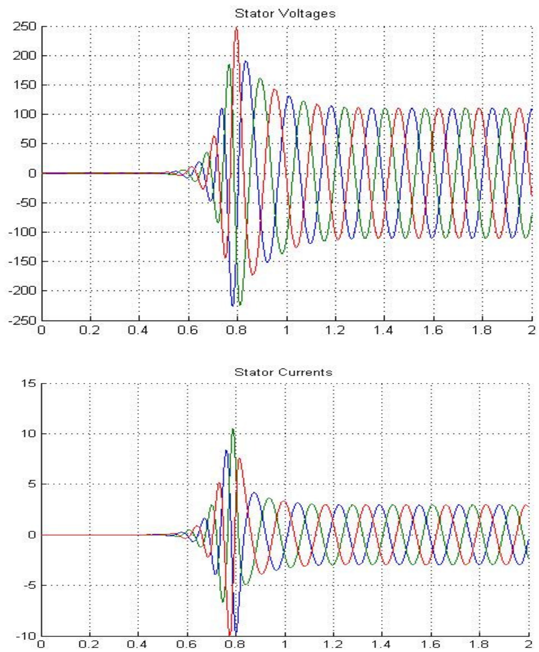

Figure 5. Stator voltages (top) and currents (bottom) during a start-up of the loaded induction generator

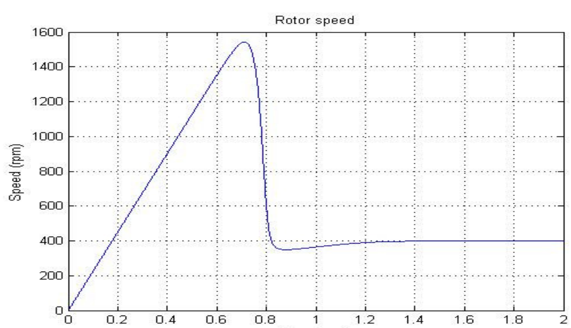

Figure 6. Rotor speed of induction drive

For the EKF algorithm the rotor voltages are considered zero. The inputs for the Kalman Filter are the stator voltages, and the stator currents are the noise measurements.

On Figure 7 the estimated state variables are shown, the stator and rotor d-q axis fluxes. The Figure 8 shows the rotor flux magnitude and the rotor flux vector angle.

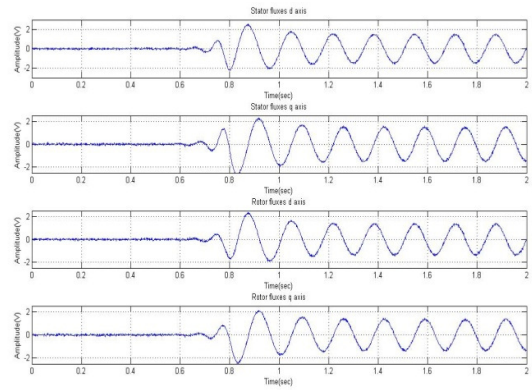

Figure 7. Induction motor estimated states with EKF estimated d-q axis stator fluxes (one-two) and estimated d-q axis rotor fluxes (three-four)

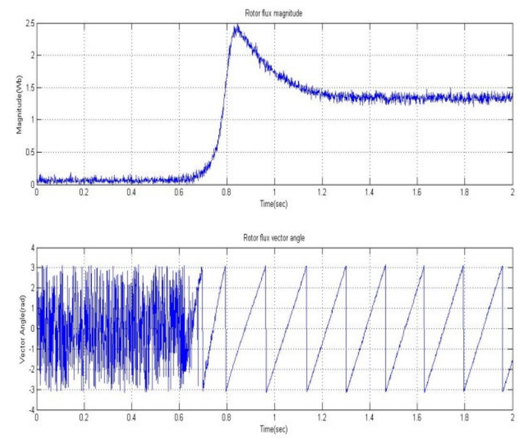

Figure 8. Estimated rotor flux magnitude and angle

V. Conclusions

In this paper simulation of a wind energy conversion system based on the induction generator was carried out using Matlab-Simulink programming environment.

Good simulation results were obtained based on real data of the wind turbine and the induction generator to validate them.

Also, a stator and rotor flux estimator model based on EKF was developed to simulate the correct achievement of the field orientation.

The block diagrams in the future will be used to simulate the system in real time using an existing dSPACE DS 1104 control board. This board is based on a floating point DSP with high speed ADC converters which makes suitable for the cross compilation of the Simulink models into the dedicated platform. With this step it will be possible to reduce the time of prototyping based on the Hardware in the Loop (HIL) technique.

Appendix

The induction generator parameters are:
Rs=7.782 Ω; Ls=0.487 H; Lm=0.45H;
Rr=7.1 Ω; Lr=0.47 H;
zp=2, pole pairs;
J=0.0106 kgm^2;

R-L generator load: R_L=7kΩ; L_L=10 H;
Excitation capacitor: C=700 μF.

List of Symbols

s, r : subscripts, denoting stator and rotor quantities.

$\omega\,(\omega_r), \omega_\lambda$: electrical (and mechanical) angular speed of the induction generator rotor, and *d-q* rotating reference frame speed.

R_s , R_r : stator, rotor, ohmic resistances.

$T_r = \frac{L_r}{R_r}$: rotor circuit time constant.

z_p : number of induction machine pole pairs.

T_e : electromagnetic torque.

$\psi_{sd}, \psi_{sq}, \psi_{rd}, \psi_{rq}$: *d-q* components of stator and rotor transient flux linkages.

L_s, L_r, L_m : stator, rotor inductances, and the magnetization inductance.

$\sigma = 1 - \frac{L_m^2}{L_s L_r}$: leakages coefficient.

u_{sd} , u_{sq} : *d-q* components of stator and rotor transient voltages.

$i_{sd}, i_{sq}, i_{rd}, i_{rq}$: *d-q* components of stator and rotor transient currents .

J_M : lumped inertia of the wind turbine rotor and the attached low-speed shaft, in Fig. 3, in [kgm^2].

K_L : stiffness of the low-speed shaft in [Nm/rad].

θ_M , θ_G : angular displacements of the low-speed shaft in [rad].

$N_2/N_1 = N$: gear ratio.

K_H : stiffness of the high-speed shaft in [Nm/rad].

θ_g, θ_r : angular displacements of the high-speed shaft in [rad].

J_g : lumped inertia of the electric generator rotor and the attached high-speed shaft in [kgm^2].

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