Optimal Networked Control Systems with State-dependent Markov Channels

Bin Hu and Tua A. Tamba

***Abstract*—This paper considers a co-design problem for industrial networked control systems to ensure both system stability and resources efficiency. The assurance of system stability and efficiency is challenging due to the fact that wireless communications in industrial environments are subject to shadow fading, and are stochastically correlated with their surrounding environments. To address such challenges, this paper first introduces a novel state-dependent Markov channel model that explicitly captures the *state-dependent features* of the wireless communications by correlating model's transition probabilities with environment states. Under the proposed channel model, sufficient conditions on *maximum allowable transmission interval* are presented to ensure *almost sure asymptotic stability* for the nonlinear networked control system. With the stability constraints, the co-design problem is then formulated as constrained optimization problems, which can be efficiently solved by SDP programs for a two-state Markov channel. Simulation results are provided to demonstrate the efficacy of the proposed co-design scheme.**

I. Introduction

Over the past decades, wireless communication technologies have evolved rapidly and became ubiquitous in our modern society. As important components to improve modern industrial automation, wireless communication protocols such as WirelessHart and WiMAX [1], have been successfully implemented in various industrial applications with the goals of building efficient, safe, and reconfigurable industrial automation systems. Building a safe and efficient industrial networked control system, however, is fairly challenging due to the fact that wireless communication channels in industrial environments are inherently unreliable and subject to *shadow fading*. Shadow fading causes significant degradation on the communication link, thereby seriously compromising system stability and performance.

Recent studies have shown that the radio communication in industrial environments often exhibits *shadow fading* that is statistically dependent on various environment states, such as large metal objects, moving machines and vehicles [2]–[4]. Such *state-dependent features* prevent conventional modeling formalisms, such as Markov chain or identically distributed independent process (i.i.d.), from being applicable to complex industrial environments [5], [6]. Research efforts have been devoted to developing effective channel models that correlate temporal variations of channel conditions with the external environment states in different industrial settings, e.g., [3]–[7].

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The proposed *state-dependent Markov channel model* in this paper differs from existing models in two aspects. First, the works in [3], [4] model the external environment (a moving vehicle) as a (semi)-Markov chain assuming that the moving vehicle cannot be controlled. This paper removes the uncontrollable assumptions and model the external environment as a Markov Decision Process (MDP). Secondly, the channel models adopted in [3], [4], [7] are confined to characterize random packet dropouts and ignore the quantize effects on the state, while this paper considers a more generalized state-dependent Markov model that takes into account of time varying data rates.

Power control has been well studied as an effective means to mitigate fading effects in wireless community. To ensure both system stability and efficiency for the whole industrial networked system, a joint design of power and control strategies must be considered. The traditional methods to solve this joint-design problem are to follow the so-called separation principle where the optimal design of communication and control strategies can be separated by assuming both systems are independent from each other. Along this line of research, numerous co-design results were developed for control and state estimation of networked control systems, see e.g., in [3], [4], [8], [9]. The independence assumption, however, has limited applicability in complex industrial process environments where the communication and control parts of the networked control system are tightly coupled with the presence of *state-dependent fading channels*.

The present paper extends prior work by developing a novel co-design framework that explores the state-dependent properties of wireless communication channels to ensure both stability and optimal performance for industrial networked systems. This paper first proposes a novel *state-dependent Markov channel model* to explicitly capture the stochastic correlation between data rates, external environments, and the transmission power. Based on the proposed channel model, this paper further presents sufficient conditions on Maximum Allowable Transmission Interval (MATI) that assure *almost sure asymptotic stability* for the nonlinear networked control system. The co-design framework is formulated as a constrained optimization problem with the stability conditions as hard constraints. The solutions to the constrained optimization problem represent optimal control and transmission power policies that minimize an average joint costs for both communication and control systems. This paper further shows that the constrained optimization problem can be efficiently solved by a SDP for a two-state Markov channel.

Notations: Throughout the paper, let $\mathbb{R}$, $\mathbb{Z}$ denote the real

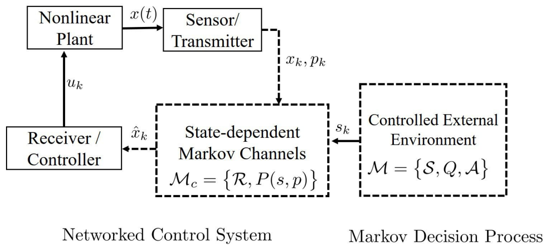

Fig. 1: Nonlinear Networked Control System with State-dependent Markov Channels

and integer number respectively, and $\mathbb{R}_{\geq 0}, \mathbb{Z}_{\geq 0}$ denote their non-negative counterparts. Let $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the n-dimensional real vector space and matrix with the dimension of $n \times m$, respectively. $\forall x \in \mathbb{R}^n$, the infinity norm of the vector is denoted by $|x| = \max_i |x_i|, 1 \leq i \leq n$ with x_i being the i^{th} element of the vector. For notation simplicity, let $|A| := \|A\|_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^{n} |a_{ij}|$ denote the infinity norm for a matrix $A \in \mathbb{R}^{n \times m}$.

II. System Framework

Fig. 1 depicts a system framework considered in this paper, which consists of a *nonlinear plant*, a *state-dependent Markov channel* $\mathcal{M}_c$, a *remote controller* and the *external environment* that is modeled by a Markov Decision process $\mathcal{M}$.

1) Nonlinear Plant: The dynamics of the nonlinear plant are modeled by the following ODE

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ represents the states of the nonlinear plant, $u \in \mathbb{R}^{n_u}$ is the input to the system that is generated by a remote controller. The vector field $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$ is a nonlinear function that is locally Lipschitz with respect to x. In this paper, we assume that the system state x can not be directly accessed by the controller, and thus must be transmitted through a wireless communication channel.

2) Sensor and Transmitter: The state x is first pre-processed by a *sensor/transmitter* module before the transmission. This paper considers both the sampling and quantization effects on the state x. Specifically, let $\{t_k\}_{k=0}^{\infty}$ denote a sequence of sampling time instants with $t_k < t_{k+1}$, and $x_k := x(t_k)$ denote the sampled state at time instant t_k. The sampled state $x(t_k)$ is then encoded by one of a finite number of symbols that are constructed based on a dynamic quantization scheme [10]. The dynamic quantization scheme maps the sampled state $x \in \mathbb{R}^{n_x}$ into the index of a finite number of symbols. Specifically, let $R \in \mathbb{N}$ denote the number of bits used to construct the symbol, and the sequence of the symbols can then labeled as $\mathcal{S} = \{1, 2, \ldots, 2^R\}$ with a total number of 2^R. The way of using the symbol to encode the state information $x(t_k)$ is by constructing a dynamic quantizer that is able to track the evolution of the $x(t_k)$ at each transmission instant. The quantizer is defined as a tuple $\mathcal{Q} = (\mathcal{S}, q(\cdot), \xi)$ where $q(\cdot) : \mathbb{R}^{n_x} \to \mathcal{S}$ is a quantization function that maps the system state into the symbol and $\xi \in \mathbb{R}_{\geq 0}$ is an anxiliary variable defining the size of the quantization regions. One typical method to implement the quantizer $\mathcal{Q}$ is the construction of a hybercubic box that is evolved dynamically to contain and track the state x. To demonstrate the mechanism of such box-based dynamic quantizer, suppose a hypercubic box is constructed at time instant t_k with $\hat{x}(t_k)$ denoted as the center of the box and $2\xi(t_k)$ as its size. Then, the box is divided equally into 2^R smaller sub-boxes with each sub-box labeled as one of the symbols $\mathcal{S}$. Among all the symbols, let $q(x) \in \mathcal{S}$ denote the symbol (sub-box) that contains the state information x. The center of that sub-box, $\hat{x}(t_k^+)$, is then used as an updated estimate of the state information x at time instant t_k. The hyper-cubic box is then updated with a new center $\hat{x}(t_k^+)$ and a new size $\xi(t_k^+) = \xi(t_k)/2^R$. The symbol representing this updated hyber-cubic is transmitted through the wireless communication channel. The following equations are used to characterize the dynamics of the dynamic quantizer

$$\hat{x}(t_k^+) = h(k, q(x(t_k)), \hat{x}(t_k), \xi(t_k), R_k) \tag{2a}$$

$$\xi(t_k^+) = \frac{\xi(t_k)}{2^{R_k}} \tag{2b}$$

where R_k is the number of bits available at time instant t_k, and is a time varying variable that depends on the wireless channel conditions in real time. As discussed in prior work [10], [11], within each time interval $[t_k, t_{k+1}), \forall k \in \mathbb{Z}_{\geq 0}$, the size of the hyper-cubic box needs to be propagated to ensure that the constructed box captures the actual state x. To be specific, we define the following differential equation to characterize the evolution of the size over time

$$\dot{\xi}(t) = g_\xi(\xi), \forall t \in [t_k, t_{k+1}) \tag{3}$$

3) Remote Controller: Under the dynamic quantizer $\mathcal{Q}$, this paper considers a model-based remote controller that maintains a "copy" of the plant dynamics in (1) defined as below,

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, u), \\ u &= \kappa(\hat{x}), \forall t \in [t_k, t_{k+1}) \end{aligned} \tag{4}$$

with the initial state $\hat{x}(t_k) = \hat{x}(t_k^+)$ over the time interval $[t_k, t_{k+1})$. The control function $\kappa(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_u}$ is a "nominal" controller that is selected to stabilize the dynamic system in (4) without considering the effect of the network.

With the definitions of system dynamics in (1), dynamic quantizer in (2) and (3), and remote controller in (4), the closed loop system can be characterized as a stochastic hybrid system defined as below,

$$\dot{x} = \tilde{f}(x, e) \tag{5a}$$

$$\dot{e} = g_e(x, e) \tag{5b}$$

$$\dot{\xi} = g_\xi(\xi), \forall t \in (t_k, t_{k+1}) \tag{5c}$$

and

$$e(t_k^+) = J_e(k, x(t_k), e(t_k), \xi(t_k), R_k) \tag{6a}$$
$$\xi(t_k^+) = J_\xi(\xi(t_k), R_k), \forall k \in \mathbb{Z}_{\geq 0} \tag{6b}$$

where $e := x - \hat{x}$ is the estimation error and $\tilde{f}(x,e) = f(x, \kappa(x-e))$, $g_e(x,e) = f(x,\kappa(x-e)) - f(x-e, \kappa(x-e))$, $J_e(k, x(t_k), e(t_k), \xi(t_k), R_k) = x(t_k) - h(k, q(x(t_k)), x(t_k) - e(t_k), \xi(t_k), R_k)$ and $J_\xi = \xi(t_k)2^{-R_k}$. The equations (5) above characterize the continuous dynamics of the closed loop system while the equations (6) describe the stochastic jump behavior of the system whose dynamics are governed by the time varying data rate R_k.

4) Controlled External Environments & State-dependent Markov Channel: The external environments in industrial settings, e.g., moving vehicles or machines, are modeled by a MDP $\mathcal{M}_{env} = \{S, s_0, A, Q\}$ where $S = \{s_i\}_{i=1}^{M_s}$ is a finite set of environment states, s_0 is an initial state, $A = \{a_i\}_{i=1}^{M_a}$ is a finite set of actions, and $Q = \{q(s|s', a)\}_{s,s' \in S, a \in A}$ is a transition matrix. Taking a forklift vehicle operating in an industrial factory as an example, the state set S in MDP represents a group of partitions for the regions in the factory floor. By taking an action $a \in A$, the vehicle moves from one region s' to another s following the transition probabilities $q(s|s', a)$. Under the environment model, the quality of wireless communication links is affected by which state/region the vehicle is located. The link quality is measured by a time varying data rate selected from a finite set $\mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$.

Let the sequence $\mathcal{I} = \{t_k\}_{k=0}^{\infty}$ denote the transmission time instants, and the random variable R_k denote the data rates at time instant t_k. Then, $\{R_k\}_{k=0}^{\infty}$ over these transmission time instants form a random process that characterizes the stochastic variations on the channel conditions. At each time instant t_k, the communication system can adjust its transmission power level to send the data through a wireless communication channel. Let $\Omega_p = \{1, 2, \ldots, M_p\}$ denote a finite set of transmission power levels with $i \in \Omega_p$ representing the power level i. The transmission power set is sorted in an ascending order such that larger number represents higher power level. Let $p_k := p(t_k) \in \Omega_p$ denote the power level selected at time instant t_k. With the external environment modeled as a MDP, the *state-dependent Markov Channel* is defined as below,

Definition 2.1: Given a power set Ω_p, an external environment modeled by a MDP $\mathcal{M}_{env}$ and a finite set of data rates $\mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$ with the data rates being arranged ascendingly, i.e., $r_i < r_j, \forall i < j$, a wireless communication channel is a *state-dependent Markov Channel* if $\forall s \in S, p \in \Omega_p$ and $\forall r_i, r_j \in \mathcal{R}$

$$\mathbb{P}\{R_{k+1} = r_i | R_k = r_j, s_k = s, p_k = p\} = P_{ij}(s,p) \tag{7}$$

where $P_{ij}(s,p)$ is a transition probability from data rate r_j to r_i given the transmission power p and environment state s. The *state-dependent Markov channel* model in (7) can be viewed as a generalization of traditional Markov channel that ignores the impact of environment state and transmission power [12], [13].

III. Problem Formulation

Given the closed-loop networked control system represented by (5), (6) and (7), the first goal is to achieve *almost sure asymptotic stability*, which is formally defined as below,

Definition 3.1 (Almost Sure Asymptotic Stability [14]): The stochastic hybrid system defined in (5) and (6) is said to be *almost surely asymptotically stable* (ASAS) if $\forall \epsilon, t' > 0$, such that:

$$\mathbb{P}\{\lim_{t' \to \infty} \sup_{t \geq t'} |x(t)| \geq \epsilon\} = 0 \tag{8}$$

Problem 3.2 (Stability Problem): The stability problem considered in this paper is to find the Maximum Allowable Transmission Time Interval (MATI) under which the networked control system with *state-dependent Markov channel* satisfies the notion of *almost sure asymptotic stability* defined in Definition 3.1.

Problem 3.3 (Co-design Problem): Let μ_m and μ_p denote the control policy and transmission power policy respectively, and for a given joint cost function $\{c(s,p,r)\}_{s \in S, p \in \Omega_p, r \in \mathcal{R}}$ the second problem is to find an optimal policy $\mu^\star := (\mu_m^\star, \mu_p^\star)$ such that the following average expected costs are minimized under the stability conditions obtained by solving Problem 3.2,

$$\begin{aligned} \min_{\mu_m, \mu_p} \quad & \lim_{\ell \to \infty} \frac{1}{\ell} \mathbb{E}_{s_0, R_0}^{\mu_m, \mu_p} \sum_{k=0}^{\ell} c(s_k, p_k, R_k) \\ \text{s.t.} \quad & \text{Stability conditions ensuring (8)} \end{aligned} \tag{9}$$

where s_0 and R_0 are initial states for the MDP system and Markov channel respectively.

IV. Main Results

This section presents the main results of this paper. Firstly, sufficient conditions on MATI are presented to ensure *almost sure asymptotic stability*. Secondly, this paper shows that the co-design problem can be solved by constrained optimizations with stability conditions as constraints.

The following assumptions are needed to derive the main results.

Assumption 4.1 ([15], [16]): Consider the stochastic hybrid system in (5) and (6). Let $\overline{e} := [e; \xi]$ denote an augmented vector for the error states e and the size of the dynamic quantizer ξ. Suppose there exist a function $W : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e+1} \to \mathbb{R}_{\geq 0}$ that is locally Lipschitz with respect to $\overline{e}$, a locally Lipschitz, positive definite, radially unbounded function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, a continuous function $H : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, a finite set of constants $\{\lambda_i\}_{i=1}^{M_R}$, real numbers $L \geq 0, \zeta > 0$, positive constants $\underline{\alpha}_W, \overline{\alpha}_W, \underline{\alpha}_V, \overline{\alpha}_V, \varrho > 0$ such that the followings hold:

1) $\forall k \in \mathbb{N}$, $\overline{e} \in \mathbb{R}^{n_x+1}$ and $r_i \in \mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$

$$\underline{\alpha}_W |\overline{e}|^2 \leq W(k, \overline{e}) \leq \overline{\alpha}_W |\overline{e}| \tag{10a}$$
$$W(k+1, \overline{J}(k, \overline{e}, r_i)) \leq \lambda_i W(k, \overline{e}) \tag{10b}$$

where $\overline{J}(k, \overline{e}, r_i) = [J_e; J_\xi]$ with the functions J_e and J_ξ defined in (6).

2) $\forall k \in \mathbb{N}, x \in \mathbb{R}^{n_x}$ and for almost all $\overline{e} \in \mathbb{R}^{n_x+1}$, such that

$$\left\langle \frac{\partial W(k,\overline{e})}{\partial \overline{e}}, \overline{g}(x,\overline{e}) \right\rangle \leq LW(k,\overline{e}) + H(x) \tag{11}$$

where $\overline{g}(x,\overline{e}) = [g_e; g_\xi]$ with the functions g_e and g_ξ defined in (5).

3) $\forall x \in \mathbb{R}^{n_x}$

$$\underline{\alpha}_V |x|^2 \leq V(x) \leq \overline{\alpha}_V |x|^2 \tag{12}$$

and $\forall k \in \mathbb{N}, \overline{e} \in \mathbb{R}^{n_x+1}$, and for almost all $x \in \mathbb{R}^{n_x}$

$$\begin{aligned}\langle \nabla V(x), \tilde{f}(x,e) \rangle \leq \varrho |x|^2 - \varrho W(k,\overline{e}) - H^2(x) \\ + \zeta^2 W^2(k,\overline{e})\end{aligned} \tag{13}$$

Remark 4.2: This assumption is essentially the same as the Assumption 1 in [16] where the inequalities (10) of part 1) are used to characterize the bounds on the function of error states $\overline{e}$ as well as its growths for different data rates at discrete time instants. It is assumed that for each given data rate $r_i \in \mathcal{R}$, there exists a corresponding positive real λ_i that bounds the growth of the error function from the above. The inequality (11) of part 2) assumes a linear growth of the error function in the continuous time domain. The inequalities (12) and (13) of part 3) are used to characterize the growth rate of the Lyapunov function with respect to the state x in the continuous time domain. The MATI bounds that ensure stochastic stability will be derived based on the parameters given in this assumption.

A. Sufficient Conditions for Stochastic Stability

Under the Assumption 4.1, sufficient conditions on the MATI are presented to ensure *almost sure asymptotic stability* for the stochastic hybrid system defined in (5) and (6) under the state dependent Markov channels.

Theorem 4.3: Consider the stochastic hybrid system defined in (5) and (6), the state-dependent Markov channel in (7), and the controlled external environment $\mathcal{M}_{env}$. Suppose Assumption 4.1 holds, let T_{MATI} denote the maximum allowable time interval, for a given joint policy $\mu = (\mu_m, \mu_p)$, the system is *almost surely asymptotically stable* if

$$T_{MATI} \leq \begin{cases} \frac{1}{L\eta} \arctan\left(\frac{\eta(1-\overline{\lambda})}{2\frac{\overline{\lambda}}{1+\overline{\lambda}}(\frac{\zeta}{L}-1)+1+\overline{\lambda}} \right) & \zeta > L \\ \frac{1}{L}\frac{1-\overline{\lambda}}{1+\overline{\lambda}} & \zeta = L \\ \frac{1}{L\eta} \operatorname{arctanh}\left(\frac{\eta(1-\overline{\lambda})}{2\frac{\overline{\lambda}}{1+\overline{\lambda}}(\frac{\zeta}{L}-1)+1+\overline{\lambda}} \right) & \zeta < L \end{cases} \tag{14}$$

with $\eta = \sqrt{\left|(\frac{\zeta}{L})^2 - 1\right|}$ for some constant $\overline{\lambda}$ that satisfies

$$\overline{\lambda} > \sqrt{\| \operatorname{diag}(\lambda_i^2)\overline{P}(\mu) \|} \tag{15}$$

where $\overline{P}(\mu) = \left[\overline{P}_{ij}(\mu)\right]_{1 \leq i,j \leq M_R}$ with $\overline{P}_{ij}(\mu) = \sum_{p\in\Omega_p, s\in S} \mathbb{P}(r_i|r_j,s,p)\mathbb{P}(s,p|r_j)$.

Proof: The idea of the proof is to first show that the stochastic hybrid system is *exponentially stable in expectation*. Then by the Borel-Cantelli Lemma, one can show that the *exponential stability in expectation* implies the *almost sure asymptotic stability* defined in (8) in Definition 3.1. Let $\overline{x} := [x^T, \overline{e}, \tau, k]^T$ denote an augmented state, and $F(\overline{x}) := [\tilde{f}(x,e)^T, \overline{g}(x,\overline{e})^T, 1, 0]^T$. Define a candidate Lyapunov function for the augmented state $\overline{x}$ as $U(\overline{x}) := V(x) + \zeta\phi(\tau)W^2(k,\overline{e})$ where the function $\phi(\tau) : [0, T_{MATI}] \to \mathbb{R}$ is the solution to the following nonlinear differential equation $\dot{\phi} = -2L\phi - \zeta(\phi^2+1), \quad \phi(0) = \overline{\lambda}^{-1}$. Following the proof of Theorem 1 in [16], for all τ and k, $\langle \nabla U(\overline{x}), F(\overline{x}) \rangle \leq -\varrho|x|^2 - \varrho W(k,\overline{e}) \leq -\varrho|x|^2 - \varrho\overline{\alpha}_W|\overline{e}|^2 \leq -(\varrho + \varrho\overline{\alpha}_W)(|x|^2 + |\overline{e}|^2) = -\tilde{\varrho}|\overline{x}|^2$ with $\tilde{\varrho} = \varrho + \varrho\overline{\alpha}_W$. By the conditions in (10) and (12), it is straightforward to show that the $\underline{\alpha}_U|\overline{x}|^2 \leq U(\overline{x}) \leq \overline{\alpha}_U|\overline{x}|^2$ with the positive constants $\underline{\alpha}_U = \underline{\alpha}_V + \zeta\overline{\lambda}\underline{\alpha}_W$ and $\overline{\alpha}_U = \overline{\alpha}_V + \zeta\overline{\lambda}^{-1}\overline{\alpha}_W$ where $\overline{\lambda}$ is defined in (15). Then, one can show that $U(\overline{x}(t,k)) \leq \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}(t-t_k)\right) U(\overline{x}(t_k,k)), \forall k \in \mathbb{N}, t - t_k \in [0, T_{MATI}]$.

Note that the randomness of the system comes from the stochastic jump equations in (6) where the sequence of the augmented state $\{\overline{e}(t_k^+)\}$ is a stochastic process whose probability distribution is governed by the state-dependent Markov channel. Let $\mathbb{1}_A$ denote an indicator function that takes value 1 when sample value falls in the set A and takes value 0 otherwise. For a given set of data rates $\mathcal{R} = \{r_i\}_{i=1}^{M_R}$, let $U_{k+1} := U(\overline{x}(t_{k+1}^+))$, then define a vector $\overline{U}_{k+1} := \Big[\mathbb{E}[U_{k+1}\mathbb{1}_{r_1}], \ldots, \mathbb{E}[U_{k+1}\mathbb{1}_{r_i}], \ldots, \mathbb{E}[U_{k+1}\mathbb{1}_{r_{M_R}}]\Big]^T$ where $\forall i,j$

$$\begin{aligned} &\mathbb{E}[U_{k+1}\mathbb{1}_{r_i}] \\ &= \mathbb{E}[V(x(t_{k+1}^+))\mathbb{1}_{r_i}] + \zeta\phi(\tau^+)\mathbb{E}[W^2(k+1, \overline{J}(k,\overline{e},r))\mathbb{1}_{r_i}] \\ &\overset{(a)}{=} V(x(t_{k+1}) + \zeta\phi(0)\mathbb{E}[W^2(k+1, \overline{J}(k,\overline{e},r))\mathbb{1}_{r_i}] \\ &\overset{(b)}{\leq} V(x(t_{k+1}) + \zeta\overline{\lambda}^{-1}\sum_{j=1}^{M_R} \mathbb{E}[\lambda_i^2 W^2(k,\overline{e})\mathbb{1}_{r_j}\mathbb{1}_{r_i}] \\ &\overset{(c)}{=} V(x(t_{k+1}) + \zeta\overline{\lambda}^{-1}\lambda_i^2 \sum_{j=1}^{M_R} \underbrace{\sum_{s\in S, p\in\Omega_p} \mathbb{P}(r_i|r_j,s,p)\mathbb{P}(s,p|r_j)}_{\overline{P}_{ij}(\mu)} \\ &\times \mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_j}] \end{aligned} \tag{16}$$

The equality (a) holds due to the fact that the state x is continuously evolved and is independent of the random jump caused by the Markov channel and MDP process. The inequality (b) holds because of the conditions (10) in Assumption 4.1. The equality (c) holds because of the *state-dependent Markov channel model* defined in (7) and the MDP process. Since the inequality (16) holds for all r_i with $\forall i = 1, 2, \ldots, M_R$,

$j = 1, 2, \ldots, M_S$, the vector $\overline{U}_{k+1}$ satisfies

$$\overline{U}_{k+1} \le V(x(t_{k+1}))\mathbf{e}_{M_R} + \zeta\overline{\lambda}^{-1} \underbrace{\begin{bmatrix} \lambda_1^2\overline{P}_{11} & \lambda_1^2\overline{P}_{12} & \cdots & \lambda_1^2\overline{P}_{1M_R} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_i^2\overline{P}_{i1} & \lambda_i^2\overline{P}_{i2} & \cdots & \lambda_i^2\overline{P}_{iM_R} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{M_R}^2\overline{P}_{M_R1} & \lambda_{M_R}^2\overline{P}_{M_R2} & \cdots & \lambda_{M_R}^2\overline{P}_{M_RM_R} \end{bmatrix}}_{\mathrm{diag}(\lambda_i^2)\overline{P}(\mu)} \times \underbrace{\begin{bmatrix} \mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_1}] \\ \mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_2}] \\ \vdots \\ \mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_{M_R}}] \end{bmatrix}}_{\overline{W^2}(k,\overline{e})} \tag{17}$$

where $\mathbf{e}_{M_RS} := [1, 1, \ldots, 1]^T$ is a column vector with M_R of 1, and the transition matrix $\overline{P}(\mu)$ is a function of the joint policy $\mu = (\mu_m, \mu_p)$. Specifically, $\forall i, j$, $\overline{P}_{ij}(\mu) = \sum_{s\in S, p\in\Omega_p} \mathbb{P}(r_i|r_j, s, p)\mathbb{P}(s, p|r_j)$ where the conditional probability $\mathbb{P}(s, p|r_j)$ can be viewed as a joint policy specifying the likelihood of selecting the MDP state $s \in S$ and transmission power $p \in \Omega_p$ when the channel date rate is $r_j \in \mathcal{R}$. By taking the infinity norm on both sides of the inequality in (17), one further has

$$\begin{aligned} &|\overline{U}_{k+1}| \\ &\overset{(d)}{\le} V(x(t_{k+1})) + \zeta\overline{\lambda}^{-1}\|\,\mathrm{diag}(\lambda_i^2\mathbb{I}_{M_S})\overline{P}^T(\mu^m, \mu^p)\||\overline{W^2}(k,\overline{e})| \\ &\overset{(e)}{\le} V(x(t_{k+1})) + \zeta\overline{\lambda}^{-1}\overline{\lambda}^2|\overline{W^2}(k,\overline{e})| \\ &\overset{(f)}{\le} |\overline{U}_k| \end{aligned} \tag{18}$$

The inequality (d) holds due to the norm condition $|y| \le |Ax| \le \|A\||x|$, $\forall x \in \mathbb{R}^n, y \in \mathbb{R}^m$ and $A \in \mathbb{R}^{n\times m}$. The inequality (e) holds due to the condition in (15). The inequality (f) holds by the claim that $|\overline{U}(\overline{x})| = V(x) + \zeta\phi(\tau)|\overline{W^2}|$. To prove the claim, note that the expectations of the positive function $U(\overline{x})$ and $W(k, \overline{e})$ are positive for all $\overline{x}, \overline{e}$. Since $\overline{U}(\overline{x}) = V(x)\mathbf{e} + \zeta\phi(\tau)\overline{W^2}(k, \overline{e})$, one can prove the claim by noting that $|\overline{U}(\overline{x})| = |V(x)\mathbf{e} + \zeta\phi(\tau)\overline{W^2}(k, \overline{e})| = V(x) + \zeta\phi(\tau)|\overline{W^2}(k, \overline{e})|$ because $V(x), \zeta, \phi(\tau) \ge 0$ are constant with respect to the expectation operator. The condition in (18) ensures that the expected value of the Lyapunov function is non-increasing at the stochastic jump states.

By combining the dynamics of both the continuous and discrete flows of the stochastic hybrid system proved above, one can show that $\mathbb{E}[U(\overline{x}(t,k))] \le \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}t\right)U(\overline{x}(0,0))$ which implies that $\mathbb{E}[|\overline{x}(t,k)|^2] \le \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}t\right)\frac{\overline{\alpha}_U}{\underline{\alpha}_U}|\overline{x}(0,0)|^2$. Following the proof of Theorem 9 in [7], one can show that the *exponential stability in expectation* implies the *almost sure asymptotic stability* by using the Borel-Cantelli Lemma. Please refer to [7], [17] for the details of the proof. ■

Remark 4.4: The MATI bounds shown in (14) are functions of the parameters ξ and L defined in Assumption 4.1, and $\overline{\lambda}$ that depends on the parameters of the SD-MC model in (7). The proposed MATI bounds differ the existing results of [7], [15]–[17] in two aspects. First, the MATI bounds in (14) generalizes the results in [15], [16] by taking into account of the impact of stochastic communication channels on the MATI. Such an impact is characterized by the selection of the parameter $\overline{\lambda}$ that must satisfy the inequality (15). Thus, the existing results can be recovered from our MATI bounds by making the parameter $\overline{\lambda}$ independent of the channel conditions. Second, the MATI results in this paper extend our prior work in [7], [17] by considering a less conservative assumption on the system structure and a more general state-dependent Markov channel.

B. Optimal Co-design of Control and Power Policies: A Constrained Optimization Problem

This section formulates the optimal co-design problem of control and power policies as a constrained optimization where the stability condition in (15) serves as constraints. The challenge of solving such constraint optimization problems lies in the difficulty of dealing with the stability constraint. In particular, the stability condition derived in (15) is equivalent to polynomial constraints in the optimization problem.

The following theorem shows that if stationary policies are considered, the co-design Problem 3.3 can be reformulated as a polynomial constrained program with a linear objective function.

Theorem 4.5: Consider the following polynomial constrained optimization problem, for given sets of MDP state S, transmission power Ω_p and data rate $\mathcal{R}$, and $\forall 1 \le i \le M_R$, let $X(s, r, p) \ge 0, \forall s, r, p$ denote the decision variables for the following optimization problem

$$\min_{\{X(s,r,p)\}} \sum_{s\in S, p\in\Omega_p, r\in\mathcal{R}} c(r, s, p)X(r, s, p) \tag{19a}$$

$$\text{s.t.} \quad \sum_{s,p} X(s, r_i, p) - \sum_{p,s,r_j} P_{ij}(s, p)X(r_j, s, p) = 0, \quad \forall r_i \in \mathcal{R} \tag{19b}$$

$$\sum_{s,p,r} X(s, r, p) = 1, \tag{19c}$$

$$\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s, p)X(s, p)\prod_{\ell\ne j} X(r_\ell) - \theta_i^2\prod_{j=1}^{M_R} X(r_j) \le 0 \tag{19d}$$

where $X(s, p) = \sum_{j=1}^{M_R} X(r, s, p)$, $X(r) = \sum_{s,p} X(r, s, p)$[1], $\theta_i = \overline{\lambda}/\lambda_i$, and $P_{ij}(s, p)$ is the transition probability of the SD-MC channel defined in (7). The optimal stationary power policy $\mu_p^\star = \{\mathbb{P}(p|r)\}_{p\in\Omega_p, r\in\mathcal{R}}$ and optimal probability

[1] $X(A)$ denote the occupation (probability) measure [18] that assigns a probability to the event A

distribution $\pi^\star = \{\mathbb{P}(s)\}_{s\in S}$ for the MDP states can then be represented as below

$$\mathbb{P}(p|r) = \frac{\sum_{s\in S} X^*(s,r,p)}{\sum_{p\in\Omega_p, s\in S} X^*(s,r,p)} \tag{20}$$

$$\mathbb{P}(s) = \sum_{p\in\Omega_p, r\in\mathcal{R}} X^*(s,r,p) \tag{21}$$

where $\{X^*(s,r,p)\}_{s\in S, r\in\mathcal{R}, p\in\Omega_p}$ are the solutions to the polynomial constrained program in (19).

Proof: The proof is based on the occupation method used in [18]. For any stationary control and power policy, let $X(r,s,p) := \mathbb{P}(r,s,p)$ denote the corresponding stationary probability distribution of the states s, r and the action p over the set $S\times\mathcal{R}\times\Omega_p$. Under the stationary policy space, the objective function can be equivalently represented by (19a) due to the following conditions

$$\begin{aligned}
&\lim_{\ell\to\infty}\frac{1}{\ell}\mathbb{E}\sum_{k=0}^{\ell} c(s_k,p_k,R_k) = \lim_{T\to\infty}\frac{1}{T}\sum_{i=0}^{T}\mathbb{E}[c(s_k,p_k,R_k)] \\
&= \sum_{\substack{s\in S, r\in\mathcal{R}\\ p\in\Omega_p}} \Bigg[\underbrace{\lim_{\ell\to\infty}\frac{1}{\ell}\sum_{k=0}^{\ell}\mathbb{P}(s_k=s,p_k=p,R_k=r)}_{X(s,p,r)} c(s,p,r)\Bigg] \\
&= \sum_{s\in S, r\in\mathcal{R}, p\in\Omega_p} X(s,p,r)c(s,p,r).
\end{aligned}$$

The conditions in (19b) and (19c) are there to ensure the Markov property of the channel model in (7) under the stationary policy as well as the probability definition of $\{X(r,s,p)\}$. To prove the equivalence between the constraint in (19d) and the stability condition derived in (15), consider $\forall 1\le i\le M_R$

$$\begin{aligned}
&\|\operatorname{diag}(\lambda_i^2)\overline{P}\| \le \overline{\lambda}^2 \iff \lambda_i^2\sum_{j=1}^{M_R}\overline{P}_{ij} \le \overline{\lambda}^2 \\
&\iff \sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)\mathbb{P}(s,p\mid r_j) \le \frac{\overline{\lambda}^2}{\lambda_i^2} \triangleq \theta_i^2
\end{aligned} \tag{22}$$

where $P_{ij}(s,p)$ is the conditional probability defined in the state-dependent Markov channel model in (7), and $\lambda_i, \overline{\lambda}$ are parameters introduced in Assumption 4.1. By Bayes's Law, one can rewrite the conditional probability $\mathbb{P}(s,p\mid r_j)$ in terms of the decision variables $\{X(s,p,r)\}$

$$\begin{aligned}
\mathbb{P}(s,p\mid r_j) &= \frac{\mathbb{P}(s,p)}{\mathbb{P}(r_j)} = \frac{\mathbb{P}(s,p)}{\sum_{s,p}\mathbb{P}(s,p,r_j)} \\
&= \frac{X(s,p)}{\sum_{s,p}X(s,p,r_j)} = \frac{\sum_{r_j}X(s,p,r_j)}{\sum_{s,p}X(s,p,r_j)}
\end{aligned} \tag{23}$$

By replacing $\mathbb{P}(s,p\mid r_j)$ in (22) with its alternative representation in (23), the constraint (22) can be equivalently rewritten as

$$\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)\frac{\sum_{r_j}X(s,p,r_j)}{\sum_{s,p}X(s,p,r_j)} \le \theta_i^2, \quad \forall 1\le i\le M_R \tag{24}$$

Let $X(r_j) := \sum_{s,p}X(s,p,r_j) > 0$ and $X(s,p) := \sum_{r_j}X(s,p,r_j)$, then multiplying $\prod_{j=1}^{M_R}X(r_j) > 0$ on both sides of inequality (24) leads to

$$\begin{aligned}
&\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)\frac{X(s,p)}{X(r_j)}\prod_{j=1}^{M_R}X(r_j) \le \theta_i^2\prod_{j=1}^{M_R}X(r_j), \forall i \\
&\iff \sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)X(s,p)\prod_{\ell\ne j}X(r_\ell) \le \theta_i^2\prod_{j=1}^{M_R}X(r_j), \forall i.
\end{aligned}$$

Thus, we have shown that the solutions to the constrained polynomial optimization problem in (19) is equivalent to the solutions to the original optimization problem introduced in Problem 3.3. The proof is complete. ■

The stability conditions in (19d) are polynomial constraints where the order of the polynomial functions depends on the number of states (data rate) in the SD-MC model. If a two-state SD-MC model is considered, it can be shown that the polynomial constraints can be reduced to quadratic constraints, which can be efficiently solved by SDP programs. The two-state SD-MC can be considered as a generalization of the well known bursty erasure channel [12].

With the optimal power policy $\mu_p^\star = \{\mathbb{P}(p|r)\}_{p\in\Omega_p, r\in\mathcal{R}}$ and optimal stationary distribution for the MDP states $\pi^\star = \{\mathbb{P}(s)\}_{s\in S}$ obtained from Theorem 4.5, the next step is to find a control policy $\mu_m = \{\mathbb{P}(a|s)\}_{a\in A(s), s\in S}$ for MDP to achieve the optimal stationary distribution $\pi^\star$. Let $\{c_m(s,a)\}_{s\in S, a\in A}$ denote the cost function for each state-action pair of the MDP process. The optimal control policy $\mu_m^\star$ is obtained by solving the following optimization problem

Problem 4.6: Consider an ergodic MDP process $\mathcal{M}_{env} = \{S, s_0, A, Q\}$ with a desired stationary distribution $\pi^\star$ obtained by solving the optimization problem (19), the objective is to find an optimal control policy $\mu_m^\star$ that minimizes the following objective function while attaining the desired stationary distribution $\pi^\star$,

$$\min_{\mu_m}\lim_{T\to\infty}\frac{1}{T}\mathbb{E}_{s_0}^{\mu_m}\sum_{i=0}^{T}c_m(s_k,a_k) \tag{25a}$$

$$\text{s.t. } Q(\mu_m)\pi^\star = \pi^\star \tag{25b}$$

where $Q(\mu_m)$ is the transition matrix of the induced Markov chain under the control policy μ.

The following Theorem 4.7 shows that the Problem 4.6 can be efficiently solved by a linear program.

Theorem 4.7: Consider an ergodic MDP process $\mathcal{M}_{env} = \{S, s_0, A, Q\}$ with the associated cost function $\{c_m(s,a)\}_{s\in S, a\in A}$, for a given stationary distribution $\pi^\star = [\pi^\star(s_1), \ldots, \pi^\star(s_{M_s})]^T$ with $\pi^\star(s)$ representing the probability distribution for the state $s \in S$, let $\{Y(s,a)\}_{s\in S, a\in A}$ denote the decision variables for the following LP problem,

$$\min_{\{Y(s,a)\}} \sum_{s\in S, a\in A} c_m(s,a)Y(s,a) \tag{26a}$$

$$\text{s.t.} \quad \sum_a Y(s,a) - \sum_{s',a} q(s|s',a)Y(s',a) = 0, \forall s \in S \tag{26b}$$

$$\sum_{s\in S, a\in A} Y(s,a) = 1, \quad Y(s,a) \geq 0, \forall s, a \tag{26c}$$

$$\sum_{a\in A} Y(s,a) = \pi^\star(s), \forall s \in S. \tag{26d}$$

Then, the Problem 4.6 can be solved by the LP formulated in (26) and the corresponding optimal control policy $\mu_m^\star$ can be obtained by $\mathbb{P}(a|s) = \frac{Y^*(s,a)}{\sum_{a\in A} Y^*(s,a)}, \forall s \in S, a \in A$. where $\{Y^*(s,a)\}$ is the solution to the LP (26).

Proof: The proof follows straightforwardly from the LP representation for constrained MDP (See equation (4.3) in Chapter 4 of [18]). The equation (26d) is equivalent to the constraint of the stationary distribution imposed in (25b). ■

V. Simulation Results

In the simulation, a linear batch reactor process described in [21] is used for the networked control system part. The state-space model for the unstable linear batch reactor process is $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

and $B = [0, 0; 5.679, 0; 1.136, -3.146; 1.136, 0]$. The process is controlled by a remote controller that uses an estimation of the state $\hat{x}$ from a model-based estimator. The system dynamics from the controller side can then be modeled as $\dot{\hat{x}} = A\hat{x} + Bu, u = K\hat{x}, \quad \forall t \in [t_k, t_k + T)$. with the state feedback controller gain selected to be

$$K = \begin{bmatrix} 0.6961 & 0.8133 & 0.5639 & -1.8492 \\ 2.6908 & 1.1764 & -1.2762 & 0.9968 \end{bmatrix}$$

In this simulation, the controlled external environment is a moving vehicle that is modeled by a two-state MDP with a state set of $\{s_1, s_2\}$ and an action set of $\{\text{Go}, \text{Stay}\}$. The MDP states represent the partitions of regions in the factory floor. Let the state s_1 denote the region that can cause shadow fading if it is occupied by the vehicle, and s_2 denote the non-shadow fading region. The transition probabilities of the states under each action are provided in Table I. Under the external environment, a two-state SD-MC model is used to simulate the *state-dependent* fading channel used for the networked batch reactor process. In the two-state SD-MC model, the states of $r_1 = 0$ and $r_2 = 2$ are selected to represent the data rates supported by the wireless communication system. A set of $\{H, L\}$ is selected for transmission power with H representing the high transmission power and L representing the low transmission power. The state-dependent transition probabilities under different environmental states and transmission power are summarized in Table II.

TABLE I: MDP $\mathcal{M}$ Transition Probability Q and Costs c_m

	s_1	s_2	$c_m(s,a)$
$s_1, Stay$	0.9	0.1	0.4
s_1, Go	0.1	0.9	0.4
$s_2, Stay$	0.9	0.1	0.6
s_2, Go	0.1	0.9	0.6

TABLE II: $\mathcal{M}_c$ Transition Probability $P_{ij}(s,p)$ and Power-Rate Costs c_p, c_r

	r_1	r_2	$c_p(p)$	$c_r(r)$
$r_1, (s_1, L)$	0.8	0.1	0.4	0.6
$r_1, (s_1, H)$	0.6	0.4	0.6	0.6
$r_1, (s_2, L)$	0.4	0.6	0.4	0.4
$r_1, (s_2, H)$	0.1	0.9	0.6	0.4
$r_2, (s_1, L)$	0.8	0.2	0.4	0.6
$r_2, (s_1, H)$	0.6	0.4	0.6	0.6
$r_2, (s_2, L)$	0.5	0.5	0.4	0.4
$r_2, (s_2, H)$	0.1	0.9	0.6	0.4

Given the process model and channel model, let $W(\bar{e}) = |\bar{e}|^2$, the parameters $L = 17.8870, \zeta = 26.5415$ and $\lambda_0 = 1, \lambda_1 = 0.5$ are determined to satisfy the Assumption 4.1. From equation (14) in Theorem 4.3, the MATI can be determined as $MATI = 0.0104s$ for $\overline{\lambda} > \sqrt{\| \operatorname{diag}(\lambda_i^2)\overline{P}(\mu)\|} = 0.6325$ with $\overline{P}(\mu) = [0.2, 0.2; 0.8, 0.8]$[2]. The transmission time interval is then selected to be $T = 0.01s \leq MATI$. Fig. 2 shows the maximum (blue dashed line) and minimum (red dash-dot line) value of the system trajectories evaluated over 1000 runs with the same initial values under $T = 0.01s$. The simulation results show that both maximum and minimum trajectories asymptotically converge to the origin, which implies the almost sure asymptotic stability.

Besides the system stability, the system performance under the proposed co-design strategy and the separation design method are compared to demonstrate the benefits and robustness of our approach over a wide range of shadow fading levels. In the simulation, different shadow fading levels are simulated by changing the transition probability $\mathbb{P}(0 \mid 0, s_1, H)$ under the shadow fading state s_1 and high transmission power H in the channel model. In the separation design method, the optimal power policy is designed to minimize only the communication costs [3] while respecting the stability constraint. The optimal control policy is determined to minimize the costs [4] induced by controlling the moving vehicle. Fig. 3 shows the comparison results of optimal joint costs generated by the separation design method (marked by black dash-dot line) and

[2]The transition matrix $P(\mu)$ is calculated based on a corresponding control and transmission power policy μ and the state-dependent Markov channel model

[3]$\min_{\mu_p} \lim_{\ell\to+\infty} \frac{1}{\ell}\mathbb{E}\sum_{k=0}^{\ell}[c_p(p_k) + c_r(R_k)]$ with c_p, c_r listed in Table II

[4]$\min_{\mu_m} \lim_{\ell\to+\infty} \frac{1}{\ell}\mathbb{E}\sum_{k=0}^{\ell}[c_m(s_k, a_k)]$ with c_m defined in Table I

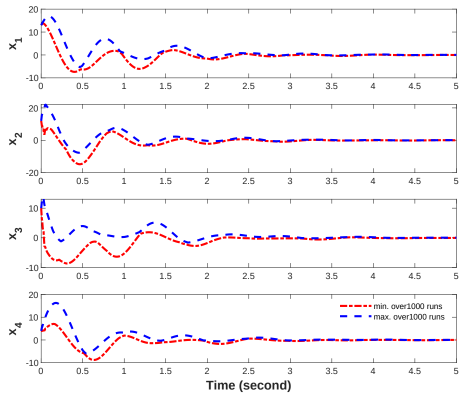

Fig. 2: Maximum and minimum value of state trajectories for the batch reactor process under the transmission time interval $T = 0.01s$.

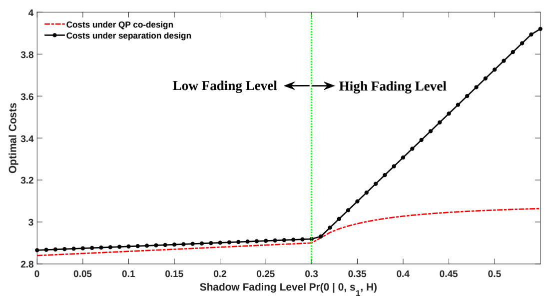

Fig. 3: Performance comparison of the proposed co-design method against the separation method under a wide range of channel conditions ranging from 0 to 0.55.

the co-design strategy (marked by red dash line). As shown by the plots, the co-design method leads to lower costs across the whole range of the shadow fading than that under the separation design. More interestingly, the co-design strategy is more robust in the high shadow fading regime (i.e., the region between 0.3 and 0.55) than the separation design in the sense that the optimal cost curve under the former is relatively flat regardless of the fading levels while the separation design method generates linearly increasing costs as the fading level is increased.

VI. Conclusions

This paper presented a co-design paradigm to ensure both stability and performance for industrial networked control systems under *state-dependent fading channels*. A novel SD-MC model was proposed to characterize the correlation between channel conditions and external environments. The proposed channel model was used to derive sufficient conditions on MATI under which the networked control system is *almost surely asymptotically stable*. The stability conditions are then imposed as hard constraints in the co-design problem whose optimal solutions can be found by solving constrained optimization problems. Numerical results were provided to demonstrate the benefits of the proposed co-design approach.

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