Sensitivity Analysis and Modeling of Symmetric Minor Hysteresis Loops Using the GRUCAD Description

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This paper presents a predictive and easily implementable hysteresis model for ferromagnetic laminations subjected to various quasi-static magnetic loads. In this model, the description of magnetic hysteresis is based on the decoupling of the magnetic field strength in a reversible and irreversible part, unlike in the Jiles–Atherton model. Thereby, the source of problems originating in the assumption that total magnetization could be split into the reversible and the irreversible component is bypassed. It should be stressed that models based on field separation principles are consistent with the laws of thermodynamics. The presented model allows for a systematic parameter identification based on standard magnetic measurements. In this paper, the variation of model parameters on the shape of the modeled hysteresis loop is presented. Furthermore, parameter identified from the major loop are applied to model symmetric minor loops that are evaluated by comparison with measurements. The structure of the model enables an inclusion of eddy-current effects, and is ready for a further exploitation in the FE modeling of macroscopic devices.

Index Terms—**Electrical steel sheets, energy conservation, magnetic hysteresis, magnetic material modeling, minor loops.**

I. Introduction

The description of ferromagnetic hysteresis advanced in [1] has gained a wide popularity in the scientific community [2]. Though perceived as less accurate than the phenomenological, mathematical Preisach approach, the Jiles–Atherton (JA) model has attracted the attention of physicists and engineers due to some of its attractive features:

1) formulation in terms of a first-order ordinary differential equation (ODE), which is simple to solve with the use of modern CAD software;
2) a few number of model parameters with a physical interpretation attributed to them, what makes it possible to carry out a qualitative analysis of the magnetization process, even in terms of underlying microstructural properties;
3) possibility to consider the effects of different physical phenomena affecting the shape of hysteresis loop using the effective field framework;
4) possibility to express the model equations analytically for the B-input case (the so-called inverse model).

A peculiar feature of the JA model is the existence of negative dM/dH slopes after a sudden field reversal [3]–[5]. This effect may hamper the model usefulness in finite element method (FEM) applications, leading to nonstability of procedures.

Therefore, it is usually suppressed by the introduction of an additional control variable δ_M, which cuts off the irreversible component of magnetization

$$\delta_M = 0.5\left[1 + \operatorname{sign}\left[(M_{\mathrm{an}} - M_{\mathrm{irr}}) \cdot \frac{dH}{dt}\ \left(\mathrm{or}\ \frac{dB}{dt}\right)\right]\right]. \quad (1)$$

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In the above-given relationship M_{an} denotes the so-called anhysteretic magnetization, whereas M_{irr} is the component of total magnetization due to irreversible magnetization processes. The difference of these quantities is multiplied by the derivative of the model input variable. Notation 'sign' denotes the sign operator, which takes the values ± 1.

It should be remarked that the introduction of δ_M may be justified theoretically, as it follows from previous analyses that after a sudden field reversal only the reversible process contributes to the total magnetization [6], [7]. On the other hand, the existence of this term cannot be traced back to the original JA model derivation, what implies its artificial character as a means to patch the JA model equations to avoid some unexpected effects and the instability of numerical codes.

As pointed out in a recent study [8], the source of problems with the JA description (regions with negative dynamic susceptibility in the quasi-static case, poor representation of minor loops, nonclosure of minor loops demonstrated, e.g., in [3] and [9]) is the conjecture inherent in model equations that the hysteresis loop branches might be obtained by the introduction of an appropriate offset from the anhysteretic curve along the M-axis.

Moreover, it should be remarked that the introduction of the effective field as a direct argument of the anhysteretic magnetization leads to a curve passing through the second and fourth quadrant of the coordinate system—contrary to a common belief that it passes through the first and third quadrants, [10].

Considering the afore-discussed problems related to the JA description and seeking for a useful macroscopic hysteresis model, which might be relatively easily implemented in FEM codes, we have turned our attention to an alternative proposal advanced several years ago by the GRUCAD research group [11]–[13]. Modifications introduced into model equations are significant enough to distinguish the description from the

JA model [14]. Most of ambiguities inherent in the JA model are avoided.

The considered description has one point in common with the JA model—it is formulated in terms of an ODE supplemented with a number of auxiliary nonlinear relationships. Such a set of equations is easy to be implemented in numerical codes and may be solved with the use of standard software libraries.

The GRUCAD description has a number of features, which make it similar to other hysteresis models, e.g., the one considered in [15], the Henrotte–Hameyer proposal [16], [17], or some more recent alternatives [18]–[20].

It should be remarked that in all afore-mentioned descriptions the hysteresis loop branches are obtained by the introduction of an offset along the H-axis. These models are consistent with the laws of irreversible thermodynamics. The resemblance of the GRUCAD model to the afore-mentioned descriptions makes it particularly interesting for engineers, who need reliable hysteresis models based on sound physical grounds and are easy to be implemented.

The papers concerning the GRUCAD model have focused mainly on its implementation in FEM codes. However, to the best of our knowledge, there are no systematic studies on the parameter sensitivity or a comparison of modeled minor loops with experimental ones available in the literature.

This paper is aimed at filling the gap. It is structured as follows. Section II recalls the model equations. Section III examines the effect of variation of model parameters on the shape of the modeled hysteresis loop. It mimics the approach proposed in [21], which carried out a similar analysis in the context of the JA model. Section IV provides exemplary modeling results of symmetric minor loops for a nonoriented electrical steel grade, where the model parameters were identified by one measured quasi-static major loop. Section V includes a discussion of the results and points out the scope for future work.

II. Model Equations

The considered description is an example of the so-called inverse models, i.e., time dependent magnetic flux density $B(t)$ plays the role of independent variable.

The concept of anhysteretic curve (truly reversible in the thermodynamic sense) is present in the GRUCAD model. The equation for the reversible field strength is

$$\begin{aligned} H_{\mathrm{an}}(t) &= B(t)/\mu_0 - M_{\mathrm{s}}\ [\coth\lambda(t) - 1/\lambda(t)] \\ &= B(t)/\mu_0 - M_{\mathrm{s}}\Lambda(\lambda) \end{aligned} \tag{2}$$

where $\Lambda(x)$ denotes the Langevin function, $\Lambda(x) = \coth x - 1/x$, whereas the quantity $\lambda(t)$ is defined

$$\lambda(t) = \frac{1}{a}\left[(1-\alpha)\, H_{\mathrm{an}}(t) + \alpha\frac{B(t)}{\mu_0}\right]. \tag{3}$$

The parameter α acts as a weighting factor between the reversible and irreversible contributions to the magnetization process and a determines the shape of the anhysteretic curve.

TABLE I
Reference Vector

Parameter	Value
α	10^{-4}
a	110 A/m
γ	0.15 A/m
H_{Hs}	270 A/m
M_{s}	$1.1\cdot 10^6$ A/m

Equations (2) and (3) may be combined into a single expression, useful for direct integration of the time dependence

$$\frac{dH_{\mathrm{an}}}{dt} = \frac{a - \alpha M_{\mathrm{s}}\Lambda'(\lambda)}{\mu_0\left[a + M_{\mathrm{s}}(1-\alpha)\Lambda'(\lambda)\right]}. \tag{4}$$

Notation $\Lambda'(x)$ denotes the derivative of Langevin function, $\Lambda'(x) = 1 - \coth^2 x + 1/x^2$. To avoid singularities of $\Lambda(x)$ and $\Lambda'(x)$ at zero, in the model implementation for arguments $x \le 0.1$ the Maclaurin series expansions were used, $\Lambda(x) \cong x/3$ and $\Lambda'(x) \cong 1/3$.

The relationships, which define the irreversible magnetic field strength are as follows:

$$\begin{aligned} \frac{dH_{\mathrm{h}}(t)}{dB(t)} &= \frac{H_{\mathrm{Hs}}\ [\coth\lambda_{\mathrm{H}} - 1/\lambda_{\mathrm{H}}] - H_{\mathrm{h}}(t)}{\delta\gamma} \\ &= \frac{H_{\mathrm{Hs}}\Lambda(\lambda_{\mathrm{H}}) - H_{\mathrm{h}}(t)}{\delta\gamma} \end{aligned} \tag{5}$$

$$\lambda_{\mathrm{H}} = \frac{H_{\mathrm{h}}(t) + \delta H_{\mathrm{Hs}}}{\gamma} \tag{6}$$

where δ is the sign of dB/dt, γ and H_{Hs} determine the irreversible magnetic field related to the pinning field. The quantities α, a, M_{s}, H_{Hs}, and γ are the model parameters [11]. Total field strength $H(t)$ is determined from the integration of the sum $dH_{\mathrm{an}}(t)/dt + dH_{\mathrm{h}}(t)/dt$. The set of ODEs may be solved using the standard variable-step Runge–Kutta method.

III. Sensitivity Analysis

To express qualitatively the effect of each parameter on the shape of the modeled hysteresis loop it is expedient to carry out a sensitivity analysis. For this purpose, a reference set of the values of model parameters is assumed. Next, the effect of variation of any value is examined; the other values are kept fixed. Such an approach might be useful for model users interested in the estimation issue, who avail of numerical codes that require a starting point in the n-dimensional space (e.g., the nonlinear Newton–Raphson method).

The assumed reference vector of model parameters is given in Table I. These values correspond to much extent to those used in [13]. Fig. 1 refers to a variation of the a parameter depicting the shape of the loop for a halved and a doubled value of the reference value. It is apparent that the anhysteretic curve skews down with an increasing value of a. Furthermore, a change in parameter a modifies the slope dB/dH and the remanence point.

Fig. 2 shows the effect of changes in the α parameter, which is a weighting factor between the reversible and irreversible parts. It becomes clear, that the irreversible part

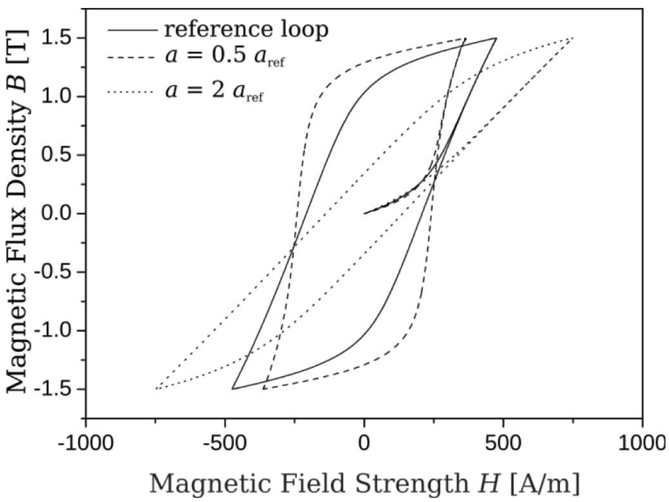

Fig. 1. Effect of variation of the α parameter. Lines: reference loop. Dashed line: loops with halved value of α. Fine-dashed line: doubled α parameter.

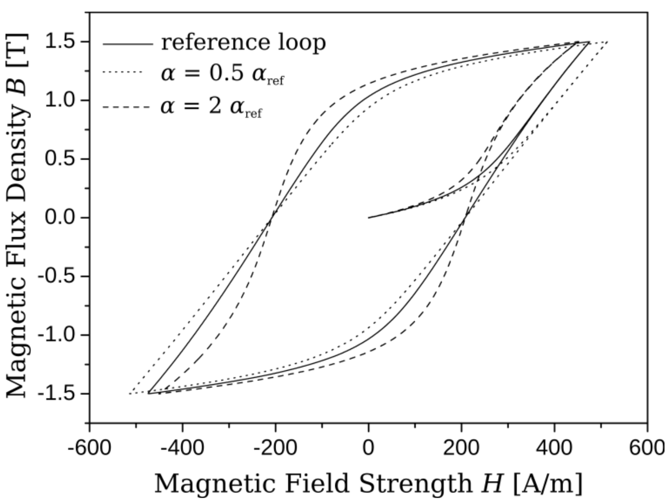

Fig. 2. Effect of variation of the α parameter. Lines: reference loop. Dashed line: loops with halved value of α. Fine-dashed line: doubled α parameter.

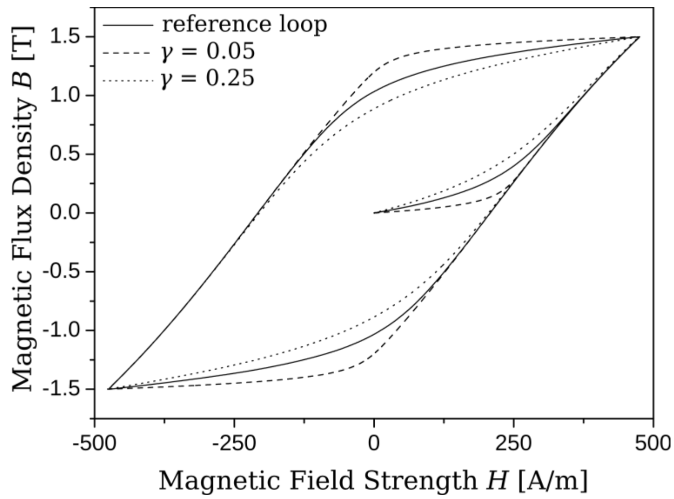

Fig. 3. Effect of variation of the γ parameter.

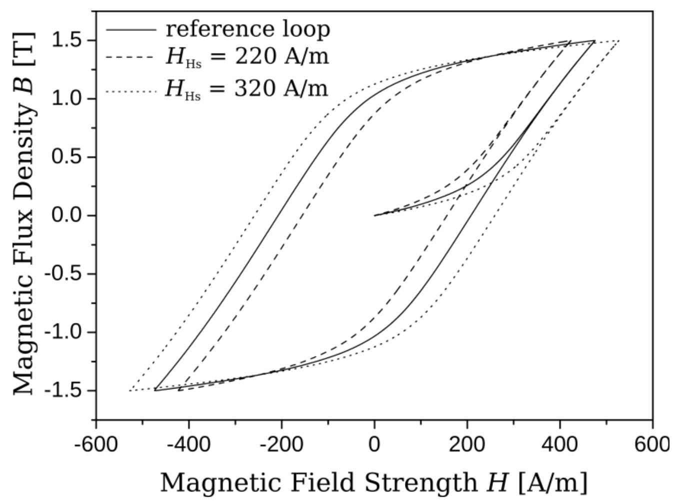

Fig. 4. Effect of variation of the H_{Hs} parameter.

of the curve slims down and the reversible part skews slightly up.

Figs. 3 and 4 consider the variations of γ and H_{Hs}, respectively. Variations in γ modifies the irreversible part of the hysteresis curve, so that the loop becomes less rectangular, more curved for increasing value of this parameter.

Changes of H_{Hs} increase the loop width, i.e., the irreversible part. This could be correlated with an increasing value of the coercivity. The effect of M_s is skipped, as this is self-evident: this variable controls the loop amplitude. The excitation amplitude assumed in simulations is $B_m = 1.5$ T.

IV. Minor Loops for a FeSi Steel Sample

A. Measurements

Measurements of hysteresis loops for a nonoriented FeSi 3.2% steel lamination, grade M270-35A (thickness 0.35 mm) have been carried out using the Epstein frame and a computer-aided setup in accordance with the international standard IEC 60404-2.

The mixed samples (cut along the rolling and the transverse directions) have been used to compensate the residual material anisotropy [22], [23].

B. Modeling

The values of model parameters α, a, M_s, H_{Hs}, and γ have been determined using experimental data for the major loop in quasi-static conditions ($B_m = 1.4$ T).

The robust trust-region-reflective algorithm implemented as MATLAB routine lsqcurvefit has been used for the estimation purposes. The trust-region-reflective algorithm is a subspace trust-region method and is based on the interior-reflective Newton method described in [2] and [24]. Each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients. The estimated set of parameters was: $\alpha = 9.48 \cdot 10^{-5}$, $a = 40$ A/m, $M_s = 1.2 \cdot 10^6$ A/m, $H_{Hs} = 50$ A/m, and $\gamma = 0.12$ A/m. The obtained parameter values have been next used in modeling symmetric minor loops.

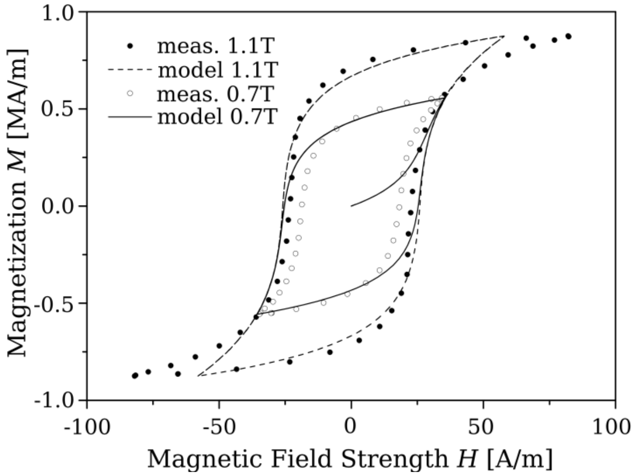

Fig. 5. Symmetric minor hysteresis loops for the nonoriented steel. Lines: modeled with the considered description. Dots: experimental data points.

Fig. 5 shows the modeling results for two representative amplitudes of flux density, $B_\mathrm{m} = 1.1$ and 0.7 T. The error predicting the shape of the hysteresis loop, i.e., iron losses, proves to be satisfactory (10%–20%).

On the basis of the presented results it can be stated that the model is able to reproduce the shape of symmetric minor loops with accuracy sufficient for engineering purposes even when the model parameters are identified just using the major loop. The accuracy could be improved considering the minor loop measurements during parameter identification.

V. Conclusion

This paper has discussed the features of a hysteresis model, based on a decomposition of the applied field into the reversible and irreversible terms, contrary to the JA approach. The deficiencies of the JA formalism, resulting from the assumption that total magnetization might be decomposed into the reversible and irreversible components have been pointed out. The alternative model is similar in spirit to some other thermodynamically consistent approaches presented in the literature. An important advantage of the description from the engineering point of view is its formulation as a B-input description.

The model sensitivity against variations of parameter values has been examined. The optimal values of model parameters have been determined of the quasi-static major loop for a sample of nonoriented electrical steel sample measured using the Epstein frame. Results of modeling symmetric minor loops using the major loop parameters with the considered description are in a satisfactory agreement with the experimental data.

In the forthcoming work, it is desirable to focus on a model extension to cover the case of increased excitation frequency as well as higher order reversal curves.

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