Consideration of Rotational Motion in the Proper Generalized Decomposition by a Sliding Interface Technique

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Consideration of moving parts in the simulation of electrical machines is a necessity to characterize the machine behaviour at various operating points. For this purpose, the Sliding Interface Technique, based on Lagrange multipliers can be utilized. To reduce the computational effort of these simulations, it is interesting to employ model order reduction techniques such as the Proper Generalized Decomposition. In this contribution, the Sliding Interface Technique, which imposes no restrictions to finite element discretization on the interface between stator and rotor, is combined with the Proper Generalized Decomposition to abolish the restriction to conformally meshed domains, while keeping the symmetry and positive definiteness of the system.

***Index Terms*—Motion, Model Order Reduction, Sliding Interfaces, Proper Generalized Decomposition**

I. Introduction

In the simulation of electrical machines, a restriction to conformally meshed geometries represents an undesired property, because in most electrical machines motion has to be considered. While in standard finite element simulation this limitation is coped with by techniques such as, e.g., overlapping elements, locked step method or Sliding Interfaces, it is still challenging if the Proper Generalized Decomposition (PGD) is used. Other techniques such as the Proper Orthogonal Decomposition (POD) do not experience this drawback [4], but the PGD, as a model order reduction (MOR) technique, shows higher computational benefits [5] and is already applied to a variety of electromagnetic field problems. However, in [2] a method based on the overlapping element method is presented to consider motion in the PGD. The overlapping element method is based on creating additional elements in the overlapping region, without introducing new degrees of freedom [1]. In this paper, an approach considering non-conforming Sliding Interfaces is employed which avoids the creation of additional elements and conserves a positive symmetric definite system [3] to take motion into account in the PGD.

II. Variational Formulation

The simulation of electrical machines involving motion by non-conforming sliding interfaces separates the geometry in two domains, namely Ω^{m} and Ω^{s}. The interface between the master domain Ω^{m} and Ω^{s} is defined as $\Gamma^{\mathrm{m}} \subset \Omega^{\mathrm{m}}$ and $\Gamma^{\mathrm{s}} \subset \Omega^{\mathrm{s}}$. Further, a mapping $p : \Gamma^{\mathrm{s}} \rightarrow \Gamma^{\mathrm{m}}$ shall be given to realize the rotational / translational motion between the domains. The variational form of the energy balance of a magnetostatic problem leads to (1), including the magnetic vector potential $\boldsymbol{A}$, the Lagrange multiplier (LM) $\boldsymbol{\lambda}$ and the excitation given by the current density $\boldsymbol{J}$ and the permanent magnets $\boldsymbol{B}_{\mathrm{PM}}$. $\boldsymbol{n}^{\mathrm{k}}$ denotes the outward normal vector of the interface k. $\boldsymbol{H}_{\mathrm{fp}}$ represents the non-linearity of the fixed point formula and ν_{fp} the fixed point coefficient [8].

$$
\begin{aligned}
\sum_{k\,=m,s} \int_{\Omega^{\mathrm{k}}} \big(\nu_{\mathrm{fp}} \nabla \times \boldsymbol{A}^k \nabla \times \partial \boldsymbol{A}^k \\
&- \boldsymbol{J}^k \partial \boldsymbol{A}^k - \nabla \times \boldsymbol{B}^k_{\mathrm{PM}} \partial \boldsymbol{A}^k\big)\, \mathrm{d}\Omega^k \\
+ \int_{\Gamma^s} \boldsymbol{\lambda} - \boldsymbol{n}^s \times (\nu_{\mathrm{fp}} \nabla \times \boldsymbol{A}^s) \mathrm{d}\Gamma^s \\
- \int_{\Gamma^m} \boldsymbol{\lambda} \circ \boldsymbol{p}^{-1} - \boldsymbol{n}^m \times (\nu_{\mathrm{fp}} \nabla \times \boldsymbol{A}^m) \mathrm{d}\Gamma^m \\
+ \int_{\Gamma^s} \partial \boldsymbol{\lambda} (\boldsymbol{A}^s - \boldsymbol{A}^m \circ p) \mathrm{d}\Gamma^s = \boldsymbol{H}_{\mathrm{fp}}
\end{aligned}
\tag{1}
$$

$\boldsymbol{A}$ as well as $\boldsymbol{\lambda}$ are approximated by nodal shape functions for two-dimensional magnetostatic field computation. Biorthogonal shape functions for λ on the slave surface are employed to reduce the computational effort of transforming the sattle point problem (1) to a symmetric positive definite problem [3]. It can be depicted that the LM ensures continuity between the domains for the tangential part of the magnetic field strength as well as the normal part of the flux density. In the following, the superscript k denotes the master / slave domain.

III. Proper Generalized Decomposition

The PGD is characterized by approximating the unknown vector $\boldsymbol{A}$ as a sum of m modes consisting of n functional products, each related to a problem parameter, e.g. the space $\boldsymbol{x}$, the relative rotor angle θ, the permanent magnet remanence $\boldsymbol{B}_{\mathrm{PM}}$ and others.

$$
\boldsymbol{A}(\boldsymbol{x}, \boldsymbol{\theta}, \boldsymbol{B}_{\mathrm{PM}}, ...) \approx \sum_{\mathrm{i=1}}^{\mathrm{m}} \boldsymbol{R}_i \cdot F_i(\theta) \cdot K_i(\boldsymbol{B}_{\mathrm{PM}}) \cdot \ldots \tag{2}
$$

In a first step, the approximation (2) is introduced into (1) and consecutively, the formulation is rearranged according to the parameter integration. Employing an alternative direction scheme (ADS) enables to solve for the different parameters [7]. The subscript depicts the mode number in the following explanations and to simplify the equations, the PGD will only contain the angle function $F(\theta)$ and the spatial component R, while the permanent magnet remanence is kept constant.

Nevertheless, the procedure is similar, if more parameters are introduced.

A. Space Mode

The evaluation of the spatial component introduces an additional splitting of the master and slave domain into a part $\Omega^{k,\text{int}}$ and $\Omega^{k,\text{air}}$; The first one has an underlying piecewise affine decomposition (PAD) and the latter one consists of elements in the airgap which are connected to either Γ^m or Γ^s, which has no underlying PAD [3],[2]. The domain $\Omega^{k,\text{int}}$ can be treated as in [5],[7]. The airgap region Ω^{air} needs to consider the connection between the slave and master domain and includes a projection operator p, which depends on the relative rotor position and due to this reason the matrix unavoidably has to be rebuilt for each angle θ_l. A weighted sum is used to build an approximation of the matrix in Ω^{air} over all positions, which is then added to the matrix of $\Omega^{k,\text{int}}$ [2]. The resulting system of equations can be solved by standard Krylov-Subspace algorithms

$$\left(\sum_{\text{k=m,s}} \mathbf{M}_{k,\text{int}} + \sum_{l} \mathbf{M}_{\text{air}}(\theta_l)\right)\boldsymbol{X} = B\ . \tag{3}$$

B. Angular Mode

To evaluate the angular mode, the spatial component $\boldsymbol{R}_m$ is assumed to be known from the previous computation, hence it is possible to evaluate the integrals in Ω_k and Γ_k, leading to a linear equation to be solved for all rotational positions. The term H_NL belongs to the non-linearity, while H_Lin extracts the information of the modes up to $m-1$, under consideration of all boundary conditions, of the system.

$$\begin{aligned} &A_\text{move}\cdot F(\theta) - B_\text{move}\cdot F(\theta) + C_\text{move}F(\theta) \\ &\quad = D_\text{move}\cdot J(\theta) + E_\text{move} - L^\text{s} \\ &\qquad + L^\text{m}(\theta) + L_\text{d}(\theta) - H_\text{Lin} - H_\text{NL} \end{aligned} \tag{4}$$

$$A_\text{move} = \sum_{k=m,s}\int_{\Omega^\text{k}} \nabla\times R_m^k \nabla\times R_m^k \mathrm{d}\Omega^k \tag{5}$$

$$B_\text{move} = \int_{\Gamma^\text{s}} \boldsymbol{n}^s \times (\nu\nabla\times\boldsymbol{R}_m^s)\boldsymbol{R}_m^s \mathrm{d}\Gamma^s \tag{6}$$

$$C_\text{move} = \int_{\Gamma^\text{m}} \boldsymbol{n}^m \times (\nu\nabla\times\boldsymbol{R}_m^m)\boldsymbol{R}_m^m \mathrm{d}\Gamma^m \tag{7}$$

$$D_\text{move} = \sum_{k=m,s}\int_{\Omega^\text{k}} J_\text{x}\boldsymbol{R}_m^k \mathrm{d}\Omega^k \tag{8}$$

$$E_\text{move} = \sum_{k=m,s}\int_{\Omega^\text{k}} \boldsymbol{B}_\text{PM}\nabla\times\boldsymbol{R}_m^k \mathrm{d}\Omega^k \tag{9}$$

$$L^s = \int_{\Gamma^s} \lambda\boldsymbol{R}_m^s \mathrm{d}\Gamma^s \tag{10}$$

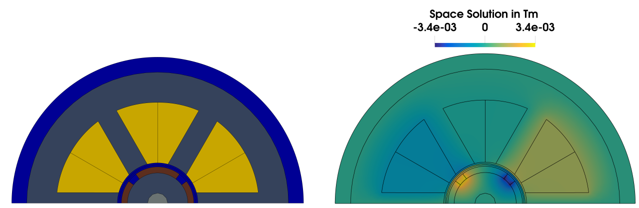
$$L^m(\theta) = \int_{\Gamma^m} \lambda\circ p^{-1}\boldsymbol{R}_m^m \mathrm{d}\Gamma^m \tag{11}$$

$$L_\text{d}(\theta) = \int_{\Gamma^s} (\boldsymbol{R}_m^s - \boldsymbol{R}_m^m\circ p)\boldsymbol{R}_m^s \mathrm{d}\Gamma^s \tag{12}$$

$$H_\text{NL} = \sum_{k=m,s}\int_{\Omega^\text{k}} \nabla\times(\nu_\text{fp}-\nu)\nabla\times\boldsymbol{A}_m \mathrm{d}\Omega^k \tag{13}$$

IV. Application

The theory of the previous section is now applied to a synchronous machine, shown in Fig. 1a, with surface magnets and the airgap of the machine having a non-conformal mesh. The first space mode of the PGD is shown in Fig 1b.



(a) Geometry of the synchronous machine including a non-conformal airgap

(b) First space mode

Fig. 1. Simulated synchronous machine with surface mounted permanent magnets.

V. Conclusions

A method for combining the PGD with non-conforming sliding interfaces based on Lagrange multipliers is presented. The limitation to conformal meshes is therefore abolished. In the full paper the theory and implementational aspects will be presented in detail. The method will be applied to simulate different characteristic operating points, such as no-load, locked rotor and nominal operating points.

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References

[1] I. A. Tsukerman, "Overlapping finite elements for problems with motion," *IEEE Trans. Magn.*, vol. 28, no. 5, pp. 2247-2249, Sept. 1992.

[2] L. Montier, T. Henneron, S. Clénet and B. Goursaud, "Proper Generalized Decomposition Applied on a Rotating Electrical Machine," *IEEE Trans. Magn.*, vol. 54, no. 3, pp. 1-4, March 2018.

[3] E. Lange, F. Henrotte and K. Hameyer, "A Variational Formulation for Nonconforming Sliding Interfaces in Finite Element Analysis of Electric Machines," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2755-2758, Aug. 2010.

[4] F. Müller, A. Siokos, J. Kolb, M. Nell and K. Hameyer, "Efficient Estimation of Electrical Machine Behavior by Model Order Reduction," *IEEE Trans. Magn.*, vol. 57, no. 6, pp. 1-4, June 2021.

[5] F. Müller, L. Crampen, T. Henneron, S. Clénet and K. Hameyer, "Model order reduction techniques applied to magnetodynamic T-Ω-formulation," *COMPEL*, vol. 39, no. 5, pp. 1057-1069, 2020.

[6] F. Chinesta, A. Ammar, A. Leygue and R. Keunings, "An overview of the proper generalized decomposition with applications in computational rheology", *J. Non-Newtonian Fluid Mech.*, vol. 166, no. 11, pp. 578-592, 2011.

[7] T. Henneron and S. Clénet, "Application of the PGD and DEIM to solve a 3D non-linear magnetostatic problem coupled with the circuit equations", *IEEE Trans. Magn.*, vol. 52, no. 3, Mar. 2016.

[8] E. Dlala and A. Arkkio, "Analysis of the Convergence of the Fixed-Point Method Used for Solving Nonlinear Rotational Magnetic Field Problems," *IEEE Trans. Magn.*, vol. 44, no. 4, pp. 473-478, April 2008.