Improved 3D Coupled Calculations of the Structural-Dynamic Behavior of Induction Furnaces Excited by Electromagnetic Forces Using Adaptive Algorithms

T. Bauer, W. Mai, and G. Henneberger

***Abstract*—The aim of this work is to make available an automatic procedure for the improved calculation of the structural-dynamic behavior of induction furnaces (Fig. 1) and handling of meshing information to the mesh generator for the following acoustic analysis. The electromagnetic forces from the electromagnetic FE-model are transferred to the initial mechanical FE-model and a loop of vibration analysis is started, which will be controlled by the maximum error rate and the number of loops. The error criterion is chosen and integrated in the meshing algorithm. The impact of the mesh refinement will be shown by applying the algorithm to a test problem explained in the following. The developed code is created in object oriented language to have the most flexibility for the implementation of the various algorithms.**

***Index Terms*—Adaptive meshing, finite element methods, object oriented design, structural-dynamic field.**

I. Introduction

INDUCTIVE heating gains more significance in recent time due to the increase in specific power density and feeding current frequency [1]. The increase of power also increases the vibrations and the noise emission of the apparatus. In order to minimize the emission several modifications have to be investigated. This should be done by numerical techniques, which are more inexpensive than to build specimen [2]. Therefore the calculation process has to be reliable regarding the solution. The most desirable way is an automatic error-estimator, which improves the solution up to an desired level. This will be performed by an adaptive algorithm described later on. Further on the paper presents the impact of the adaptive calculation on the acoustic noise emission.

II. Flow of Analysis

The calculation is divided into three parts shown in Fig. 2. First, the electro-magnetic calculation determines the input data for the force post-processing. There exist two components of forces. The Lorentz force caused by the currents in the coil and surface forces at regions with large differences in magnetic permeability act on the structure of the furnace and this leads to respectively high sound pressure levels. The next step is to transform the forces on the structural-dynamic model to evaluate the vibrations upon the surface of the structure. Finally the sound emission of the vibrating structure is determined by a second order boundary element method with curved elements to achieve more exact results [3].

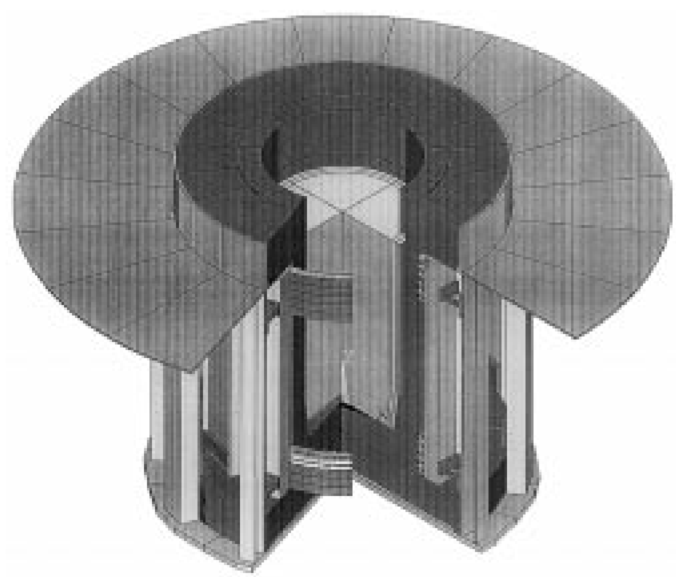

Fig. 1. Structural-dynamic model of an induction furnace.

III. Adaptive Calculation

A. Overview

The quality of the numerical results considering the finite element method depends on the mesh density used in regions where large gradients of the solution quantity exist. These regions can not be determined *a priori* in general for each application, which is calculated. To overcome this problem an adaptive meshing algorithm is used to remesh the structure, after the calculation with an initial coarse mesh. The criterion used to determine regions of large differences in the gradients of the solution is the difference of the mechanical stress calculated from the displacements in every element. The flowchart in Fig. 2 shows the calculation process in detail.

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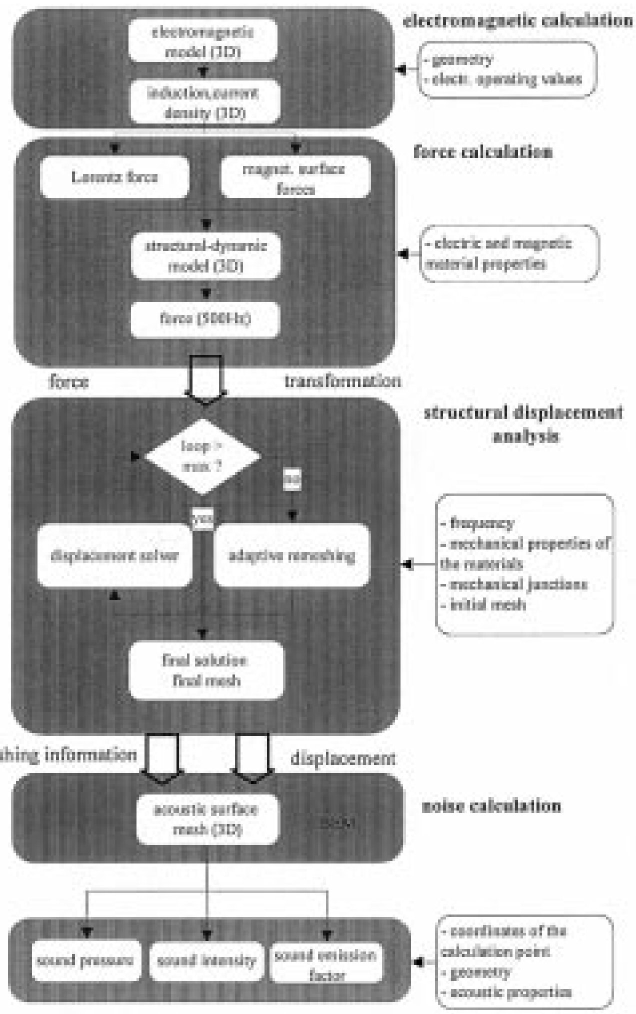

Fig. 2. Structural-dynamic model of an induction furnace.

The adaption function is established and gives instructions to the remeshing algorithm to subdivide elements in the mentioned regions. The adaptive algorithm does not depend on the element shape or order because of the object-oriented design. The number of cycles of meshing and calculation can be controlled and also the smallest size of the elements or the percentage of elements to work on.

B. Mechanical Equations

The basic equation for the numerical calculation of the structural-dynamic behavior is given by applying Hamiltons principal to the equation of motion considering the time harmonic nature of the forces [4], [5]:

$$(\underline{K} - \omega_{mech}^2 \cdot \underline{M}) \cdot \underline{U} = \underline{R} \tag{1}$$

where ω_{mech} is the excitation frequency of the problem. This is twice the amount of the electrical frequency. $\underline{U}$ stands for the unknown vector representing the displacement of each field node. The vector $\underline{R}$ is the global force vector. This excitation

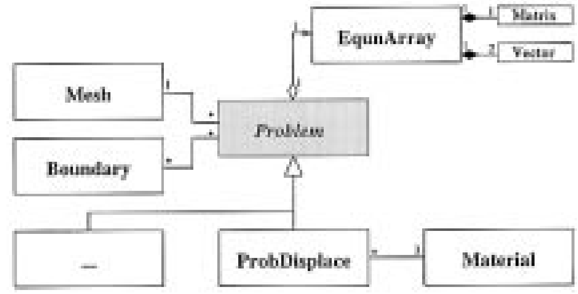

Fig. 3. The local adaption criterion obtained from the mechanical solution (adaptive step 2).

of the mechanical problem is derived from the electromagnetic field solution. $\underline{K}$ is the global stiffness matrix given by

$$\underline{K} = \sum_{i=1}^{n} \int_{\Omega_i} \underline{B}_i^T \cdot \underline{H}_i \cdot \underline{B}_i \, d\Omega_i \tag{2}$$

with $\underline{B}$ as the differentiation matrix, which contains the derivatives in the three dimensions. The Hook matrix $\underline{H}$ incorporate the elasticity coefficients.

The matrix $\underline{M}$ is the global mass matrix calculated with

$$\underline{M} = \sum_{i=1}^{n} \int_{\Omega_i} \rho_i \underline{N}_i^T \cdot \underline{N}_i \, d\Omega_i \tag{3}$$

with ρ_i and $\underline{N}_i$ as the mass density and the interpolation function of the element respectively.

C. Error Estimation

The local mechanical stress distribution is chosen as the criteria for the solution quality. In order to be independent of both the element size and shape the stress is weighted by the total stress energy.

The stress $\underline{\tau}$ is calculated according to Hook's law

$$\underline{\tau} = \underline{H} \cdot \underline{B} \cdot \underline{U} + \underline{\tau'} \tag{4}$$

where $\underline{\tau'}$ is the initial stress.

The adaption criterion δ_i of the element i is now calculated with

$$\delta_i = |\tau| \cdot \frac{\frac{1}{2} \int_{\Omega_i} \tau^T \cdot \underline{H} \cdot \tau \, d\Omega_i}{\frac{1}{2} \sum_{i=1}^{N} \int_{\Omega_i} \tau^T \cdot \underline{H} \cdot \tau \, d\Omega_i} \tag{5}$$

with N as the number of elements. Based on this adaption criterion distribution, 10% of the elements with the largest error values are marked for adaptive refinements. The refinement strategy itself divides the marked elements at their longest edges. Possible unmarked neighboring elements are also split to keep the mesh consistency [8].

D. Object Oriented Implementation

The structural displacement analysis is developed using the object oriented paradigm. The mesh handling, the information flow and the remeshing algorithm is implemented with a maximum of design and source code reuse. The mechanical problem is described in the new class `ProbDisplace`, shown in Fig. 3.

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This is a subclass of `Problem` which takes care of all communications between the mesh and the equation array [9]. Also, for the error calculation existing design concepts are used. The existing superclass `ErrorEstimator` is specialized by the new class `EstimateErrorStress`. Note, that the code for the adaptive refinement in [9], including the automatic interpolation of the result vector, is reused without any modifications because of the inheritance principle.

This allows a simple substitution of the order of the interpolation function, the adaptive meshing algorithm and the error estimation criterion by exchanging objects.

IV. Results

A. Structural-Dynamic

The electromagnetic force is calculated using an 18°-model, which is the smallest symmetry with regard to the electro-magnetic field, presented in [2]. The forces will be transformed to an 90°-model constructed for the structural-dynamic calculation. Note, this model respects the construction parts, which have a symmetry to 90°. The forces in the windings cause radial vibrations of the yoke and the crucible. This leads to a toroidal movement of the upper construction of the furnace, which the working platform is attached to.

Due to the low weight and the extensions of this platform a large number of mechanical modes are stimulated. This leads to high deflections. This part is the main source of the emitted noise. This is the reason why this part of the model has to be modeled carefully. Because of the less thickness of the platform the original mesh is coarse and has to be refined locally.

For verification purposes a bending bar is taken into consideration and it is explained, how the error affects the solution quality of the structural calculation. Considering the dimensions of the bar which have the same proportion as the yoke of the furnace the height is 0.3 m and the thickness is 0.03 m. At the left side the yoke is bounded and on the right side a total force of 1000 N acts on the bar. The solution for a bending bar is well known and results for this problem to 0.635 mm. The initial mesh starts with 530 nodes and 1698 elements and gives an error of deflection at the end of the beam of about 23%.

B. Error Distribution and Adaptive Refinements

The mechanical solution is now investigated. Fig. 4 shows the local adaption criterion distribution based on (4) and (5). The maximum values occur in the regions of highest stress, which are lying in the near of the bounded nodes. At the end of the bar, where the deflection rises to a maximum, the stress and the values become small. Also it is noticeable, that the neutral phase of the bar (in the symmetry axis) is nearly error free. Thus it is proved, that the adaption criterion is suitable to improve the Finite Element mesh locally in regions of high errors. Now the elements with the highest error values are refined and the mechanical problem is solved again. In Fig. 6 the topology of the resulting equation system after inserting boundary conditions and reordering is presented.

The most common algorithm to reduce bandwidth and wavefront are the GPS strategy presented by Gibbs, Poole, and Stockmeier [6] and the renumbering technique of

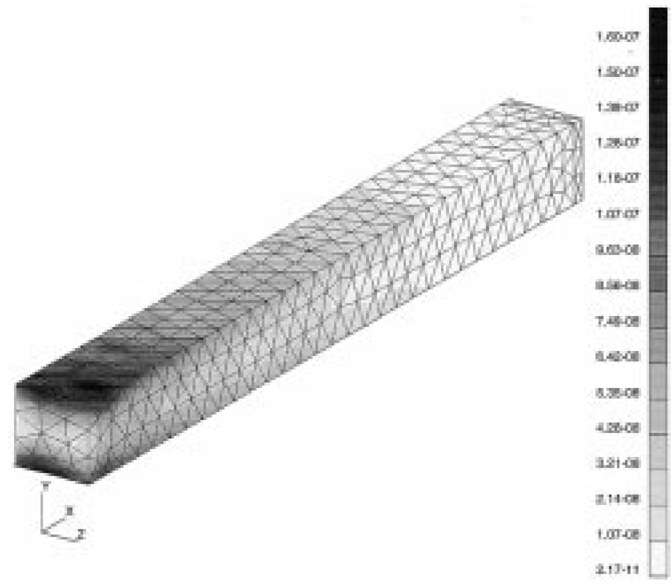

Fig. 4. Class diagram of the problem definition.

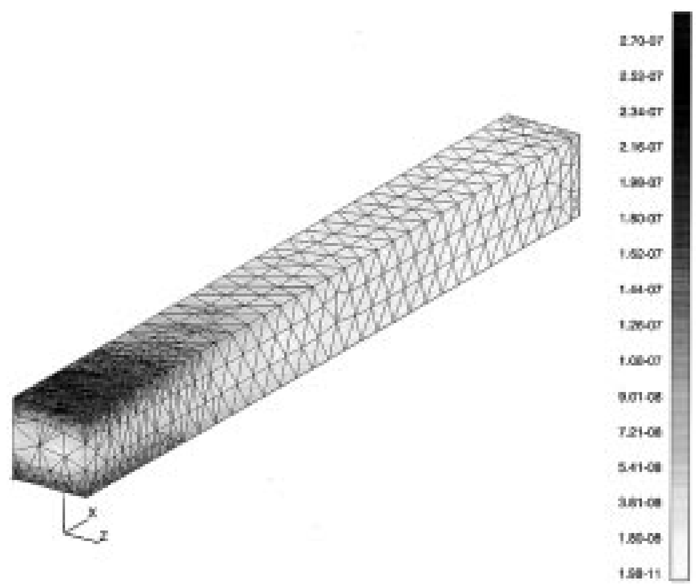

Fig. 5. The local adaption criterion obtained from the mechanical solution.

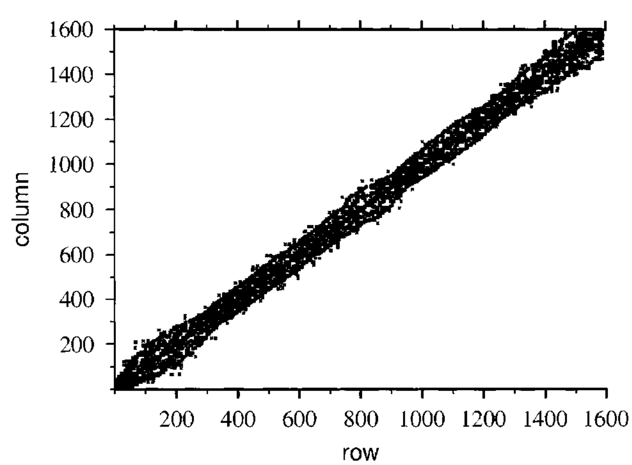

Fig. 6. The matrix topology after the second adaptive step.

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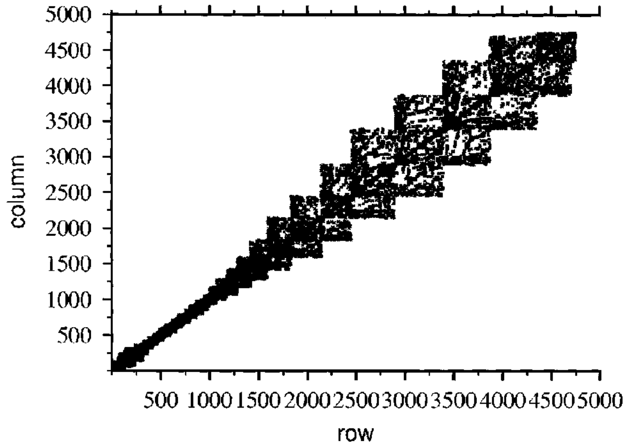

Fig. 7. The matrix topology of the initial mesh.

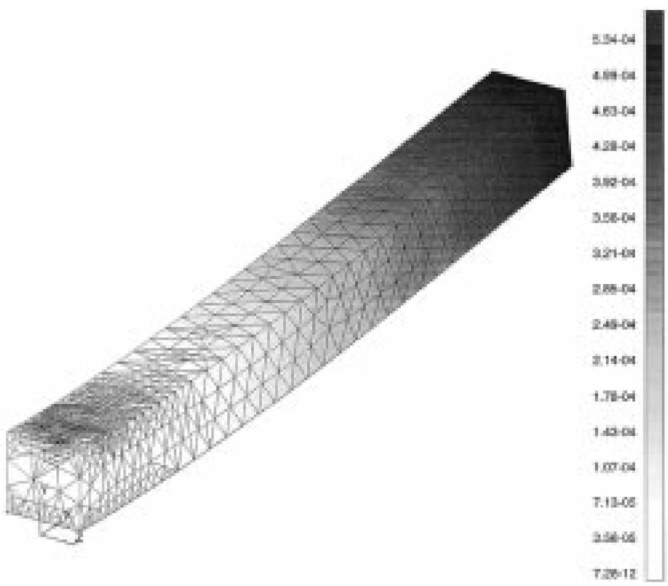

Fig. 8. The displacement solution for the adapted mesh.

Cuthill–McKee [7]. In this solver we use the Cuthill–McKee method. After the second adaptive step the number of nodes grew up to 1582 and the number of elements is enlarged to 6540. The maximum error is reduced from 23% to 16% after 3 calculations. Notice that the bandwidth is enlarged and can not be compressed in a better way, which takes great impact on the solution time of the system of equation. This effect is caused by the increasing number of nodes, which enlarge the number of diagonal elements.

The model of the induction furnace, Fig. 9, shows the regions in which the mesh density is increased (bright color). It is remarkable, that the regions, where the forces act on the structure, are remeshed.

V. Conclusions

The main idea of the presented paper is the improvement of the most important calculation step for the coupled calculation of the electro-magnetic, structural-dynamic and acoustic

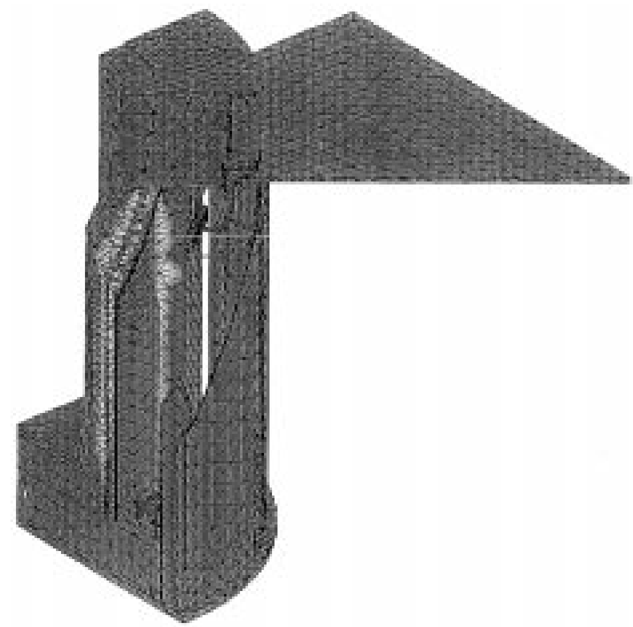

Fig. 9. The distribution of the adaption criterion for the induction furnace.

problem. It has been shown, that the stress in the element, weighted by the elastic energy, is a suitable tool to reduce the error in the Finite Element calculation regarding to structural-dynamic problems.

Performing two adaption steps the amelioration of the solution results to 7%. The bandwidth increases noticeable due to the additional nodes, which were added to the initial mesh.

Further more the use of object oriented coding techniques and the extensive use of inheritance and specialization makes it possible to extend this procedures easily to other Finite Element problems.

References

[1] T. Bauer, S. Dappen, G. Henneberger, and R. Jordan, “Reduction of electromagnetic field emission of heating devices,” in *Proceedings of 3rd Int. Workshop on El. and Mag. Fields*, Liege, 1996.

[2] T. Bauer and G. Henneberger, “Three-dimensional calculation and optimization of the acoustic field of an induction furnace caused by electromagnetic forces,” *IEEE Trans. Magn.*, vol. 35, no. 3, pp. 1598–1601, May 1999.

[3] R. D. Ciskowski and C. A. Brebbia, *Boundary Element Methods in Acoustic*, ser. Computational Mechanics Publications Elsevier Applied Science Southampton Boston, 1991.

[4] S. G. Lekhnitskii, S. W. Tsai, and T. Cheron, *Anisotropic Plates*: Gordon and Breach, Science Publishers.

[5] O. C. Zienkiewicz and R. L. Taylor, *The Finite Element Method*, Fourth ed. London: McGraw-Hill Book Company, 1991, vol. I and II.

[6] N. E. Gibbs, W. G. Poole, and P. K. Stockmeier, “An algorithm for reducing the bandwidth and profile of a sparse matrix,” *SIAM J. Numer. Anal.*, no. 13, pp. 236–250, 1976.

[7] E. Cuthill and J. M. McKee, “Reducing the bandwith of sparse matrices,” in *Proc. 24th Nat. Conf. Assoc. Comput. Mach.*, New York, NY, 1969, pp. 157–172, ACM Pub. P69, Edition, Vol. I and II, McGraw-Hill Book Company, London, 1991.

[8] K. Tani and T. Yamada, “H-version adaptive finite element method using edge element for 3D nonlinear magneto-static problems,” *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 1756–1759, March 1997.

[9] W. Mai and G. Henneberger, “Object-oriented design of finite element calculations with respect to coupled problems,” in *Proceedings of COMPUMAG Sapporo*, 1999.

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