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Drawing with divergent perspective, ancient and modern

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Abstract. Before methods for drawing accurately in perspective were developed in the 15th century, many artists drew with divergent perspective. But we found that many university students draw with divergent perspective rather than with the correct convergent perspective. These experiments were designed to reveal why people tend to draw with divergent perspective. University students drew a cube and isolated edges and surfaces of a cube. Their drawings were very inaccurate. About half the students drew with divergent perspective like artists before the 15th century. Students selected a cube from a set of tapered boxes with great accuracy and were reasonably accurate in selecting the correct drawing of a cube from a set of tapered drawings. Each subject's drawing was much worse than the drawing selected as accurate. An analysis of errors in drawings of a cube and of isolated edges and surfaces of a cube revealed several factors that predispose people to draw in divergent perspective. The way these factors intrude depends on the order in which the edges of the cube are drawn.

1 Introduction

Methods for drawing in accurate perspective were developed in 15th-century Florence (Richter 1970; Edgerton 1975). A correct perspective drawing of an object creates the same image in an eye as the object when the eye is in the location from which the object was drawn. Consider a cube viewed with one eye with one face in a frontal plane. A projection of the cube from the nodal point of the eye onto a sheet of paper parallel to the front surface of the cube defines a drawing of the cube in one-point perspective, as shown in figure 1. The drawing is said to be in polar projection and has the following features: (i) The front surface is square. (ii) The receding edges converge on a vanishing point on the horizon directly opposite the eye. This convergence specifies linear perspective. (iii) Diagonals of receding horizontal surfaces converge on two distance points on opposite sides of the vanishing point. The distance between each distance point and the vanishing point equals the distance of the eye from the drawing. Convergence of the diagonals specifies foreshortening in the drawings of the receding surfaces (aspect-ratio perspective).

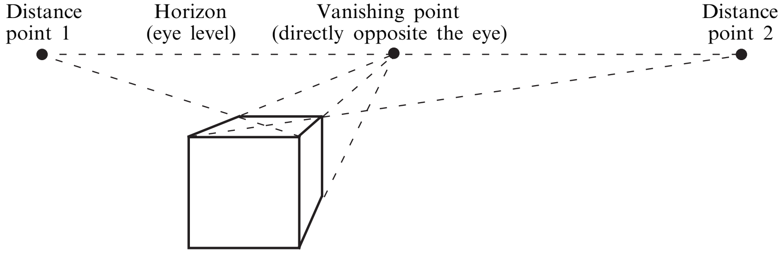

Figure 1. A drawing of a cube with one surface parallel to the drawing surface. The eye is opposite the vanishing point at a distance from the drawing equal to the distance between the vanishing point and either of the two distance points.

In an orthographic drawing the projection lines connecting each point in the object and the corresponding point in the drawing are parallel. An orthographic drawing of a cube with one surface parallel to the plane of the drawing is simply a square. An orthographic drawing of a cube with no surface parallel to the plane of the drawing has no linear perspective (parallels in the cube are parallel in the drawing), but the drawings of the surfaces are foreshortened (they have aspect-ratio perspective). The surfaces are also sheared into parallelograms. The polar projection of a cube becomes more orthographic as its distance from the projection plane increases.

A plan view, a front elevation, and a side elevation of a cube are three orthographic drawings made from three orthogonal directions. Each drawing has parallel edges and no foreshortening, as shown in figure 16.

Before the 15th century, the receding edges of rectangular objects were often drawn either parallel or diverging. We looked for drawings made before the 15th century of rectangular objects, such as pedestals and tables with the front surface parallel to the picture plane. We found 15 examples from China, Persia, ancient Greece, Pompeii, and medieval Europe. In every case the top of the object was drawn with parallel or near-parallel sides, and the side of the object was drawn with divergent perspective. Figure 2 shows some examples with lines added and the convergence angles of the tops and sides of the rectangular objects indicated in the caption.

We report here that a large proportion of university students draw with parallel or divergent perspective, like pre-15th-century artists. None of the students we tested came close to drawing with correct convergent perspective. One can understand drawing in parallel perspective. The receding edges of a rectangular object are indeed parallel, and we see them as parallel in 3-D space. Our visual system evolved to allow us to see in 3-D, not to make 2-D drawings or to perceive the 2-D layout of the retinal image. Only architects and artists need to transfer the 3-D layout of a scene into 2-D. But why did early artists and many people today draw in divergent perspective, particularly when drawing the side of a rectangular object? Receding rectangular surfaces do not diverge into the distance. One possibility is that a receding surface drawn with parallel edges appears to diverge. For example, the drawing of a table in figure 2f has parallel sides that appear to diverge. This is because a horizontal surface would have to diverge to produce a projection on the frontal plane with parallel sides. Perhaps some early artists drew with divergence because they copied drawings made with parallel perspective. But we report here that many university students drew with divergent perspective even when drawing an actual cube.

It is well known that, when selecting a frontal shape to match with a shape inclined in depth, people select a frontal shape that lies between the image of the shape and the actual shape of the inclined stimulus. Thouless (1930) called this perceptual effect "regression to the real object". The real object is the shape of the inclined stimulus when viewed with the visual axis orthogonal to the stimulus. We will use the term "regression to the orthogonal view". We show that, when selecting a drawing that most resembles a cube, people show regression to the orthogonal view. However, we show that, when drawing a cube, people make errors that are far larger than the effect of regression. The five following experiments were designed to reveal why many people draw in divergent perspective, and why divergent perspective is more evident in drawings of the vertical side of an object than in drawings of the top of an object.

These experiments were approved by York University Ethics Committee in accordance with the World Medical Association Declaration as revised in 2008.

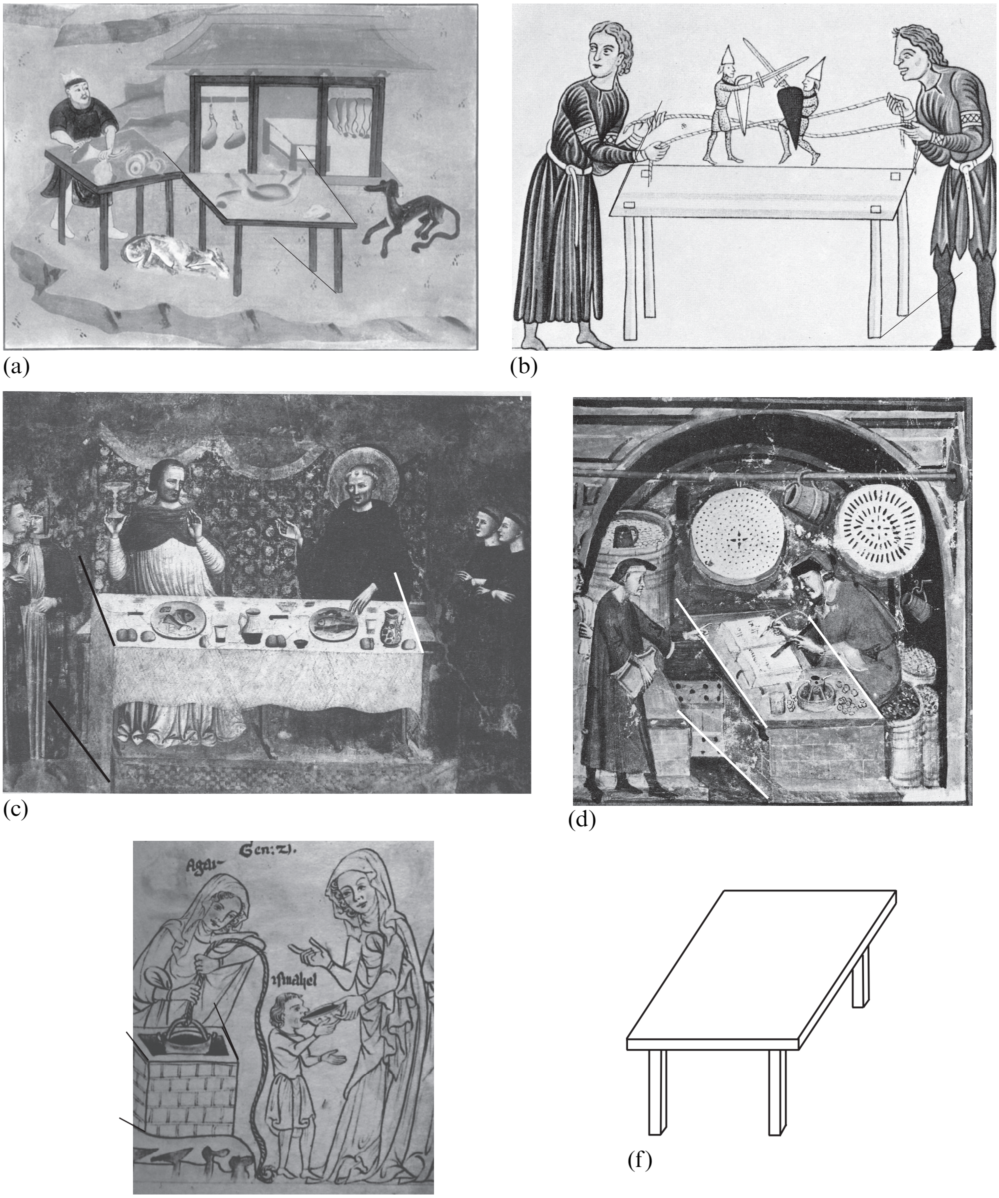

Figure 2. Examples of early divergent perspective. Added lines indicate the angles of perspective. (a) A fresco from the grotto of Touen Houang, China, Tang dynasty (618 – 906). The top of the table diverges 6°, while the side diverges 12° (from Fourcade 1962). (b) Medieval Puppeteers. (c) *Miracle of St Guido* by Jacopo da Bologna. In Pomposa Abbey, Ferrara, ca 1350. Fototeca Berenson. The edges of the top of the table diverge 2°, while the edges of the side diverge 12°. (d) From *Life in the Middle Ages* by R Delort, Universe Books, New York, 1972, page 46. Side diverges 11°, top converges 1°. (e) From the Velislav Bible, ca 1340 (Prague University Library). Top diverges 13°, side diverges 22°. (f) Illusory divergence of parallel lines on the receding sides of the drawing of a table.

2 Experiment 1. Drawing a cube and a 2-D projection of a cube

This experiment was designed to reveal how accurately young adults produce a perspective drawing of a cube and how accurately they copy a correct perspective drawing of a cube.

2.1 *Method*

2.1.1 *Subjects.* The subjects were eighty university students aged 18 to 22 years. None of them had been trained to draw in perspective. They all had normal or corrected-to-normal vision and a stereoacuity of at least 60 min of arc as tested by the stereotest circles of the Stereo Optical Company.

2.1.2 *Stimuli.* The stimuli were a 15.2 cm cube and a correct full-scale drawing of the cube, which will be referred to as the 2-D cube. The stimuli were made from stiff black card. The nine visible edges of each stimulus were white lines about 3 mm wide. Since the rear edges of the cube were not visible, it appeared like a solid black cube with white edges. The stimuli were presented one at a time in a box lined with black velvet. They were illuminated by a dimmed tungsten lamp so that nothing other than their white edges was visible. There were no visible shadows or shading. The near surface of the cube and the 2-D cube was vertical and parallel to the subject's interocular axis. The stimuli were viewed with both eyes. Each stimulus was below and to the left of a point directly in front of the centre of the interocular axis. This point defined a vanishing point that was a compromise between the distinct vanishing points of the two eyes. The vanishing point was used to define a correct drawing of the cube in one-point perspective. The head of each seated subject was supported on a chin-rest, with the eyes 76 cm from the plane containing the near surface of the cube or the 2-D cube. Figure 3a shows the cube drawn in one-point perspective relative to the vanishing point. It also shows relevant distances, the angle of convergence of a correct drawing of the cube, and labels of the relevant edges. Cross-fusion of the images in figure 3b creates an impression of the cube seen by the subjects.

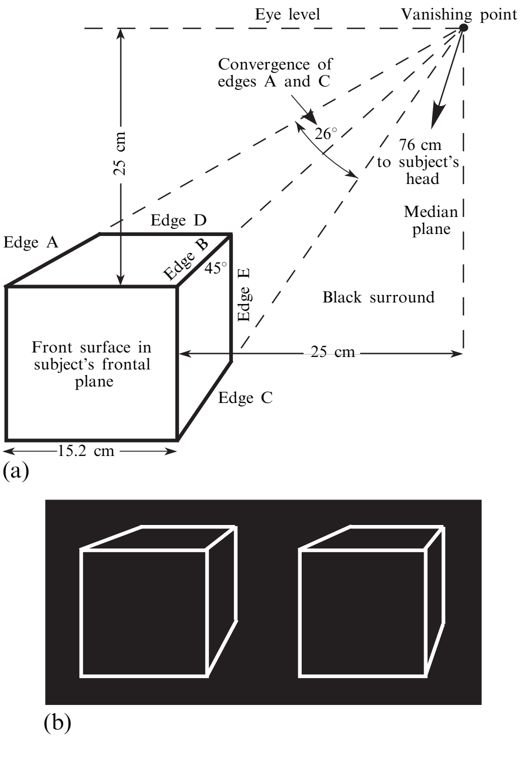

Figure 3. (a) A 2-D projection of the cube used in all the experiments. The point midway between the subject's eyes was 76 cm directly in front of the vanishing point. Subjects drew the cube on a sheet of paper to the right of the cube at a viewing distance of 25 cm. The front face of the cube was pre-drawn. Subjects drew the remaining five edges (A, B, C, D, and E). (b) Cross fusion of the two images creates an impression of the cube seen by the subjects.

2.1.3 *Procedure.* All eighty subjects drew the cube. The first forty subjects drew with a pencil on a vertical sheet of white paper just to the right of the cube at a viewing distance of 25 cm. All the other drawings in all experiments were done with a white crayon on black paper. Subjects looked at the cube with both eyes and then rotated the head on the chin-rest 25° so as to look squarely at the sheet of paper. They could look back and forth between the cube and drawing as often as they wished. The sheet of paper contained a 7.4 cm square in such a position that the vanishing point of a correct drawing of the cube based on the square was in its correct location on the subject's visual horizon and median plane. Thus, the image of the cube and a correct drawing of the cube produced the same images in the eyes. The pre-drawn square standardised the size and location of all the drawings.

A subset of sixteen students also drew the 2-D cube. Half of them drew the cube first and half of them drew the 2-D cube first.

2.2 *Results and discussion*

Figure 4 shows the two extreme examples of the drawings of the cube and the mean drawing derived from the eighty subjects. Table 1 shows the mean errors derived from drawings of the cube produced by the eighty subjects. For edges A, B, and C a positive angular error indicates that the drawn edge was rotated towards the vertical (counterclockwise) from its position in a correct drawing. For edges D and E a positive linear error indicates how far, in millimetres, the edge was displaced out from its correct position. Table 1 also shows the mean divergence errors for each pair of receding edges. A positive divergence error indicates how far the angle between a pair of edges diverges from the correct angle of convergence. In a correct drawing, edges A

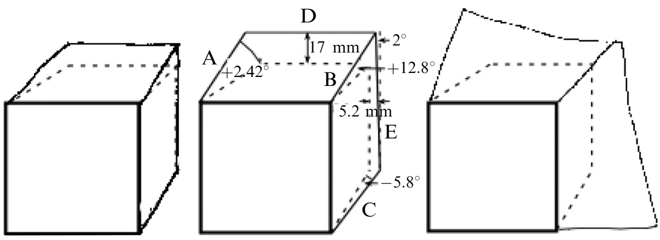

Figure 4. Drawings of a cube. The front surface was pre-drawn. Dashed lines indicate the accurate drawings relative to a vanishing point opposite the eye. (a) The best drawing. (b) The mean of the drawings produced by the eighty students. The error for each edge is with respect to the accurate drawing. (c) The worst drawing. Edges A and C diverge 96° relative to the true value and diverge 70° relative to parallel. Note how this subject drew edges D and E curved in order to connect them to edge B while approximating a right angle between them.

Table 1. Mean errors of eighty subjects in drawing the edges of a 3-D cube. A positive angular error indicates that the edge was drawn more vertical than in the correct drawing. Divergence errors A – B and B – C are the angles between the drawings of edges A and B, and B and C with respect to the correct angle of convergence of 13°. A divergence error of 13° indicates that the edges were drawn parallel. Divergence error A – C signifies the angle between edges A and C with respect to the true angle of convergence of 26°. A convergence error of 26° indicates that the edges were drawn parallel. All pairs of edges were drawn more divergently than the correct value.

Item	Angular errors/°			Divergence errors/°			Linear errors/mm	
	edge A	edge B	edge C	edges A – B	edges B – C	edges A – C	edge D	edge E
Means	24.19	12.83	−5.76	11.36	18.59	29.95	16.93	5.2
SD	11.38	7.75	10.54	9.38	9.31	14.26	10.07	6.25
SEM	1.27	0.87	1.18	1.05	1.04	1.59	1.59	2.59

and B converge 13°, and edges B and C converge 26°. In the statistical analysis of the errors, the Holm–Bonferroni procedure (Holm 1979) was used to adjust the p values for multiple comparisons. In the mean drawing, edge A is displaced 24.2° towards the vertical from its correct value ($p < 0.001$), and edge B is almost parallel to edge A. The 1.4° deviation from parallel is not significantly different from 0° ($p = 0.14$). Edge C is displaced 5.76° towards the horizontal, which is significantly different from 0° ($p < 0.001$). Edges B and C diverge 18.6° relative to the correct 13° angle of convergence. This means that edges B and C diverge 5.6° relative to parallel, which is significantly different from 0° ($p < 0.001$). Edges A and C diverge 29.95° from their correct angle of convergence ($p < 0.001$), but deviate only 3.94° from parallel, which is not significantly different from 0° ($p > 0.08$). Several subjects drew edge E tilted to the left. Figure 5 shows the distribution of divergence errors for edges A and C in 5° intervals. All eighty subjects drew both sides of the cube with more divergence than the correct value. Out of the eighty subjects, forty-three drew edges A and C diverging with respect to parallel, and fifty-six drew edges B and C diverging with respect to parallel. None of the subjects drew any of the surfaces more convergent than the correct value.

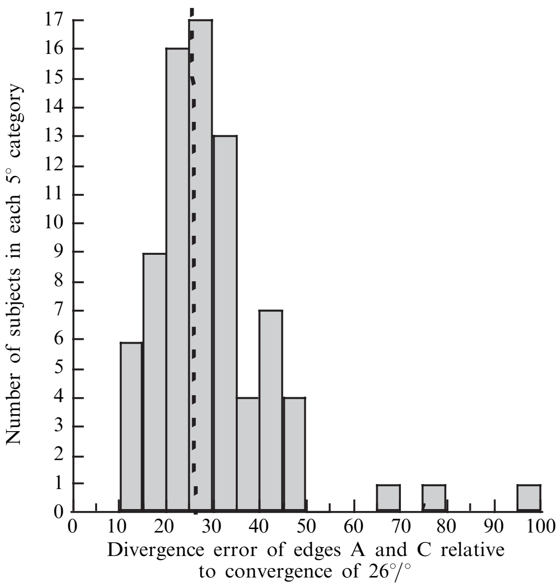

Figure 5. The distribution of divergence errors in the drawings of edge A relative to edge C. The errors are with respect to the 26° of convergence of these edges in a correct drawing. The dashed line indicates where the edges were drawn parallel.

Edge D was displaced up 17 mm from its accurate location, which is a significant error ($p < 0.001$). The centre of edge E was displaced outward only 5.2 mm, which is significantly more than 0 mm ($p < 0.001$), but significantly less than the displacement of edge D ($p < 0.001$). Subjects were asked whether they thought their drawings were good copies of the cube. They all admitted that their drawings were not accurate, but very few of them attempted to correct the drawing when asked to do so.

Figure 6 shows the mean drawing of the cube and of the 2-D projection of the cube derived from sixteen subjects. Table 2 shows the mean errors of the sixteen subjects for each of the edges of the two stimuli and the divergence errors for each pair of receding edges with respect to the correct angle of convergence. In the mean drawing of the cube, the divergence errors are 16.3° for edges A and B, 18.6° for edges B and C, and 24.9° for edges A and C. All these errors are significantly different from zero ($p < 0.001$). All sixteen subjects drew both sides of the cube with more divergence than the correct value. Nine of the sixteen subjects drew edges A and C diverging with respect to parallel. None of the subjects drew surfaces that converged relative to the true value.

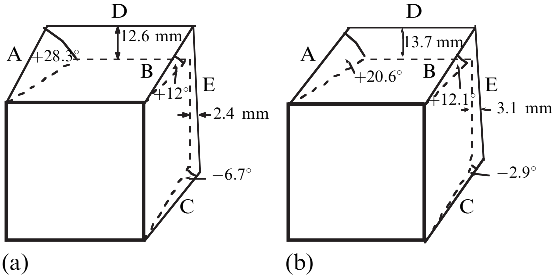

Figure 6. (a) The mean drawing of the cube made by sixteen subjects. (b) The mean drawing of the 2-D cube. Dashed lines indicate accurate drawings. For edges A, B, and C, 'plus' indicates they were drawn more vertical than the accurate drawing. 'Plus' for edges D and E indicates they were drawn displaced away from the accurate drawing (in mm).

Table 2. The upper table shows the mean errors for each edge in the drawings of the cube and of the 2-D projection of the cube with respect to the correct drawing of the edges. Errors for edges A, B, and C are in degrees. A positive error indicates that the edge was drawn more vertical than the correct drawing. Errors for edges D and E are in millimetres. The lower table shows the mean divergence errors for each pair of receding edges in the drawings of the cube and of the 2-D projection of the cube. The errors are with respect to the correct convergence, which is 13° for edges A – B and B – C, and 26° for edges A – C. All the divergence errors are positive, which means that each pair of edges was drawn with more divergence than the correct angle of convergence.

Item	Errors in drawing each edge of the 3-D cube					Divergence errors of pairs of edges of the 3-D cube		
	edge A	edge B	edge C	edge D	edge E	edges A – B	edges B – C	edges A – C
Mean	28.3	12.0	−6.7	12.6	2.4	16.3	18.6	34.9
SD	14.8	8.0	6.9	7.3	5.5	12.3	5.38	12.6
SEM	3.81	2.06	1.78	1.88	1.43	3.17	1.39	3.26
	Errors in drawing each edge of the 2-D cube					Divergence errors of pairs of edges of the 2-D cube		
Mean	20.6	12.1	−2.9	13.7	3.1	8.4	15.0	23.5
SD	7.1	8.2	5.5	7.4	4.8	6.9	5.6	6.8
SEM	1.84	2.11	1.43	1.9	1.24	1.78	1.44	1.77

In the mean drawing of the 2-D projection of a cube, the divergence errors for the three pairs of receding edges are significantly different from zero ($p < 0.002$). However, the divergence error for edges B and C (15°) is significantly less than the error of 18.7° in the drawing of the cube ($p < 0.01$). The divergence errors for edges A and B, and A and C were larger in the drawing of the cube than in the drawing of the 2-D cube. But the differences were not quite significant ($p = 0.07$). Only three subjects drew edges A and C with divergent perspective.

The main finding from experiment 1 is that adults with no training in drawing in perspective draw a cube very inaccurately. In all the drawings made by eighty university students the receding edges of the top surface were rotated towards the vertical and were, on average, parallel. The lower edge of the side surface was rotated towards the horizontal. Each pair of receding edges converged much less than in an accurate drawing and many of the students drew receding edges diverging rather than converging. Divergence was greater in the drawings of the side surface than of the top surface. The 2-D projection of a cube was drawn more accurately but still with significant divergence with respect to the accurate convergence. We are left with the question of why many people draw a cube with divergent perspective, particularly when drawing the side of an object.

The first thing to check is how accurately people recognise a cube as a cube and how correctly people recognise a correct drawing of a cube.

3 Experiment 2. Selecting a cube and a drawing of a cube

Experiment 2 was designed to answer two questions:
(i) How accurately do adults select a cube from a set of tapered boxes?
(ii) How accurately do adults select an accurate perspective drawing of a cube from a set of drawings with too much or too little perspective?

The subjects for both these tasks were a new group of sixteen university students (eight male and eight female) selected from the pool of eighty students used in experiment 1.

3.1 *Stimuli and tasks*

The stimuli for selecting a cube were a set of 11 boxes with a 15.2 cm $\times$ 15.2 cm front face but with various degrees of convergent or divergent taper of the receding surfaces. The boxes were made from black card with white visible edges. There were no visible shadows or shading, so that perspective and binocular disparity provided the only information about their structure. They were presented in the black box in the location shown in figure 3a. They were shown one at a time to each subject in random order six times. Subjects indicated whether each box tapered away from them or tapered towards them relative to a cube.

The stimuli for selecting the most accurate drawing of a cube consisted of a set of 21 white-line drawings on a black background with various degrees of taper of edges A and C. The taper of the edges varied in 1° intervals with respect to the accurate convergence of 26°. The stimuli were PowerPoint slides presented on a monitor in the same location as the tapered boxes. They were shown to each subject 10 times in random order. Subjects indicated whether edges A and C diverged more than or less than the edges of a correct drawing of a cube. Figure 7 shows some of the tapered drawings.

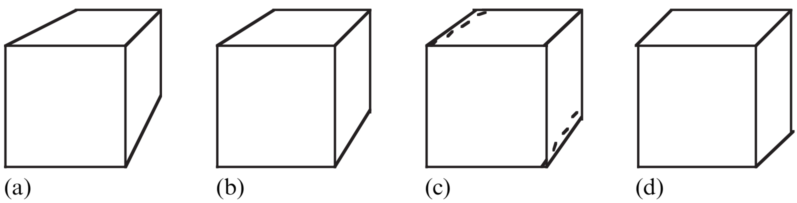

Figure 7. Examples of stimuli used to measure the ability to recognise the correct drawing of a cube. (a) A drawing with too much convergence. (b) The correct drawing of a cube for the specified vanishing point. (c) The mean of drawings selected by twelve subjects as a drawing of a cube. The dashed lines indicate the correct drawing. (d) A drawing with parallel receding edges.

The students were finally shown a series of drawings of a cube that had the correct convergence but varied in the depth dimension in 1 mm intervals. The series was presented on the drawing board 10 times, 5 times in ascending order and 5 times in descending order with randomly varying initial depths. For each sequence, subjects selected the drawing they judged to be the best drawing of a cube.

3.2 *Results and discussion*

The psychometric function in figure 8a shows that subjects selected the cube from among the tapered boxes with almost perfect accuracy. No subject had a mean error greater than $\pm 1.1°$ from the correct value. The mean error across all subjects is $-0.125°$ (SEM 0.185°). The mean JND defined as the difference between the 50% and 75% points is 0.66° (SEM 0.138°). Therefore, it is clear that errors in drawing a cube did not arise from errors in perceiving the cube as a cube.

The psychometric function in figure 8b shows that subjects selected a drawing of a cube with a mean divergence error for edges A and C. The error ranges from near zero to more than $+14°$ (SEM 1.17°). None of the subjects had a mean negative error

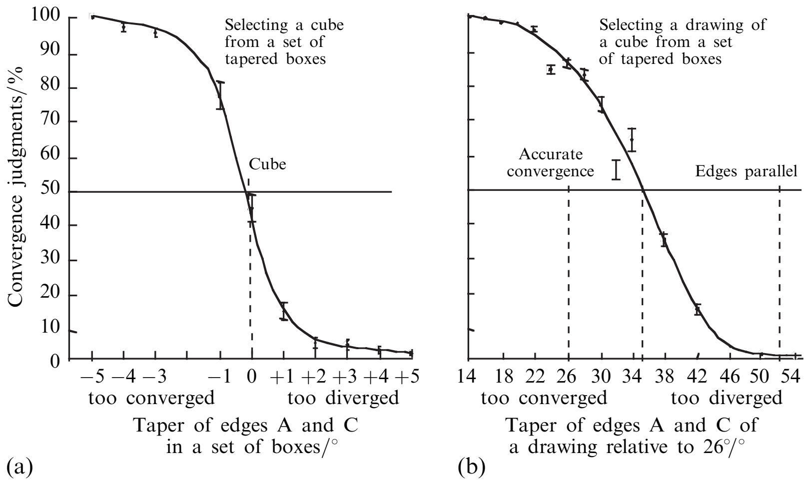

Figure 8. (a) Subjects selected a cube from a set of boxes with various degrees of taper. (b) Subjects selected the best drawing of a cube from a set of tapered drawings.

(too much convergence). The mean error over all subjects is $+8.5°$ of divergence with respect to a correct drawing. The divergence error from the selection of the cube is significantly less than the error from the selection of the drawing of a cube ($t = 7.2$, $p < 0.001$). Figure 7c shows the mean 2-D projection of the box that was selected as most like a drawing of a cube. Note that the mean divergence error that these subjects made when drawing the 3-D cube was $+34.9°$! They were therefore much better at selecting a cube and at selecting a drawing of a cube than they were at drawing a cube.

The mean drawing selected from a set of drawings with variable depth had a mean error of 1.1 mm, which is much less than the mean error of 13.7 mm in drawing the 2-D projection of a cube shown in figure 6b.

These results show that subjects had no difficulty recognising a cube as a cube and that they selected a drawing of a cube that was much more accurate than their drawings. Why don't subjects draw a cube that is at least as accurate as the drawing that they select as correct? Subjects commented that their drawings were not correct, but they did not know how to correct them. Thus the ability to recognise a correct drawing does not determine how well subjects draw a cube.

Clearly, most of the errors in drawing a cube do not arise from an inability to recognise a cube or the correct drawing of a cube. Errors in drawing a cube must arise mainly from the act of converting a 3-D cube into a 2-D drawing. The following experiments were designed to investigate the sources of these errors.

4 Experiment 3. Drawing the isolated receding edges of a cube

This experiment was designed to investigate errors in drawing the edges of a cube in more detail. The mean drawing of the cube in figure 4b shows that subjects drew edges A and B rotated towards the vertical, and edge C rotated towards the horizontal. It is hypothesised that, when asked to draw an isolated line that is orthogonal to the frontal plane, people have a tendency to rotate the drawn line towards the vertical within the sagittal plane in which the receding line lies. In other words, with respect to the 3-D line, the drawn line is rotated about a horizontal axis towards the frontal plane. The correct drawing should lie within the plane passing through the eye and the receding line, because all lines in this visual plane project the same image. Figure 9 appears in 3-D when the two images are cross-fused. The thick line indicates a line orthogonal to the frontal plane. The thin line is the correct drawing of the receding line. It lies at the intersection of the frontal plane and the visual plane containing the receding line.

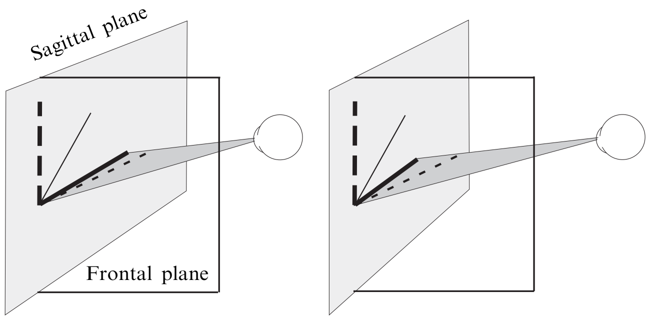

Figure 9. When cross-fused, the two images create a 3-D figure. The thick line is a horizontal receding line viewed by an eye in front of the frontal plane. The thin dashed line is the correct drawing of the receding line rotated into the frontal plane within the visual plane containing the receding line. The thick dashed line is the receding line rotated in the sagittal plane to the vertical. The thin line is a typical drawing of the receding line. It is a compromise between a correct drawing and a vertical line.

The thicker line indicates the receding line rotated in the sagittal plane to the vertical. The thin line is a typical drawing of the receding line. It is a compromise between a correct drawing and a vertical line.

The tendency to rotate a receding line within a sagittal plane into the frontal plane should be evident for both edges A and C when each edge is seen in isolation. When seen as part of the cube, edge A should manifest the same tendency because the drawing of the whole top surface swings out towards the frontal plane. However, when edge C is seen as part of a cube, the tendency to rotate it within the sagittal plane is opposed by a tendency to swing the side surface of the cube about a vertical axis into the frontal plane. The two tendencies should tend to cancel and thereby reduce the error in drawing edge C when it is part of a cube compared with when it is seen in isolation.

4.1 *Stimuli and tasks*

A fresh set of twelve students participated in this experiment. The stimuli were (i) the cube, (ii) receding edge A alone, (iii) receding edge C alone, (iv) edge A projected into the frontal plane, (v) edge C projected into the frontal plane. The stimuli were constructed of white lines and presented one at a time in random order in the black box in the position indicated in figure 3a. Subjects drew each stimulus with a white pencil on a black sheet of paper on the vertical drawing board in the same location as in experiment 1. The front surface of the cube was pre-drawn on the drawing paper. A white spot on the drawing paper indicated the lower end of each edge seen alone.

4.2 *Results and discussion*

Table 3 shows the results for the twelve subjects. Figure 10 shows the mean of the 12 drawings. The Holm–Bonferroni procedure was used to adjust the p values for multiple comparisons. Consider first the mean drawings of edge A relative to accurate drawings. Edge A presented in the frontal plane is rotated 7.8° towards the vertical ($p = 0.02$). When presented alone in 3-D, it is rotated 27.8° towards the vertical and, when part of a cube, it is rotated 21.6° towards the vertical. Two subjects drew edge A to within 5° of the vertical. The errors for the 3-D edges are not significantly different ($p = 0.36$), but they are both significantly greater than the error for the 2-D edge ($p < 0.001$). Now consider edge C. Edge C presented in the frontal plane was rotated 4.3° towards the vertical ($p < 0.01$). When presented alone in 3-D it was rotated 19.9° towards the vertical ($p < 0.001$). But, when part of a cube, it was rotated 2.7° towards the horizontal, which is not significantly different from zero error.

Table 3. The mean errors in degrees in drawings of edges A and C with respect to the correct drawings. Each edge was displayed alone receding in depth, alone as a 2-D projection of the receding edge onto the frontal plane, or as part of a complete 3-D cube.

Item	Edges seen alone				Edges of cube	
	A in 3-D	C in 3-D	A in 2-D	C in 2-D	A	C
Mean	27.8	19.9	7.8	4.3	21.6	−2.7
SD	16.6	9.3	6.9	3.9	14.2	10.8
SEM	5.01	2.80	2.07	1.18	4.27	3.26

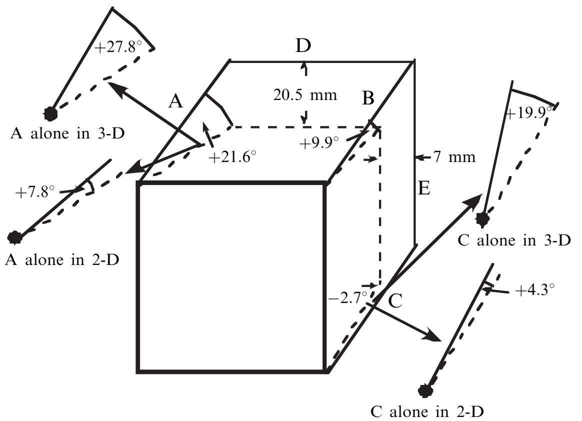

Figure 10. The mean drawings of the cube and of each receding edge seen in isolation. The front face of the cube and the spots were pre-drawn. The thin lines indicate the mean positions of the drawn edges. The dashed lines indicate the correct drawings.

This means that, when drawing an isolated line that was orthogonal to the frontal plane, subjects tended to rotate the drawing towards the vertical. As predicted, the error for edge A when it was part of the cube is not significantly different from the error when receding edge A was presented alone. Receding edge C was rotated 19.9° towards the vertical when presented alone, but it was rotated 2.7° towards the horizontal when part of a cube. This supports the idea that edge C is subject to two competing tendencies. One is a tendency for the 3-D edge presented alone to be rotated within a sagittal plane into the frontal plane. The other is a tendency for edge C as part of the right surface of the cube to be rotated about a vertical axis towards the frontal plane. When edge C was part of the cube, these two tendencies tended to cancel, leaving a small residual error of only −2.7°.

These results confirm that, when people draw an isolated line that is orthogonal to the frontal plane, they rotate the drawing towards the vertical. However, when the line is an edge of the side surface of a cube, this tendency is opposed by a tendency to rotate the surface about a vertical axis into the frontal plane.

5 Experiment 4. Drawing the isolated receding surfaces of a cube

This experiment was designed to test the following hypothesis:
If subjects are shown only the top surface of the 3-D cube, they will draw it rotated about a horizontal axis with sides A and B more or less parallel. If they are shown only the side surface, they will draw it rotated about a vertical axis with sides B and C more or less parallel.

5.1 *Stimuli and tasks*

A fresh group of twenty-four students drew (i) the cube, (ii) the top surface of the cube presented alone, (iii) the side surface of the cube presented alone. As in the previous experiments, the stimuli had white edges and subjects drew with a white pencil on a black sheet of paper. The front face of the cube, the near horizontal edge of the top surface, and the near vertical edge of the side surface were pre-drawn on the drawing paper. The stimuli were presented in different orders.

5.2 *Results*
Table 4 shows the results for the top and side surfaces presented alone. Figure 11 shows the mean of the drawings of the cube and of the two surfaces seen alone. The Holm–Bonferroni procedure was used to analyse the results. Each surface seen alone is drawn rotated towards the frontal plane. The drawing of the top surface is rotated about a horizontal axis and the side surface is rotated about a vertical axis. Edges A and B converge 3.4° from parallel ($p < 0.01$). Edges B and C diverge 5.9° from parallel ($p < 0.01$). While these departures from parallel are significant, they are small. For both surfaces, edges D and E are displaced from their correct position (34 mm and 19.8 mm) by a significantly greater amount than in the drawing of the cube ($p < 0.02$ and $p < 0.001$, respectively). Thus, both faces were rotated into the frontal plane to a significantly greater extent when seen alone than when seen as part of a cube.

Table 4. The mean errors (°) in drawings of the top and side surfaces of a cube displayed in isolation.

Item	Top surface			Side surface		
	edge A	edge B	edges A–B	edge B	edge C	edges B–C
Mean	34.3	24.6	9.6	5.6	24.4	18.9
SD	12.5	11.1	3.9	14.3	15.5	9.2
SEM	5.32	4.07	0.82	1.88	0.42	1.92

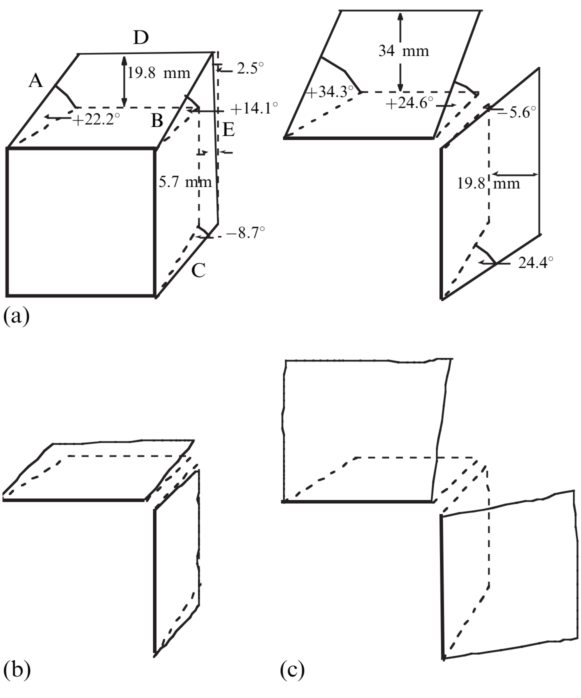

Figure 11. (a) Mean drawings made by twenty-four students of the cube and the top and side surfaces shown in isolation. (b) The best drawing of the surfaces. (c) The worst drawing of the surfaces. The bold lines were pre-drawn. The dashed lines indicate the correct drawings.

These results support the hypothesis that isolated receding surfaces tend to be drawn as if rotated towards the frontal plane, with the receding edges more or less parallel. This hypothesis implies that horizontal receding surfaces are rotated around a horizontal axis into the drawing, while vertical receding surfaces are rotated about a vertical axis. If the horizontal and vertical surfaces of a complete cube were drawn rotated this way, they would no longer be connected because edge B would have to be drawn twice. Figure 12 shows how children solve this problem when drawing a cube. Some children draw the receding surfaces of a cube in the frontal plane with the edges not connected. Other children draw the faces in the frontal plane and connect the edges by extending one surface or by distorting the surfaces. Adults retain the tendency to rotate receding surfaces into the frontal plane. However, when the surfaces are part of a cube, adults reduce this tendency and solve the problem of connecting the edges by drawing the last side with divergent perspective. Van Sommers (1984) described a similar strategy but did not produce any quantitative evidence.

Figure 12. Examples of drawings of an obliquely viewed cube by children aged 5 to 8 years (from Chen 1985).

6 Experiment 5. The effect of the order in which a cube is drawn

We suggest that our subjects applied the following strategies in drawing a cube, although not necessarily consciously. In experiments 1 to 4 subjects started their drawing with edge A. They rotated edge A towards the vertical and drew edge B more or less parallel to edge A. Consequently, the top surface was drawn rotated about a horizontal axis towards the frontal plane. They then drew the side surface rotated about a vertical axis towards the frontal plane. However, edge B could not be drawn parallel to edge C because edge B was already drawn parallel to edge A. This produced divergent perspective into the side surface. It also left the problem of connecting edges B, D, and E. If edges B and C were drawn the same length, edge E would have to be tilted to the left. Several subjects did tilt edge E to the left (see figure 6).

Experiment 5 was designed to test the following hypothesis:

If subjects start by drawing edge A of a cube, they should produce a drawing like figure 13a. If subjects start by drawing edge C, they should draw edge B parallel to edge C. Edge A will be rotated about a horizontal axis towards the frontal plane as before, but it will not be parallel to edge B because edge B is already drawn parallel to edge C, as shown in figure 13b. This results in the top face of the cube being drawn with divergent perspective rather than the side face.

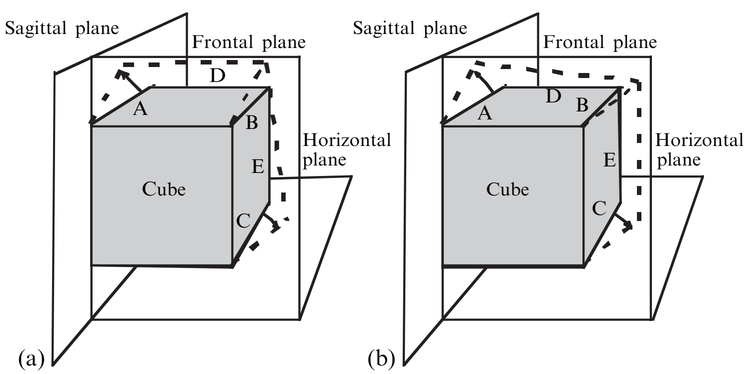

Figure 13. (a) When the edges of a cube are drawn in the order A, B, C, D, E, it is predicted that edge B will be drawn parallel to edge A and edges B and C will diverge, as indicated by the dashed lines. (b) When the edges are drawn in the order C, B, A, D, E, it is predicted that edge B will be drawn parallel to edge C and edges A and B will diverge.

6.1 *Stimuli and task*

The same group of twenty-four university students used in experiment 4 drew the 3-D cube with the edges drawn in the order A, B, C, D, E (A-first condition), and in the order C, B, A, D, E (C-first condition). The front surface of the cube was pre-drawn. Half the subjects drew the cube in one order first and the other half drew the cube in the reverse order first.

6.2 *Results*

Table 5 shows the results for the drawings of the cube in the two orders. Figure 14 shows the mean of the drawings of the cubes in the two conditions. A repeated-measures linear mixed model analysis was performed for the angular and linear errors. Comparisons of interest were tested using the multiple comparison technique of Hothorn et al (2008).

Table 5. Mean errors of twenty-four subjects in drawing a cube starting with edge A and starting with edge C. The errors for edges A, B, and C are in degrees. The errors for edges D and E are in millimetres. Mean divergence errors for edges A and B and for edges B and C are with respect to the correct convergence of 13°. The divergence error for edges A and B in 'start with edge A' is clearly less than that in 'start with edge C'. The divergence error for edges B and C in condition 'start with edge A' is clearly more than that in condition 'start with edge C'.

Item	Start with edge A					Start with edge A	
	edge A	edge B	edge C	edge D	edge E	edges A–B	edges B–C
Means	22.2	14.1	−8.7	19.8	5.7	8.1	22.8
SD	8.0	8.0	8.2	10.5	7.0	6.2	8.3
SEM	1.67	1.67	1.72	2.18	1.46	1.29	1.73
	Start with edge C					Start with edge C	
Means	18.8	0.8	−8.2	13.2	9.2	18.6	8.9
SD	11.2	9.9	8.7	10.7	9.0	9.8	10.3
SEM	2.34	2.06	1.81	2.23	1.88	2.05	2.17

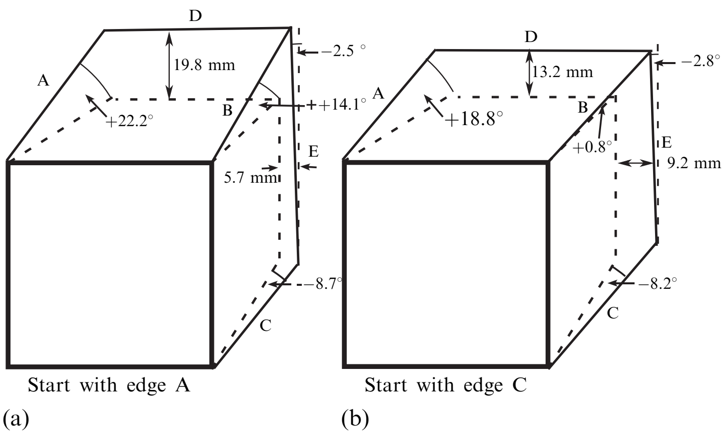

Figure 14. The mean drawings of the cube by twenty-four subjects. The thick lines were pre-drawn. The dashed lines indicate the correct drawings. (a) The cube was drawn in the order A, B, C, D, E. (b) The cube was drawn in the order C, B, A, D, E.

The angle data showed a significant interaction between order and edge ($F_{2,115} = 11.095$, $p < 0.0001$), and significant main effects of edge ($F_{2,115} = 181.783$, $p < 0.0001$) and order ($F_{1,115} = 18.813$, $p < 0.0001$). Edges A and C had significant angular errors for both starting orders ($p < 0.001$). Edge A was drawn rotated towards the vertical, and edge C was rotated towards the horizontal. As predicted, the drawings of edges A and C were not significantly different in the two conditions ($p > 0.5$). Also, as predicted, edge B was drawn approximately parallel to edge A in the A-first condition but more parallel to edge C in the C-first condition. Edge B had a significant angular error in the A-first condition ($p < 0.001$) but not in the C-first condition ($p > 0.5$). The difference in the error of edge B (13.4°) between the two conditions was highly significant ($p < 0.001$). As predicted, edges A and B were drawn significantly more parallel in the A-first condition than in the C-first condition (difference = 10.0°, $p = 0.013$) and, conversely, edges C and B were drawn significantly more parallel in the C-first condition than in the A-first condition (difference = 13.89°, $p = 0.013$).

Also, as predicted, edges A and B were drawn significantly more parallel than edges B and C in the A-first condition ($p < 0.001$). However, edges B and C were not drawn significantly more parallel than edges A and B in the C-first condition ($p = 0.135$).

Overall, these results support the hypothesis that the way people draw a cube depends on the order in which the edges are drawn. When the top surface is drawn first, it is rotated towards the frontal plane about a horizontal axis with its receding edges more or less parallel. The side face then tends to be drawn with divergent perspective. When the side surface is drawn first, it is rotated about a vertical axis towards the frontal plane with its edges more-or-less parallel. The top surface then tends to be drawn with divergent perspective.

7 General discussion

We have shown that many adults draw a cube with the receding edges diverging rather than converging. Note the similarity between the mean drawing of a cube produced by our subjects in figure 4b and the divergent perspective evident in the pre-15th-century paintings of rectangular objects in figure 2. Not one of eighty randomly selected university students came close to drawing a cube with the correct convergence. Subjects were aware that their drawings were not correct but were unable to correct them. We suspect that pre-15th-century artists also realised that their paintings were not correct but they too did not know how to correct them. As soon as drawings in correct perspective were produced in the 15th century everyone viewing them realised that they were far better than what had come before and drawing in correct perspective became the norm.

The 3-D structure of the cube was specified by linear perspective and binocular disparity. The problem did not arise from an inaccurate registration of the 3-D structure of the cube, because subjects selected a cube from a set of tapered boxes with great accuracy. They showed almost perfect shape constancy. When selecting a correct drawing of a cube, they selected a drawing that was, on average, about 9° too divergent. In this case, the perspective of the drawing indicated a 3-D cube, but the zero binocular disparity indicated a 2-D stimulus. This effect is a manifestation of what Thouless (1930) called "regression to the real object" or what we refer to as regression to the orthogonal view. Thouless asked subjects to select a shape in the frontal plane to match a shape inclined in depth at the same distance. For example, when subjects selected an ellipse in the frontal plane to match an inclined circle, they selected an ellipse that was intermediate between a circle and the image of the inclined circle. This is due to a tendency to perceive a frontal ellipse as inclined and therefore longer in its perceived depth dimension relative to its lateral dimension, coupled with a tendency to perceive an inclined circle as an ellipse. The same tendency is evident in Shepard's tables shown in figure 15. The in-depth dimension of each drawing appears elongated. This is because each drawing is perceived as a table in 3-D. The orthogonal view of the 3-D table that would create the drawing would indeed be elongated in depth relative to the 2-D image. The perceived shape of the drawing regresses to this orthogonal view.

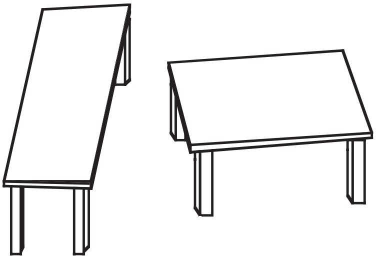

Figure 15. The two tabletops are the same shape and size. In each case, the depth dimension appears elongated relative to the frontal dimension (after Shepard 1990).

Regression to the orthogonal view is evident in the errors made by our subjects when selecting the best drawing of a cube. This tendency is also evident in the drawings made by our subjects of isolated edges or faces of the cube. Generally, the drawings were between the correct perspective and parallel perspective. However, regression to the orthogonal view does not explain why many subjects drew the sides of the whole cube with divergent perspective. We conclude that drawing a cube with divergent perspective arises in the act of drawing.

We propose that people draw with divergent perspective because they do not register the whole 3-D object and then draw it. Instead, they produce the drawing piecemeal according to the following tendencies.

(i) A single line that is orthogonal to the frontal plane tends to be drawn rotated towards the frontal plane within the sagittal plane containing the line. For a correct drawing the line should be rotated in the visual plane containing the line.

(ii) Receding parallel lines tend to be drawn parallel. In other words, they are drawn as if viewed orthogonally.

(iii) A horizontal receding surface tends to be drawn rotated about a horizontal axis towards the frontal plane or, equivalently, as if the surface were viewed orthogonally in plan view.

(iv) A vertical receding surface tends to be drawn rotated about a vertical axis towards the frontal plane or, equivalently, as if the surface were viewed orthogonally in side elevation.

(v) When drawing a cube, people draw one receding side before they draw the other receding side. This produces divergent perspective in the surface that was drawn second, as explained below.

The first four tendencies can be understood as manifestations of a general tendency to draw receding lines and surfaces as if they were viewed orthogonally (in the frontal plane). They reflect the effect of regression to the orthogonal view. The fifth tendency is responsible for producing divergent perspective, in which receding edges are drawn diverging rather than converging.

The drawings of the cube were not simple orthographic drawings made from a single vantage point. An orthographic drawing of a cube with one surface parallel to the picture plane is a square. In an orthographic drawing of a cube rotated out of the picture plane (a dimetric or trimetric projection) all edges are parallel, but the foreshortening (aspect ratio) of receding surfaces is the same as in a correct drawing in polar perspective, as shown in figure 16. Very few subjects drew all three receding edges parallel, and they all drew receding surfaces with too little foreshortening.

It is as if our subjects attempted to flatten receding surfaces by drawing them as they would appear when viewed orthogonally (in a frontal plane). However, if each surface of a cube were drawn as if viewed orthogonally, the drawing would consist of three equal squares representing the plan, front elevation, and side elevation of the cube. It would be three orthogonal drawings, as in figure 16b. For many purposes, this is a perfectly valid way to represent the 3-D structures. Architects use it, as they did

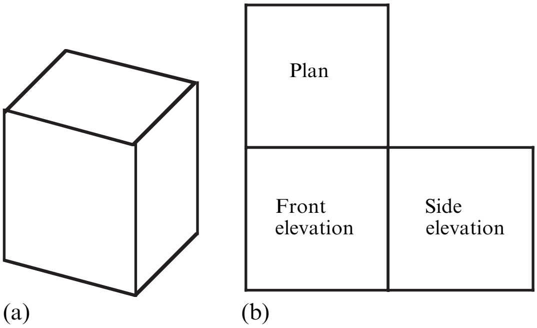

Figure 16. (a) A dimetric orthographic drawing of a cube. All receding lines are drawn parallel but foreshortening is correct. (b) Three orthogonal drawings of a cube. All sides are squares.

in ancient times, long before drawing in polar perspective was developed. This way of drawing is very evident in young children, as can be seen in figure 12. Most adults retain a tendency to draw in this way, which is most evident when they draw an isolated receding surface. However, when drawing a cube, adults have a competing tendency to keep the edges of the drawing connected. If the top surface of a cube is drawn first, that surface tends to be drawn with parallel edges and rotated about a horizontal axis towards the frontal plane (regression to the orthogonal view). But this means that only the lower edge of the side surface can be rotated about a vertical axis towards the frontal plane. This produces divergent perspective in the side surface. If the side surface of a cube is drawn first, it is rotated about a vertical axis toward the frontal plane with its two edges more or less parallel. But now, only one edge of the top surface can be rotated about a horizontal axis, which produces divergent perspective in the top surface. These tendencies account for the fact that many people draw one or more surfaces of a cube with divergent perspective, just as artists did before the 15th century.

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