# PROCEDURE FOR THE NUMERICAL COMPUTATION OF MECHANICAL VIBRATIONS IN ELECTRICAL MACHINES

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## ABSTRACT

The present paper describes a way for calculating the mechanical vibrations of the stator and case in the two dimensional modelling of electrical machines. The mechanical vibrations are the result of the magnetic forces acting on the surfaces of the stator. The proceeding of the vibration calculation can be divided into three steps:

1. Finite Element Method (FEM) calculation of the magnetic field,
2. Local force density calculation and its Fourier decomposition,
3. Calculation of the dynamic displacements of the electrical machine's stator and case.

## MAGNETIC FIELD

The calculation of the magnetic field will be done using the Finite Element Method with a vector potential description. Therefore this procedure isn't discussed here in detail.

## SURFACE FORCE DENSITY

The magnetic field and the mechanical force distribution in synchronous and asynchronous machines can be regarded as rotating ones. The magnetic field is cyclic in one pole-pair. As soon as these forces act on the machine's stator together with the case, dynamic deformation occurs on the surface.

The FE-method will be used for the calculation of the magnetic field distribution as well as for the calculation of the mechanical vibrations. On the one hand a magnetic model describes one pole of a synchronous machine, which will be used here for illustrating the method. This model can be used for the calculation of the magnetic field as well as for the surface force density. On the other hand a mechanical model including the whole stator and the case will be used for the calculation of the mechanical vibrations.

Following [1] the method used for the calculation of the surface force density is a derivative of the stress tensor method. For a linear approximation of the ferromagnetic material the surface force density $\vec{\sigma}$ takes the form as below:

$$\vec{\sigma} = \vec{H}_1 (\vec{B}_1 \cdot \vec{n}_{12}) - \frac{1}{2}\vec{n}_{12}(\vec{B}_1 \cdot \vec{H}_1) - [\vec{H}_2 (\vec{B}_2 \cdot \vec{n}_{12}) - \frac{1}{2}\vec{n}_{12}(\vec{B}_2 \cdot \vec{H}_2)] \quad (1)$$

The values $\vec{H}_1, \vec{B}_1, \vec{H}_2, \vec{B}_2$ describe the magnetic field at both sides of the boundary surface. An illustration can be taken from Fig. 1.

The unit normal vector $\vec{n}_{12}$ is in the direction from region 2 to 1. It can be seen from equation (1) that the product of the field strength and the flux density is essential for the calculation of the surface force density. In [1] it is shown that the global force $\vec{F}$ acting upon a body can be calculated by integrating the stress tensor $\bar{T} \cdot \vec{n}$ over a closed surface S

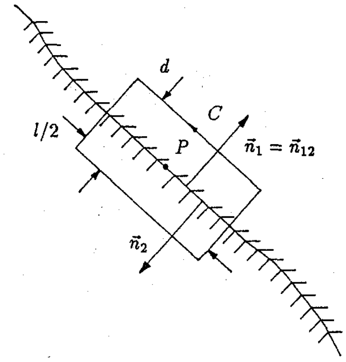


**Fig. 1:** The surface force density

$$\vec{F} = \oint_S \bar{T} \cdot \vec{n} \, da \quad (2)$$

For the expression $\bar{T} \cdot \vec{n}$ it can be written

$$\bar{T} \cdot \vec{n} = \vec{H}(\vec{B} \cdot \vec{n}) - \frac{1}{2}\vec{n}_{12}(\vec{B} \cdot \vec{H}) \quad (3)$$

supposing to regard linear material properties. Regarding the definition of the stress tensor (2) for a small part of volume on the surface of a body (Fig. 1), one gets

$$\vec{\sigma} = \lim_{d,l \to 0} \frac{1}{l} \oint_C \bar{T} \cdot \vec{n} \, ds = \bar{T}_1 \cdot \vec{n}_1 + \bar{T}_2 \cdot \vec{n}_2 \quad (4)$$

Using (3) and the unit vector $\vec{n}_{12}$, (4) becomes (1).

Regarding further on the electrodynamic field-conditions on the body's border

$$\mathrm{Div}\, \vec{B} = 0 \ \text{ and } \ \mathrm{Rot}\, \vec{H} = 0 , \quad (5)$$

an easy calculation leads to the expression

$$\vec{\sigma} = \frac{1}{2}\vec{n}_{12}[B_n(H_{1n} - H_{2n}) - H_t(B_{1t} - B_{2t})] . \quad (6)$$

having set $B_n = B_{1n} = B_{2n}$ and $H_n = H_{1n} = H_{2n}$ .

See also Fig. 2 and take into account that the surface force density is always in the direction of the normal unit vector on the surface.

Returning to the application of the synchronous machine, the force is a cyclic function in one pole. This surface force density function has to be approximated by a number of models, each

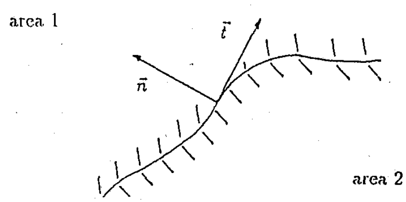


**Fig. 2:** Decomposition of the vectors in normal and tangential direction

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with another rotor position. The synchronous machine, here used as an example, includes 12 poles. There have been chosen 30 models with a 1 degree (mech.) difference of the rotor position from one model to the next one.

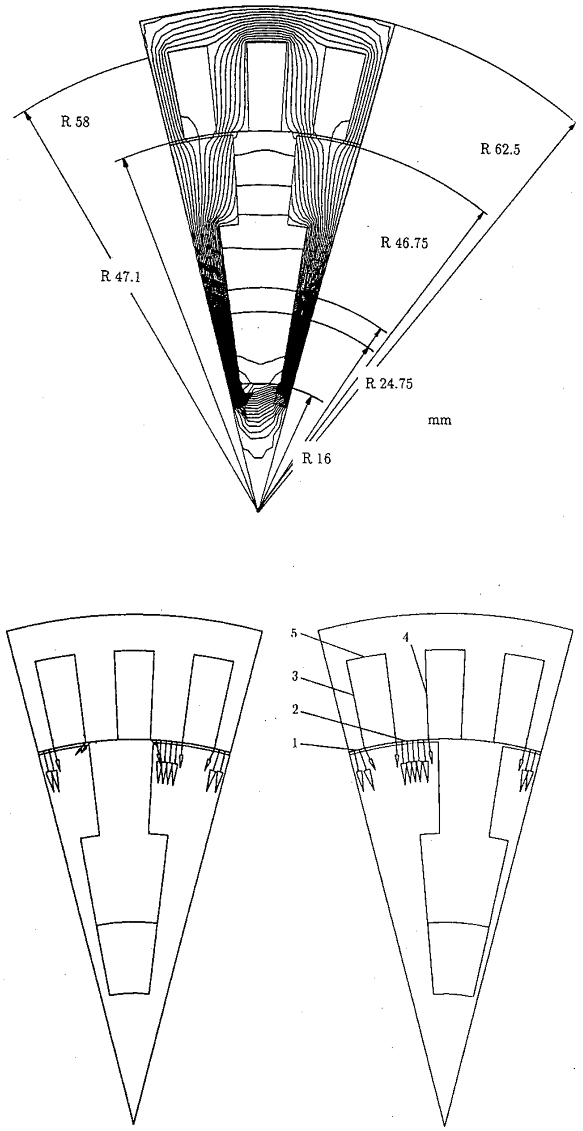



**Fig. 3:** Geometric data and some models with their surface force density

Fig.3 shows some geometric data about the synchronous machine and a selection of 3 models with the surface force densities acting on the stator. The arrows show the direction and the relative magnitude of the force density. The calculated force densities are in a range of 13 N/cm$^2$.

In Fig. 4 and Fig. 5 there is shown a set of time functions of the force density at several points described in Fig. 3. A discrete Fourier decomposition of the surface force time dependent functions in every surface element, using the following equations

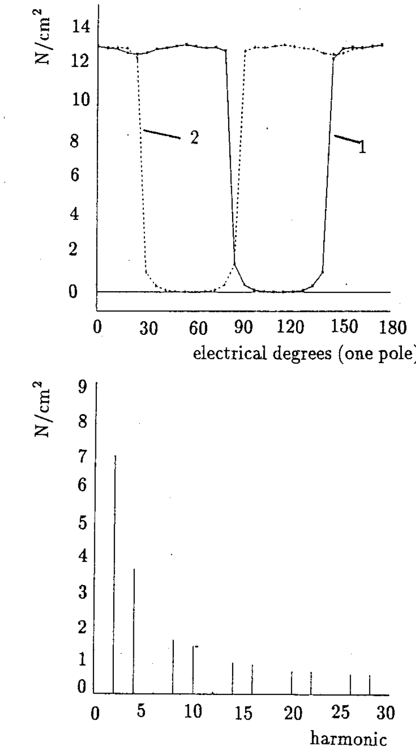



**Fig. 4:** Time dependent functions of force density at some chosen points (see also Fig. 3) and a spectrum for the points 1 and 2

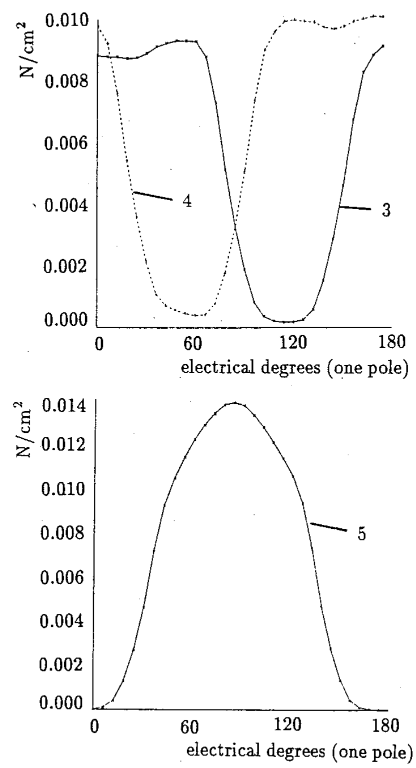



**Fig. 5:** Time dependent functions of force density at some chosen points (see also Fig. 3)

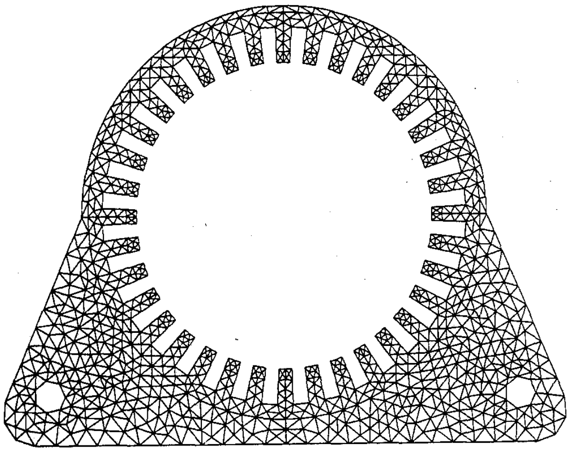
$$\sigma(\omega t) \approx a_0/2 + \sum_{n=1}^{q} a_n \cos(n\omega t) + b_n \cos(n\omega t) \quad (7)$$

$$a_0 = \frac{2}{N} \sum_{i=1}^{N} \sigma(\omega t_i) \quad , \quad a_n = \frac{2}{N} \sum_{i=1}^{N} \sigma(\omega t_i) \cos(n\omega t_i), \quad (8)$$

$$b_n = \frac{2}{N} \sum_{i=1}^{N} \sigma(\omega t_i) \sin(n\omega t_i) \; , n = 1, 2, \ldots, (\frac{N}{2} - 1), \quad (9)$$

produces a spectrum for each surface element of the mesh. $N$ is the double number of models used for one period of the surface force function, e.g. $N = 60$ for this example. Such a spectrum, as shown in Fig. 4, has to be made for each surface element of the mesh. The frequencies of the harmonics are related to the frequency of the stator current. The surface force distribution has to be transfered from the magnetic to the mechanic model (see Fig. 4 and Fig. 5) for each harmonic. Then the displacement calculations can be made.

## MECHANICAL COMPUTATION



1100 nodes and 1776 elements

**Fig. 6:** The mechanical model

The FE-calculation of the dynamic displacements is based upon the principle of Hamilton, which prescribes to find the minimum of the difference from the kinetic energy and the elastic potential of the whole structure [2]. Using this principle one gets the following system of equations

$$\underline{M} \cdot \ddot{\underline{D}} + \underline{K} \cdot \underline{D} = \underline{R} \quad (10)$$

$\underline{D}$ is the global vector of node displacements, $\underline{M}$ is the global mass matrix including the informations of inertia and $\underline{K}$ is the global stiffness matrix, describing the elastic features of the structure. Because of the harmonic time dependence of the node displacements one gets

$$\ddot{\underline{D}} = -\omega_{mech}^2 \cdot \underline{D}. \quad (11)$$

Combined with the equation (10) it follows

$$(\underline{K} - \omega_{mech}^2 \cdot \underline{M}) \cdot \underline{D} = \underline{R}. \quad (12)$$

The vector $\underline{R}$ includes the amplitudes of the exciting force densities.

The models for both the magnetic and mechanic computation consist of triangular elements. The mechanic model is shown in Fig. 6. The model includes the stator and its case with 1100 nodes and 1776 elements. The two bore-holes have, as a result of the Dirichlet boundary conditions, no displacements. The following table includes some parameters which were used for the mechanic computation.

| | stator | Al-case |
|---|---|---|
| E-modul. | $1.2 \cdot 10^{11}\mathrm{N/m^2}$ | $7.1 \cdot 10^{10}\mathrm{N/m^2}$ |
| density | $7.8 \cdot 10^3\mathrm{kg/m^3}$ | $0.9 \cdot 10^3\mathrm{kg/m^3}$ |
| poisson's number | 0.25 | 0.25 |

For solving the equation (10), the 'Incomplete Cholesky - Conjugate Gradient' method [3] has been used. The number of iteration steps for the desired precision is equal about 50 - 60 percent the number of nodes. It has been also noticed that the distribution of the surface force density is uneven.
The density of the Al-case is reduced to $0.9 \cdot 10^3\mathrm{kg/m^3}$ because of the uneven length of machine's stator and case.

## SOME RESULTS

The mechanical vibrations were computed at 1500 rpm and 3000 rpm. The machine with 12 poles signifies a stator-frequency of $f_{St} = 150$Hz at a speed of 1500 rpm. Therefore the stator's frequency at a speed of 3000 rpm is $f_{St} = 300$Hz. The following table presents an overview of the harmonics surface-force-densities and their displacements.

| harmonic order | frequency 1500 rpm [Hz] | displacement 1500 rpm [$\mu$m] | cog's force densities [N/cm$^2$] |
|---|---|---|---|
| 2 | 300 | $1.86 \cdot 10^{-2}$ | 6.88 |
| 4 | 600 | $6.43 \cdot 10^{-3}$ | 3.67 |
| 6 | 900 | $2.25 \cdot 10^{-2}$ | 0.15 |
| 8 | 1200 | $5.23 \cdot 10^{-3}$ | 1.47 |
| 10 | 1500 | $2.46 \cdot 10^{-3}$ | 1.36 |
| 12 | 1800 | $5.99 \cdot 10^{-4}$ | 0.22 |
| 14 | 2100 | $3.81 \cdot 10^{-3}$ | 0.92 |
| 16 | 2400 | $2.34 \cdot 10^{-3}$ | 0.89 |
| 18 | 2700 | $2.57 \cdot 10^{-4}$ | 0.09 |
| 20 | 3000 | $8.86 \cdot 10^{-4}$ | 0.74 |
| 22 | 3300 | $1.65 \cdot 10^{-3}$ | 0.71 |
| 24 | 3600 | $2.01 \cdot 10^{-4}$ | 0.04 |
| 26 | 3900 | $1.26 \cdot 10^{-3}$ | 0.69 |
| 28 | 4200 | $6.53 \cdot 10^{-4}$ | 0.68 |

Following there shall be presented some results for a speed of 1500 rpm. Fig. 7 displays the resulting deformation for the 2nd harmonic (300 Hz). It is quasistatic, i.e. the local deformation follows consequently the surface force distribution as a result of the fact that the frequency is far away from an eigenvalue.
Fig. 8 shows the nodes displacements for the 6th harmonic (900 Hz).One recognizes an eigenform of the order 1.
Fig. 9 presents the deformation results of the 24th harmonic (3000 Hz). Here can be found the third eigenform of the structure.

Fig. 10 and 11 finally display at a speed of 3000 rpm the deformations of the 16th and 24th harmonics. The eigenforms of the order 4 and 5 can be recognized.

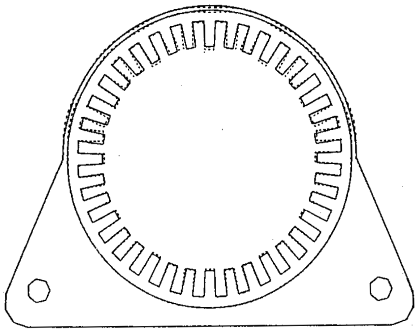


**Fig. 7:** Structure deformation - 2nd harmonic - 300Hz

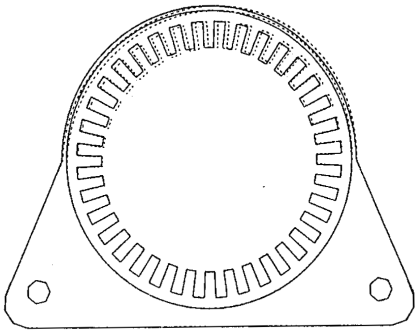


**Fig. 8:** Structure deformation - 6th harmonic - 900Hz

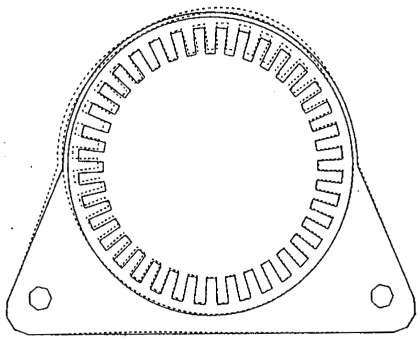


**Fig. 9:** Structure deformation - 24th harmonic - 3600 Hz

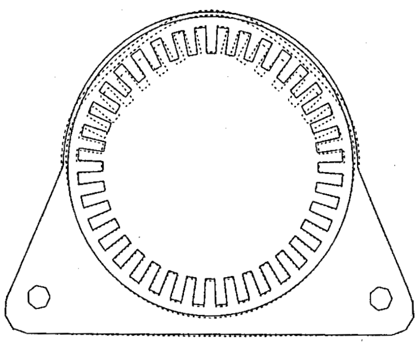


**Fig. 10:** Structure deformation - 16th harmonic - 4800Hz

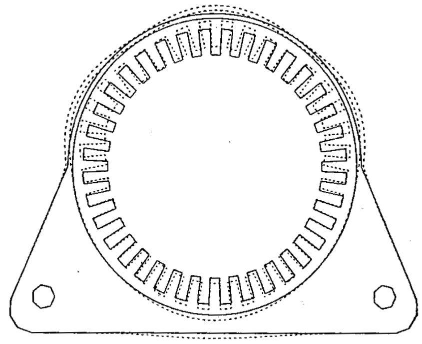


**Fig. 11:** Structure deformation - 24th harmonic - 7200Hz

## CONCLUSIONS

1.) The Finite Element Method is a usefull tool to compute

- the machine's magnetic field,
- the machine's mechanic deformations even in case of time harmonic magnetic fields;

2.) The method used here for calculating the surface force density being a derivative of the Maxwell Stress Tensor method permits the combination of computing magnetic field and mechanic deformation.

3.) The computed results of structure deformations point out the importance of eigenvalues and eigenforms. An investigation of mechnical vibrations has to pay attention to the resonance frequencies.

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