Combined Numerical and Analytical Method for Geometry Optimization of a PM Motor

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For the reduction of the cogging torque of permanent-magnet synchronous machines several approaches are known. But cogging torque predictions for arbitrary types of machines using the same approach are still troublesome and imprecise. Therefore, a fast optimization process is developed and presented in this paper. Here, the combination of both numerical and analytical simulation results in such a fast method.

***Index Terms*—Cogging torque, notches in teeth, optimization, permanent-magnet synchronous-machine, two-dimensional (2-D) finite elemenet method (FEM).**

I. Introduction

In the past few years, the hydraulic power steering in several cars has been replaced by electrical power-steering systems. The standard drive is a permanent-magnet synchronous machine. One major task in developing permanent-magnet machines is to minimize the cogging torque. Several methods are known [1]–[3]. They are divided into three groups depending on the main effect applied. The first group controls the function of the magnetization manipulating the shape of the magnets, the magnetization of the magnets themselves, the pole-arc to pole-pitch ratio, and the shape of the iron core [4]. The second group considers the relative air–gap permeance by modeling the shape of the slots, the tooth width, or using teeth pairing, notches in the teeth, or extra slots [5]. Finally, the third group compensates the cogging torque by skewing, pole shifting, or an asymmetric distribution of the magnets [6].

For the basic machine regarded here skewing was applied. The compensation of the cogging torque by skewing is very sensible to aberrations of the skewing angle by manufacturing tolerances. Therefore, notches in the teeth are applied instead. In this paper, the optimization of the notch shape using a combined numerical and analytical method is described.

II. Analytical Model

The chosen approach for the reduction of the cogging torque is the insertion of extra notches in the stator teeth. The notches produce supplementary cogging torque which is phase shifted due to the location of the notches [6]. The sum of all cogging-torque contributions result in a very low total cogging torque. As the notches act like regular slots, the number of stator slots N_s is virtually increased. The fundamental order of the cogging torque, which reaches the highest magnitude of all orders by far, increases if an adequate number of notches is inserted. With an adequate choice of the number of notches per tooth N_n the peak-to-peak value of the cogging torque is reduced successfully. The machine regarded here consists of $N_p = 2p = 8$ rotor poles and $N_s = 18$ stator slots. With the number of cogging-torque periods per slot[1]

$$N_c = \frac{\mathrm{LCM}(N_s, 2p)}{N_s} = \frac{\mathrm{LCM}(18, 8)}{N_s} = \frac{72}{18} = 4 \qquad (1)$$

the optimal number of notches per tooth is derived to

$$N_n = \min\{N_c \neq m(N_n + 1), \forall m \in \mathbb{N}\} = 2. \qquad (2)$$

In order to find the optimal shape of the notches, the energy-method is applied and modified [7]

$$T = \frac{dW}{d\Theta} \qquad (3)$$

where T is the torque, W the magnetic energy in the air gap with respect to the relative position of the stator and the rotor Θ. With the flux density in the air gap B_δ depending on Θ and the location in the air gap α the magnetic energy reads

$$W(\Theta) = \frac{1}{2\mu_0}\int_V B_\delta^2 dV = \frac{1}{2\mu_0}\int_V B_m^2(\alpha)\Lambda^2(\Theta, \alpha) dV. \qquad (4)$$

$B_m(\alpha)$ is the magnetization function produced by the magnets in the air gap and $\Lambda(\Theta, \alpha)$ is the relative air–gap permeance function. The square of the flux density produced by the magnets and the square of the relative air–gap permeance function are discomposed with Fourier's transformation

$$\Lambda^2(\theta, \alpha) = \sum_{i=0}^{\infty} a_{i \cdot N_s} \cos(i \cdot N_s(\theta + \alpha)) \qquad (5)$$

$$B_m^2(\alpha) = \sum_{j=0}^{\infty} b_{j \cdot N_p} \cos(j \cdot N_p \alpha) \qquad (6)$$

where $a_{i \cdot N_s}$ and $b_{j \cdot N_p}$ are the Fourier coefficients of the square of the relative air–gap permeance function and the magnetization function, respectively. To simplify the expression for the

[1]LCM represents the least common multiple of two natural numbers.

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calculation of the cogging torque, it is considered that the relative air–gap permeance function does not change with the radius [1]. Thus, the cogging torque can be written as

$$T_{\mathrm{cog}}(\theta) = \frac{l_m \pi}{4\mu_0}\left(R_s^2 - R_r^2\right) \cdot \sum_{n=0}^{\infty}(n \cdot N_{CT} \cdot a_{n \cdot N_{\mathrm{CT}}} \cdot b_{n \cdot N_{\mathrm{CT}}} \cdot \sin(n \cdot N_{\mathrm{CT}} \cdot \theta)) \quad (7)$$

where l_m is the machine length, R_s the stator radius, R_r the rotor radius and N_{CT} the least common multiple (LCM) of the number of stator slots and rotor poles. It is sufficient to optimize the permeance function or the magnetization function for the most significant harmonics of the cogging torque. For example, the optimization of the studied machine with notches in the tooth head can be performed in theory by minimizing the 72nd component of the square of the relative air–gap permeance function.

With the assumption that Λ is a square function

$$\begin{aligned} \Lambda &= 1, \quad \text{below stator teeth} \\ \Lambda &= 0, \quad \text{below notches and slots} \end{aligned} \quad (8)$$

the relative air–gap permeance function reads

$$\Lambda^2(\theta, \alpha) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos(nN_s(\theta + \alpha)) \quad (9)$$

$$a_0 = \frac{N_s}{\pi} 3w_t \quad (10)$$

$$a_{nN_s} = \frac{2}{\pi n}\left(\sin\left(nN_s \frac{w_t}{2}\right) - \sin\left(nN_s\left(\frac{w_t}{2} + w_n\right)\right) + \sin\left(nN_s\left(\frac{3w_t}{2} + w_n\right)\right)\right), \quad n \geq 1 \quad (11)$$

where w_n is the angular width of the notches, w_s the angular width of the slots and w_t the angular distance between the end of a slot or notch and the beginning of the next one. Depending on the width of the slot Fig. 1 shows the resulting behavior of the 72nd order of (11). The minimum, in fact "0," is reached for $w_n = w_s$. This corresponds to the result [6] obtained.

Fig. 2 shows a zoom of the lamination with $w_n = w_s$. It is obvious, that this notch shape will result in very poor load-torque performance. To proof the cogging-torque behavior a static two-dimensional finite-element model (2-D FEM) is applied comparing the notched with the basic machine model (without notches). The cogging-torque behaviors are depicted in Fig. 3. Contrary to the results [6] achieved the peak-to-peak values of the cogging torque rise to about three times the value as for the basic machine. Thus, the analytical model applied in [6] does not suite here and a numerical optimization is applied instead.

III. Numerical Model

The analytical model introduced in the previous section does not give suitable results. Therefore, a step-by-step numerical optimization of the notch shape is used. Parameters are the width w_n and height h_n of the rectangular notch. h_n is varied in the range of $0.364 < h_n/h_s < 0.727$, with h_s being the height of the slot opening. w_n is regarded in the interval of $0.0 < w_n/w_s < 1.0$ as above. Fig. 4 depicts the maximal peak-to-peak value resulting from the FEM simulation. The cogging torque does not depend strongly on the height of the notch in the lower range of w_n/w_s. In this region, the local optimum is located as well. The resulting load torque will be the highest for the smallest notch. Therefore, the smallest height of the notch is chosen: $h_n = 0.364h_s$. A smaller notch height will be hard to manufacture in large series.

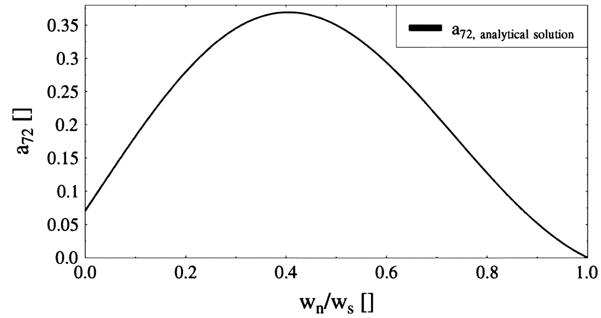

Fig. 1. Optimal notch width estimated with 72nd order using the rectangular relative permeance function.

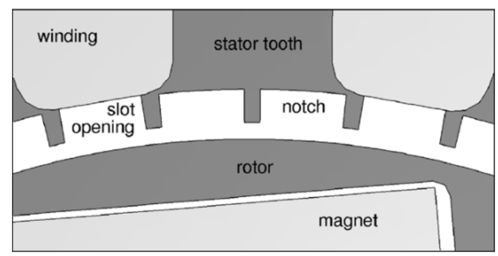

Fig. 2. Stator tooth with notches shaped like the slot opening.

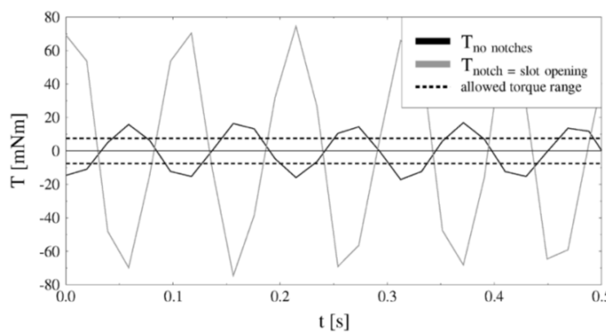

Fig. 3. Cogging torque of machine without notches and machine with notches shaped like the slot opening.

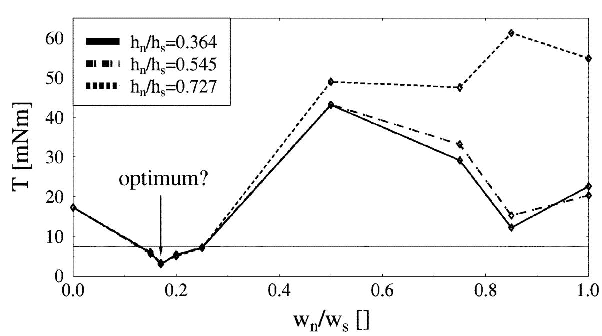

Fig. 4. Optimization of the height of the notch.

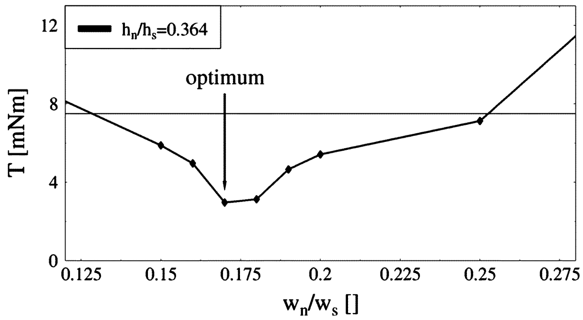

Fig. 5. Optimization of the width of the notch.

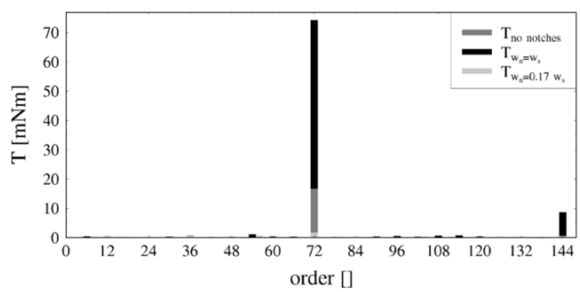

Fig. 6. Harmonics of the cogging torque: without notches, with $w_n/w_s = 1.0$, and with $w_n/w_s = 0.17$.

TABLE I
TORQUE VALUES FOR 72nd ORDER COMPUTED WITH FE MODELS

model	$T_{peak-to-peak}$	comparison
no notches	16.63 mNm	100.0 %
large notches	74.25 mNm	446.5 %
optimal notches	1.69 mNm	10.2 %

For the minimization of load-torque loss the smallest width w_n should be chosen. Therefore, a finer exploration in the region marked with "optimum?" in Fig. 4 is performed and shown in Fig. 5. The optimal width of the notch is found to be $w_n = 0.17w_s$. Obviously this does not correspond to the analytical solution found and disproved in the previous section. Fig. 6 shows the orders of the cogging torque for the three models: basic machine, with $w_n/w_s = 1.0$, and with $w_n/w_s = 0.17$. The main detected order is as expected the $72\text{nd} \cong \text{LCM}(18, 8)$. This order is 4.46 times as high for the large notches compared to the case without notches where the optimal notch shape found in this section reduces the 72nd order to the 0.1 part as Table I. shows.

The optimal notch shape is depicted in Fig. 7. The notches are by far smaller and will not effect the load torque significantly which the large notches from the previous section do.

IV. COMBINED MODEL

Pure numerical approaches for geometry optimization show very good results but are rather time-consuming. There are other approaches like genetic algorithms varying selected geometric parameters [8]. For a first try a fast method is favorable. Therefore, a combined numerical and analytical method is developed, here.

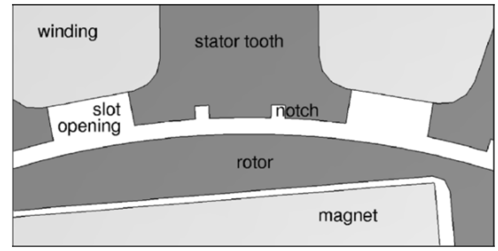

Fig. 7. Stator tooth with optimal notch shape.

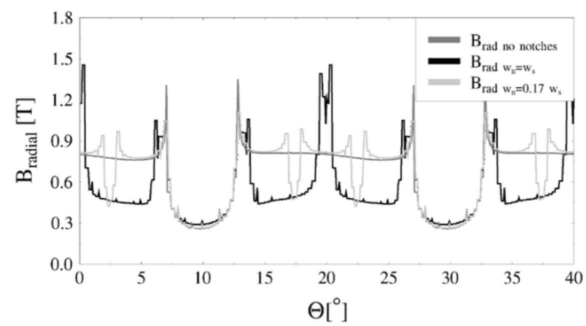

Fig. 8. Permeance function for the three models, two slot pitches.

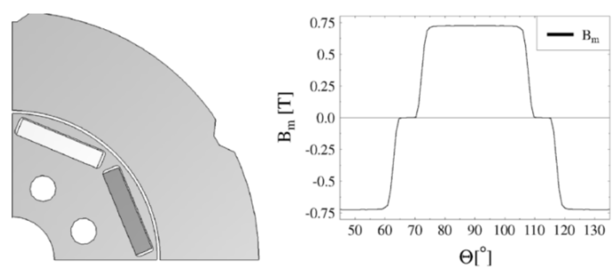

Fig. 9. FE model and magnetization function of rotor magnets.

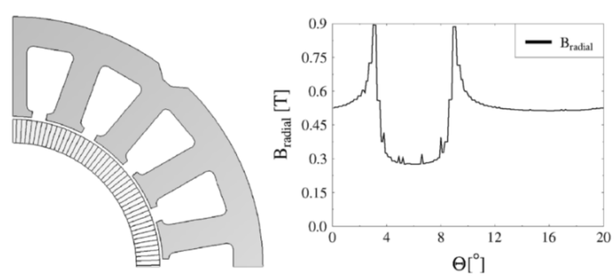

Fig. 10. FE model and radial flux-density function of the stator.

The relative air–gap permeance function, which is related to the radial component of the flux density in the air gap, derived from the FEM models is not a rectangular function at all, as assumed in the analytical model before (Fig. 8). The notches and the slot opening can be detected very easily. At those positions B_{rad} drops to values $B_{\text{rad}} < 0.5\text{T}$. The average value in the iron is about $\bar{B}_{\text{rad}} \approx 0.8\text{T}$. At the edges of the notches and the slot openings very high peaks arise. Especially in the case of the large notches the very narrow iron bridges are saturated with up to $B_{\text{rad}} > 1.4\text{T}$.

The combined model exists of two FE models using 2-D static simulation [9] and (9). From the first model depicted in Fig. 9 the magnetization function of the permanent magnets is extracted from the air–gap flux-density. In order to obtain a

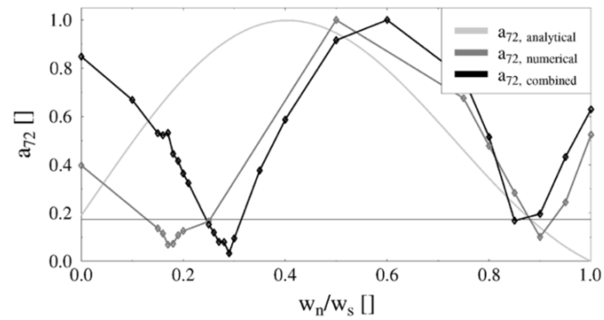

Fig. 11. Comparison of estimated relative cogging torque.

good approximation of the relative air–gap permeance function, a second model is applied. The "permeance" model consists of the stator with slots and e.g., notches and a rotor which is uniformly magnetized in radial direction. Fig. 10 shows the model and the air–gap flux-density for one stator tooth which is related to the permeance function. The simulations are very fast (≈5 min each). With the two functions extracted and (9) the main order of the cogging torque 72nd is calculated for a variation of different notch widths. The two functions are regarded in the frequency domain [10]. The nth component of the cogging torque depends only on the nth component of the square of the magnetization function and the nth component of the square of the relative air–gap permeance function. In this way, the optimization process shrinks to the following steps:

1) calculation of one position of the rotor field model;
2) extraction of the air–gap flux-density (magnetization function of the rotor magnets);
3) fast Fourier transform (FFT) of the square of the magnet flux density (b_i) calculated in step 2;
4) calculation of one position of the stator model (with the radial magnetized rotor);
5) extraction of the air–gap flux-density (relative air–gap permeance function);
6) FFT of the square of the relative air–gap permeance function (a_j);
7) repetition of steps 4–6 for different geometries (notch shapes);
8) comparison of the fundamental component of $a_n \cdot b_n$ for the different geometries.

The comparison of the relative cogging torque estimated for different notch widths regarded here are depicted in Fig. 11. With the assumption that the numerical optimization gives the best result, the analytical and the combined approach are compared to it. As mentioned, the analytical approach does not sufficiently suite the numerical results. The local minimum at $w_n = 0.17w_s$ is not detected. With the new combined approach the cogging torque behavior depending on the width of the notch is represented very good. The minimum at $w_n = 0.87w_s$ is found and there is a minimum above $w_n = 0.17w_s$ at $w_n = 0.28w_s$. With the new approach a first try of optimization can be performed in a very short time. The region of interest (minimal cogging torque) is detected and the notch width does not have to be varied in the complete range for the numerical simulation process any longer.

V. Conclusion

In this work, the geometric optimization of a PMSM is performed with 2-D FEM simulations to achieve a reduction of the cogging torque. The approach is the insertion of notches in the stator teeth. It is deduced that the optimal number of notches is two. The optimal shape of the notches are found to be $h_n = 0.364h_s$ high and $w_n = 0.17w_s$ wide. The peak-to-peak value of the cogging torque of the original machine is reduced by 90%.

As the reduction of the cogging torque is obtained mainly by the reduction of its 72nd harmonic (LCM of N_s and N_p), it is possible to speed up the optimization process when optimizing this component. The exclusive use of analytical methods has resulted to be not sufficiently accurate, here. As a result, a faster optimization process that includes the use of FEM is proposed. In this process, two models are simulated: a model with a massive stator, which allows the deduction of the magnet's flux-density function, and a model with a radial magnetized rotor, which gives a good estimation of the relative air–gap permeance function. The combination of the results from both models gives a good estimation of the behavior of the cogging torque. The advantage is that only one rotor position per model has to be calculated, saving a lot of time.

The approach is tested with the studied machine. The optimization function is well approximated. Therefore, it can be used to have a quick view of the optimization function and detect the region of minimal cogging torque. Then the exact value of the optimum can be found using pure FEM simulations. The combination of both methods allow for the optimization to be fast and accurate here saving 98% of time.

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