Monopedal Hopping with a Leg and a Tail

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1 Introduction

We explore[1] how a tail can be used in a planar hopping robot to inject energy in the leg spring. We present our control strategy as anchoring [1] to appropriately selected "compartments"[2] of the full dynamics a suite of 1-DOF templates which we couple together in the style of [3]. We present initial empirical evidence and conjecture (without proof) that the limit behavior of the coupled system is conjugate to a cross product of the constituent templates' limit behaviors.

2 Model and Compartmentalized Dynamics

Our model (the "tailed monoped" or TM) nominally has 6 DOF (unconstrained) as shown in Figure 1. It is constrained to 4 DOF in stance by the assumption of a pinned toe, and dynamically reduced to 4 DOF in flight in consequence of an assumed massless toe which renders the (q_3, q_4) dynamics degenerate. We note that the COM of the body segment, and the axes of rotation of the legs and tail are all coincident at the "hip" by design. Additionally we assume that the tail mass is light, i.e. $m_2 \ll m_1$. Our task, hopping, is a cyclic behavior with alternating 4-DOF hybrid modes of stance and flight.

Note that the flight system has a non-integrable angular momentum conservation $b^T \begin{bmatrix} \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = 0$, where

$$b := \begin{bmatrix} -\frac{1}{I_b} \\ \frac{1}{I_b} + \frac{m_1+m_2}{m_1 m_2 l_2^2} \end{bmatrix}, \quad B := \frac{1}{\|b\|} \begin{bmatrix} b_\perp^T \\ b^T \end{bmatrix}, \quad (1)$$

are *constant* because the COM is at the hip [4]. Define the task coordinates (II.G of [5]) $(s,a) \in (\mathbb{R}_+ \times S^1) \times T^2$,

$$s := \begin{bmatrix} q_3 \\ q_4 \end{bmatrix}, \quad a := \begin{bmatrix} \phi \\ \theta \end{bmatrix} := B \cdot \begin{bmatrix} q_5 \\ q_6 \end{bmatrix}, \quad (2)$$

where s and a are the "SLIP" and "attitude" (comprised of "net pitch," ϕ, and "relative tail angle," θ) DOF's respectively.

2.1 Attitude Dynamics $(a, \dot{a})$

It can be shown that $(a, \dot{a})$ is invariant to the reset maps ($\ddot{a}$ is defined continuously across transitions). Standard derivations (e.g. [5]) yield (in flight and stance modes, respectively),

$$\ddot{a}\big|_{\text{flight}} = \begin{bmatrix} 0 \\ \tau_{\text{tail}} \end{bmatrix}, \quad \ddot{\phi}\big|_{\text{stance}} \approx \frac{-1}{\sqrt{I_b^2 + (I_t + I_b)^2}} \tau_{\text{hip}} \quad (3)$$

Figure 1: The 6-DOF model of the tailed monoped (**left**) with actuators (red), system parameters (blue) and degrees of freedom (black), together with snapshots of the robot platform in stance (**middle**) and flight (**right**) modes, attached to a planarizing boom.

where $I_t := m_2 l_2^2$, and the approximation is $\frac{m_2}{m_1+m_2} \approx 0$ by the "light tail mass" assumption to eliminate coupling from the translational (SLIP) dynamics. We only seek to control one of the two DOF in each mode: ϕ in stance and θ in flight.

2.2 Translational (SLIP) Dynamics $(s, \dot{s})$

In flight, we use τ_{hip} to arbitrarily set the leg touchdown angle as in [3]. In stance, again following [3], we will reserve τ_{hip} to control the attitude; thus, our actuation of the SLIP subsystem relies on the direct coupling from the tail torque,

$$\ddot{s} = \begin{bmatrix} \frac{-\cos\xi}{l_2 m_1} \\ \frac{\sin\xi}{l_1 l_2 m_1} \end{bmatrix} \tau_{\text{tail}} + n \implies \ddot{s}_1\big|_{\xi=0} = -\frac{1}{l_2 m_1} \tau_{\text{tail}} + n, \quad (4)$$

where cross-compartment coupling through the relative tail angle, $\xi := q_4 - q_5 - q_6$, directs the tail affordance, τ_{tail}, and the "noise" term, n, (inertial and coriolis coupling forces) will be neglected by the controller (and thus "rejected" by the coupled closed loop system rather than explicitly cancelled).

3 Control Strategy: Anchor Three Templates

Raibert's hopper [3] pioneered a "parallel composition" of two translational DOF with one rotational DOF, by artfully neglecting their highly coupled dynamics. Notwithstanding independent correctness proofs for each feedback-compensated 1-DOF subsystem in isolation [6, 7], we know of no formal results yielding conditions sufficient for the forward limit set of the highly coupled 3-DOF closed loop dynamics to coincide with the cross product of the three isolated limit sets—a result empirically observed in [3]. In this paper, we pursue the same idea, but instantiate the Raibert vertical hopping template by coupling a 1-DOF leg-spring excitation controller (physically acting through the tail). In flight, the tail actuator grants us a new affordance that we only[3] use here

[2]We use this term to connote the long tradition of modeling dynamics in terms of subsystems (here, distinct subsets of the physical DOF's of the anchor) [2] that exchange a resource flow (here, energy).

[3]We omit a detailed design discussion here, but a revolute tail avoids the morphological specialization of a dedicated prismatic actuator and can be

Mode	Templates		
	Planar hopping (2-DOF)	Body pitch (1-DOF)	Tail (1-DOF)
	m_1, l_1, s_1, s_2, (τ_{shank})	I_b, ϕ, τ_{hip}	I_b, θ, l_2, m_2, τ_{tail}
Flight	**(RH)** Toe placement	$\ddot{\phi} = 0$	**(IR)** $\ddot{\theta} = -k_\theta^T \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$
Stance	**(RH)** Timed τ_{shank} thrust	**(RH)** $\ddot{\phi} = -k_\phi^T \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}$	**(RH)** $\ddot{s}_1 = k_{shank}\dot{s}_1 + n$
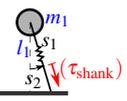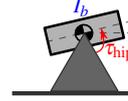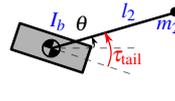
Table 1: Three different templates anchored by the tailed monoped, using controllers from Section 3 in the dynamics given in (3) and (4). The Raibert hopper (**RH**) is a composition of the first two columns, and the grey cells represent the substitution of τ_{tail} in place of τ_{shank} for leg-spring excitation. In flight, the "tail" subsystem anchors the "inertial reorientation" (**IR**) template [4].


Figure 2: A window of data from a hopping trial (abcissa is time, stance phases have a dark background). The plots show roughly that (a) our controller is attempting to regulate ϕ in stance (with large overshoot due to suboptimal gains), and (b) θ is destabilized in stance, and regulated in flight.

to regulate the added DOF, θ. The compartmentalization of the dynamics in Section 2—though not an exact decoupling—promotes this simple parallel composition of templates (Table 1) whose correctness (that the limit set commutes through the composition) we hope to prove.

The leg-spring excitation controller of Table 1 (a) assumes[4] $\xi \approx 0$, and (b) is not designed to maximize jumping height: rather we conjecture (as established empirically in [3] and formally in [6]) that steady-state hopping height should be monotonic with k_{shank} in some range. In implementation, we use the modified leg-spring energizing control $\tau_{tail} = k_{shank}(t_b - t)$ since the robot has no sensors for s_1, where t_b is the "bottom" time (as estimated based on [3]). Note as well that while the attitude subsystem anchors the G.A.S. "inertial reorientation" template [4] in flight, the template coordinate θ is destabilized in stance. A formal hybrid systems argument is in preparation, but empirically we find that "high enough" PD gains k_θ relative to k_{shank} suffice.

4 Preliminary Experimental Results

The experiments were performed on a new tailed biped robot platform (Figure 1), which inherits compliant 1-DOF half-circle legs from RHex [9] (in parallel for sagittal plane behaviors), and introduces a 1-DOF revolute point-mass tail [4] driven by two parallel motors. The robot weighs 2 Kg (with onboard battery) and each motor is (theoretically) capable of producing 2.85 N-m of torque. An IMU, motor encoders and current sensors comprise the sensing capabilities, and control is performed on a STM32F3 microcontroller. Figure 2 shows some data from preliminary hopping experiments (with a fixed touchdown angle in place of the full horizontal speed control) performed on a planarizing boom.

5 Discussion

Tails are known to be used in lizards for inertial control in free-fall [10] or turning [11], and demonstrably increase ma-

neuverability in terrestrial robots' aerial phase of locomotion as well [4, 12]. Here, the tail functions similarly in flight, but acquires a new function in stance as an energetic "pump," moving joules from the battery through the attitude compartment and into the leg spring—and, thereby, into the body compartment through the inertial coupling. We are not aware of prior locomotion work wherein a tail is used to help power the stance phase, and we wish to both extend this novel affordance to a wider range of walking and running behaviors as well as further explore the diverse roles tails may play in legged locomotion. In so doing, it will be important to develop a more principled approach to identifying the appropriate templates and associated "compartmental" coordinate systems in the anchoring morphology.

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reprised for other uses such as static standing, reorienting the body in free fall [4], directing reaction forces through ground contact for leaping [8], etc.

[4] We make the reasonable assumption that the "inertial reorientation" and "body pitch" controllers attract to the goal state $a = 0$, which corresponds to $\xi = q_4 \approx 0$ for small sweep angles on touchdown.