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Brief paper

Robust tracking control of an array of nanoparticles moving on a substrate[☆]

Guoqiang Hu[a,1], Warren Dixon[b], Han Ding[c]

[a] EXQUISITUS, Centre for E-City, School of Electrical and Electronics Engineering, Nanyang Technological University, 639798, Singapore

[b] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611, USA

[c] State Key Lab of Digital Manufacturing Equipment and Technology of China, Huazhong University of Science and Technology, Wuhan 430074, China

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ABSTRACT

Control of nanosystems with frictional dynamics using feedback control methods is important to a wide range of applications of nanotribology. This paper studies the tracking control problem of an array of nanoparticles moving on a substrate with friction between the substrate and the particles. The focus of this study is on control design and stability analysis. The major challenges in this problem include nonlinearities and uncertainties in the frictional dynamics and limited availability of measurable states in nanosystems. The particle–substrate interaction is considered to be unknown, and the unknown effect of unmodeled particle dynamics on the dynamics of the center of mass of the array is also considered. A nonlinear identifier is first developed to identify these unmodeled dynamics. A feedback controller is then developed based on the identifier to control the center of mass of the particles to track a desired trajectory. Boundedness of the closed-loop states and semiglobal asymptotic stability of the tracking error are proven using Lyapunov theory for the case of linear inter-particle interactions. An example with more general Morse-type inter-particle interactions is included to provide some level of confidence that the results are general but not assuredness that they are. Numerical simulation results are provided to demonstrate the performance of the developed identification and control law.

1. Introduction

Modeling and control of nanosystems in the presence of friction is an interesting and challenging research topic in nanotechnology and nanotribology (Carpick, 2006; Park et al., 2005; Park, Ogletree, Thiel, & Salmeron, 2006; Socoliuc et al., 2006; Urbakh, Klafter, Gourdon, & Israelachvili, 2004). The advances on altering and controlling nanoscale friction will lead to more reliable nanoelectromechanical devices (Carpick, 2006). The ability to compensate for and control friction would also have interesting technical applications in the motion of nanoobjects in patterned semiconductor substrates (Park et al., 2006). In addition, the study of nanoscale friction has a technological impact in reducing energy loss in machines (Guo, Qu, Braiman, Zhang, & Barhen, 2008). Despite the broad potential applications and impact, control of nanoscale frictional dynamics is challenging due to the inherent model complexities including nonlinearities and uncertainties (Urbakh et al., 2004). This paper aims at controlling frictional dynamics from the perspective of feedback control design and analysis based on the existing nanotribology research results. The studies on control design and stability analysis provide tools and insights to study nanotribology. To investigate the problem of control and manipulation of frictional properties during sliding, the work in this paper leverages the method development in controlling an array of particles moving on a substrate, as a typical nanosystem (Braiman, Barhen, & Protopopescu, 2003; Guerra, Vanossi, & Urbakh, 2008; Guo & Qu, 2008; Guo et al., 2008). Specifically, this paper studies the tracking control problem of an array of nanoparticles moving on a substrate with friction between the substrate and the particles.

The problem of controlling the array of particles in the presence of friction was introduced by Braiman et al. (in Braiman et al. (2003)) based on the Frenkel–Kontorova (FK) model (Reiter, Demirel, & Granick, 1994). In Braiman et al. (2003) and Protopopescu, Barhen, Amselem, and Dahan (2006), an innovative global controller based on the concepts of non-Lipschitzian dynamics and terminal attractor was developed to regulate the velocity of the center of mass (CM) of the array to a desired value. However, it could not eliminate the persistent fluctuations of the controlled variable around the desired value (Guo, Qu, & Zhang, 2006). In Guo and Qu (2008), a Lyapunov-based nonlinear control approach is used to control the CM of the array of

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E-mail addresses: gqhu@ntu.edu.sg (G. Hu), wdixon@ufl.edu (W. Dixon), hding@sjtu.edu.cn (H. Ding).

[1] Tel.: +65 6790 4362; fax: +65 6792 0415.

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particles. Specifically, the authors of Guo and Qu (2008) studied the stability of equilibrium points of the array in the presence of linear and nonlinear inter-particle coupling, and they further designed global control laws to regulate the velocity of the array CM to any given constant desired velocity with uniformly ultimately bounded (UUB) error. In addition to this controller, the authors of Guo and Qu (2008) introduced a discontinuous controller that enabled asymptotic stability.

This paper will address the following question: *can a continuous controller be designed to achieve asymptotic position tracking control for the array of nanoparticles in the presence of unknown frictional dynamics?* In this paper, a nonlinear dynamic identifier is developed to identify the effect of the unmodeled particle dynamics. This identification method is built upon the prior work in Makkar, Hu, Sawyer, and Dixon (2007) and Xian, Dawson, de Queiroz, and Chen (2004). A nonlinear feedback controller is then developed based on the identifier to control the array of particles via a Lyapunov-based synthesis method. The developed controller yields semiglobal asymptotic tracking results for the CM. A contribution of this paper is that a *continuous* controller is developed to achieve semiglobal asymptotic position tracking (as opposed to other methods in literature that yield UUB) of the CM of an array of particles in the presence of uncertain frictional dynamics. A rigorous stability analysis is provided for the semiglobal asymptotic tracking results using a Lyapunov-based analysis method for the case of linear inter-particle interactions.

The rest of this paper is organized as follows. The equations of motion of an array of particles are presented in Section 2. The tracking control problem of the array of particles is formulated in Section 3. An identifier for unmodeled dynamics and a feedback control law are developed in Section 4. The stability analysis of the developed controller is conducted in Section 5 by using Lyapunov theory. Numerical simulation results are provided in Section 6 to demonstrate the performance of the developed control method.

2. Dynamic models of an array of nanoparticles moving on a substrate

A schematic of a one-dimensional array of N identical nanoparticles moving on a substrate is depicted in Fig. 1. The driving dynamics of this array of particles are given by a set of coupled nonlinear equations (Braiman et al., 2003; Guo & Qu, 2008)

$$m\ddot{x}_i + \nu\dot{x}_i = -\frac{\partial P_s}{\partial x_i} - \frac{\partial V_p}{\partial x_i} + u_i + \eta, \quad i = 1, \dots, N. \tag{1}$$

In (1) and Fig. 1, $x_i(t) \in \mathbb{R}$ is the coordinate of the i-th particle, $m \in \mathbb{R}^+$ is its mass, $\nu \in \mathbb{R}^+$ is the linear friction coefficient representing the single particle energy exchange with the substrate, $u_i(t) \in \mathbb{R}$ is the external force applied to the i-th particle, and $\eta(t) \in \mathbb{R}$ denotes additive noise. The particles interact with each other via a pair-wise inter-particle potential $V_p\left(x_i - x_j\right) \in \mathbb{R}$. The array is subject to a potential $P_s(x_i) \in \mathbb{R}$ applied by the substrate. If $P_s(x_i) \in \mathbb{R}$ is periodic, then $P_s(x_{ref} + a) = P_s(x_{ref}) \in \mathbb{R}$ where $x_{ref}(t) \in \mathbb{R}$ denotes a position on the substrate with respect to the origin O_{ref}, and the constant $a \in \mathbb{R}^+$ denotes the period of the substrate potential. The misfit length between the substrate potential period a and the distance $d_e \in \mathbb{R}^+$ between the equilibrium points of two neighboring particles is denoted by $b \triangleq a - d_e \in \mathbb{R}$.

It is challenging to control an array of nanoparticles due to the following two technological restrictions. First, it is difficult to apply separate control input forces to the particles at the nanoscale, so the same external force is applied to all the particles, i.e., $u_i = u, i = 1, \dots, N$. Second, it is difficult to measure the positions of all the particles, so the measurements of the CM of the array will be used. To facilitate the subsequent control development and stability analysis, two assumptions are made for the general model in (1), see Braiman et al. (2003) and Guo and Qu (2008).

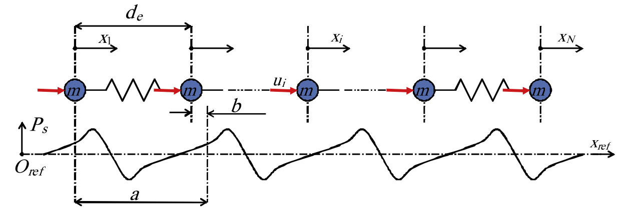

Fig. 1. Schematic of an array of N nanoparticles moving on a substrate.

Assumption 1. The noise $\eta(t)$ is negligible, i.e., $\eta = 0$.

Assumption 2. The measurements of the position and velocity of the CM of the array are available.

Let $f_{si}(x_i) = \frac{\partial P_s}{\partial x_i} \in \mathbb{R}$ denote the force applied to the i-th particle, which is induced by the periodic substrate potential. Let $F_i(x_{i-1}, x_i, x_{i+1}) = -\frac{\partial V_p}{\partial x_i} = F(x_{i+1} - x_i) - F(x_i - x_{i-1})$ where $F(x_{i+1} - x_i) \in \mathbb{R}$ represents the inter-particle force exerted by the $(i+1)$-th particle to the i-th particle and $-F(x_i - x_{i-1}) \in \mathbb{R}$ represents the inter-particle force exerted by the $(i-1)$-th particle to the i-th particle.

By normalization from (1) and continuing using the symbols in (1), we get the following dimensionless model:

$$\ddot{x}_i + \nu\dot{x}_i + f_{si}(x_i) = u + F(x_{i+1} - x_i) - F(x_i - x_{i-1}), \tag{2}$$

where $i \in \{1, \dots, N\}$. In (2), two auxiliary states, $x_0 \triangleq x_1 \in \mathbb{R}$ and $x_{N+1} \triangleq x_N \in \mathbb{R}$, are defined to avoid writing separate equations for the first and last particles. Typically, the inter-particle force is modeled as either a linear interaction (Braiman et al., 2003) (i.e., $F(x) = c_{ip}x$ where $c_{ip} \in \mathbb{R}^+$ is the stiffness coefficient of the inter-particle force), or a Morse interaction (Braiman et al., 2003; Urbakh et al., 2004) (i.e., $F(x) = \frac{c_{ip}}{\beta}\left(e^{-\beta x} - e^{-2\beta x}\right)$ where $\beta \in \mathbb{R}^+$ is a stiffness coefficient). In (2), $F(x)$ is considered as a general unknown function satisfying the following property:

Property 1. $|F(x)| \le g(|x|)$ *where* $g(\cdot) \in \mathbb{R}$ *is a non-decreasing real positive function.*

Remark 1. Property 1 is satisfied for both linear and Morse inter-particle interactions because $|c_{ip}x| \le c_{ip}|x|$ and $\left|\frac{c_{ip}}{\beta}\left(e^{-\beta x} - e^{-2\beta x}\right)\right| \le \frac{2c_{ip}}{\beta}e^{2\beta|x|}$.

The unknown function $f_{si}(x_i)$ is assumed to satisfy the following assumption.

Assumption 3. The substrate force function $f_{si}(\cdot)$ and its first and second derivatives exist and are bounded, i.e.,

$$|f_{si}| \le U_{f_s}, \qquad \left|f'_{si}\right| \le U_{f'_s}, \qquad \left|f''_{si}\right| \le U_{f''_s}, \tag{3}$$

where $U_{f_s}, U_{f'_s}, U_{f''_s} \in \mathbb{R}$ are positive constant scalars.

Remark 2. In the existing literature (Braiman et al., 2003; Guo & Qu, 2008), the substrate potential is assumed to be of a harmonic form (i.e., $f_{si}(x) = \sin(x)$), which satisfies Assumption 3.

3. Control problem formulation

The control objective is to enable the CM of the array of particles, denoted by $x_{cm} \triangleq \frac{1}{N}\sum_{i=1}^{N} x_i \in \mathbb{R}$, to track a desired trajectory $x_d(t) \in \mathbb{R}$. The motion of the array is controlled by applying an external force on the particles.

For the array of N particles, taking the sum of both sides of (2) gives the equation of motion of the CM

$$\ddot{x}_{cm} + \nu \dot{x}_{cm} + \frac{1}{N}\sum_{i=1}^{N} f_{si}(x_i) = u. \tag{4}$$

The subsequent control development relies only on the states $x_{cm}(t)$ and $\dot{x}_{cm}(t)$. The states of individual particles are not required. This strategy follows the precedence in the literature and it is not known how such a measurement would be done experimentally. This paper focuses on theoretical control design and stability analysis, so the experimental evaluations of the measurement methods will not be addressed.

To facilitate the subsequent development, a coordinate transformation is defined as

$$z_i \triangleq x_i - x_{cm}, \quad i = 1, \ldots, N. \tag{5}$$

Based on the definition of $x_{cm}(t)$,

$$\sum_{i=1}^{N} z_i = 0, \qquad \sum_{i=1}^{N} \dot{z}_i = 0. \tag{6}$$

The CM equation of motion in (4) can be rewritten as

$$\ddot{x}_{cm} + \nu \dot{x}_{cm} + f(x_{cm}, z) = u, \tag{7}$$

where $z \triangleq [z_1, \ldots, z_N]^T \in \mathbb{R}^N$ and $f(x_{cm}, z) \triangleq \frac{1}{N}\sum_{j=1}^{N} f_{sj}(x_{cm} + z_j) \in \mathbb{R}$. Applying the coordinate transformation (5) to the dynamic model in (2) gives

$$\begin{aligned} &\ddot{z}_i + \ddot{x}_{cm} + \nu \dot{z}_i + \nu \dot{x}_{cm} + f_{si}(x_{cm} + z_i) \\ &\quad = u + F(z_{i+1} - z_i) - F(z_i - z_{i-1}), \quad i = 1, \ldots, N. \end{aligned} \tag{8}$$

Subtracting (4) from (8) results in

$$\ddot{z}_i + \nu \dot{z}_i + f_{zi}(x_{cm}, z) = F(z_{i+1} - z_i) - F(z_i - z_{i-1}), \tag{9}$$

where $f_{zi}(x_{cm}, z) \in \mathbb{R}$ is given by

$$f_{zi}(x_{cm}, z) = f_{si}(x_{cm} + z_i) - \frac{1}{N}\sum_{j=1}^{N} f_{sj}(x_{cm} + z_j). \tag{10}$$

The tracking control problem is defined as follows.

Output tracking control: Design a control input $u(t)$ such that the CM tracks a desired trajectory $x_d(t)$ in the sense that

$$x_{cm}(t) - x_d(t) \to 0 \quad \text{as } t \to \infty,$$

and the states $z_i(t)$, $i = 1, \ldots, N$ are bounded (i.e., $z_i(t) \in \mathcal{L}_\infty$).

4. Output tracking control development

In this section, a nonlinear identifier is developed to compensate for the unknown dynamic term $f(x_{cm}, z)$ in (7), and a robust tracking controller is developed based on this identifier. The control development is based on the following two assumptions.

Assumption 4. The derivatives $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ of the desired trajectory exist and are bounded as $|\dot{x}_d(t)| \le U_{\dot{x}_d}$ and $|\ddot{x}_d(t)| \le U_{\ddot{x}_d}$, where $U_{\dot{x}_d}$ and $U_{\ddot{x}_d}$ are two constant positive scalars.

The position tracking error for the CM, denoted by $e_1(t) \in \mathbb{R}$, is defined as

$$e_1 \triangleq x_d - x_{cm}. \tag{11}$$

To facilitate the subsequent design and analysis, the filtered tracking errors $e_2(t), r(t) \in \mathbb{R}$ are defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \tag{12}$$

$$r \triangleq \dot{e}_2 + \alpha_2 e_2, \tag{13}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}$ denote positive constant scalars. Using (7), (11) and (12), the filtered tracking error in (13) can be expanded as

$$r = \ddot{x}_d(t) + \nu \dot{x}_{cm} + f(x_{cm}, z) - u(t) + \alpha_1 \dot{e}_1 + \alpha_2 e_2. \tag{14}$$

The filtered tracking error $r(t)$ is not measurable because (14) depends on $f(x_{cm}, z)$. In addition, based on (11)–(13), $r(t)$ can also be shown to be not measurable because it depends on $\dot{e}_2(t)$, and therefore, depends on the acceleration term $\ddot{x}_{cm}(t)$, which is not measurable.

Based on (14), the control input $u(t)$ is designed as

$$u = \ddot{x}_d(t) + \nu \dot{x}_{cm} + \alpha_1 \dot{e}_1 + \alpha_2 e_2 + \hat{f}(t), \tag{15}$$

where $\hat{f}(t) \in \mathbb{R}$ denotes a subsequently designed identification term. Substituting (15) into (14) gives

$$r = f(x_{cm}, z) - \hat{f}(t). \tag{16}$$

It can be concluded from (16) that if $r(t) \to 0$, then $\hat{f}(t)$ will identify the unmodeled particle dynamics. To facilitate the design of $\hat{f}(t)$ to ensure that $r(t) \to 0$, we differentiate (16) as

$$\dot{r} = \Psi(t) - \dot{\hat{f}}(t), \tag{17}$$

where $\Psi(t) \in \mathbb{R}$ denotes the unmeasurable auxiliary term

$$\Psi(t) \triangleq \frac{d}{dt} f(x_{cm}, z). \tag{18}$$

Based on (17) and the subsequent stability analysis, $\dot{\hat{f}}(t)$ is designed as (Makkar et al., 2007; Xian et al., 2004)

$$\dot{\hat{f}}(t) = (k_1 + 1)r + k_2 \operatorname{sgn}(e_2), \tag{19}$$

where $k_1, k_2 \in \mathbb{R}$ are positive constant scalars. Since $r(t)$ is not measurable, the following equations will be used to implement $\hat{f}(t)$ as

$$\begin{aligned} \hat{f}(t) &= (k_1 + 1)(e_2(t) - e_2(0)) \\ &\quad + \int_0^t (k_1 + 1)\alpha_2 e_2(\tau) d\tau + \upsilon(t) \end{aligned} \tag{20}$$

$$\dot{\upsilon}(t) = k_2 \operatorname{sgn}(e_2(t)). \tag{21}$$

Remark 3. The standard signum function $\operatorname{sgn}(\cdot)$ is discontinuous. However, the generalized solution to (21) is continuous. The control input synthesized based on $\upsilon(t)$ is therefore a continuous signal.

After substituting (19) into (17), the following closed-loop error system can be obtained:

$$\dot{r} = -(k_1 + 1)r - k_2 \operatorname{sgn}(e_2) + \Psi(t). \tag{22}$$

To facilitate the subsequent analysis, another unmeasurable auxiliary term $\Psi_c(t) \in \mathbb{R}$ is defined as

$$\begin{aligned} \Psi_c(t) &\triangleq \frac{d}{dt} f(x_d, z) = \frac{d}{dt}\frac{1}{N}\sum_{j=1}^{N} f_{sj}(z_j + x_d) \\ &= \frac{1}{N}\sum_{j=1}^{N} (\dot{z}_j + \dot{x}_d) f'_{sj}(z_j + x_d). \end{aligned} \tag{23}$$

The time derivative of $\Psi_c(t)$ is

$$\begin{aligned} \dot{\Psi}_c(t) &= \frac{1}{N}\sum_{j=1}^{N} (\dot{z}_j + \dot{x}_d)^2 f''_{sj}(z_j + x_d) \\ &\quad + \frac{1}{N}\sum_{j=1}^{N} \left(\ddot{z}_j + \ddot{x}_d\right) f'_{sj}(z_j + x_d). \end{aligned} \tag{24}$$

Based on (23) and (24), Assumption 4, and Theorem 1, the following inequalities can be obtained:

$$|\Psi_c(t)| \leq \left(U_{\dot{z}} + U_{\dot{x}_d}\right) U_{f_s'} \leq U_{\Psi_c} \tag{25}$$

$$\left|\dot{\Psi}_c(t)\right| \leq \left(U_{\dot{z}} + U_{\dot{x}_d}\right)^2 U_{f_s''} + (U_{\ddot{z}} + U_{\ddot{x}_d})U_{f_s'} \leq U_{\dot{\Psi}_c},$$

where $U_{\Psi_c}, U_{\dot{\Psi}_c} \in \mathbb{R}$ are known positive constant scalars.

After subtracting $\Psi_c(t)$ from $\Psi(t)$ and then adding $\Psi_c(t)$ to the right-hand side of (22), the closed-loop error system in (22) can be expressed as

$$\dot{r} = -(k_1+1)r - k_2 \mathrm{sgn}(e_2) + \tilde{\Psi}(t) + \Psi_c(t), \tag{26}$$

where the unmeasurable auxiliary term $\tilde{\Psi}(t) \in \mathbb{R}$ is defined as

$$\tilde{\Psi}(t) \triangleq \Psi(t) - \Psi_c(t). \tag{27}$$

5. Main results and stability analysis

In this section, the closed-loop stability of the system determined by (9) and (26) will be analyzed. Boundedness of $z(t)$ and $\dot{z}(t)$ will first be discussed. It will be proven in Theorem 1 that $z(t)$ and $\dot{z}(t)$ are bounded when the inter-particle interaction is linear. Boundedness of the closed-loop states and semiglobal asymptotic stability of the tracking error will be analyzed and proven in Theorem 2.

In the case of linear inter-particle interaction, (9) can be written as

$$\ddot{z} + \nu\dot{z} + f_z(x_{cm}, z) = -c_{ip}Lz, \tag{28}$$

where

$$f_z(x_{cm}, z) \triangleq [f_{z1}, f_{z2}, \ldots, f_{zN}]^T \in \mathbb{R}^N, \tag{29}$$

and $L \in \mathbb{R}^{N\times N}$ is a constant matrix given by

$$L = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & -1 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \tag{30}$$

Based on (10) and (29), and Assumption 3, it can be obtained that $\|f_z(x_{cm}, z)\| \leq 2U_f\sqrt{N}$.

Algebraic graph theory can be used to study the properties of L in (30). If we use an undirected chain graph to represent the connections between the nanoparticles of the array, then L in (30) is the Laplacian matrix of the graph (Merris, 1994). Based on the notion of algebraic connectivity (Godsil & Royle, 2001) or Eq. (17) in Olfati-Saber and Murray (2004), the following lemma can be obtained.

Lemma 1. *Under constraint (6), the term z^TLz can be lower and upper bounded as $\lambda_2\|z\|^2 \leq z^TLz \leq \lambda_N\|z\|^2$, where λ_2 and λ_N are the smallest and largest positive eigenvalues of the matrix L, respectively.*

Theorem 1. *Suppose that the inter-particle interaction is linear, then $z(t)$ and $\dot{z}(t)$ are bounded for any $x_{cm}(t)$.*

Proof. Define a Lyapunov function candidate $W(z, \dot{z})$ as

$$W = \frac{1}{2}\dot{z}^T\dot{z} + \frac{1}{2}(\nu z + \dot{z})^T(\nu z + \dot{z}) + c_{ip}z^TLz$$

where ν is a positive constant scalar. The function $W(z, \dot{z})$ is positive definite and radially unbounded and it satisfies $W(0,0) = 0$. Based on Lemma 1, $W(z, \dot{z})$ can be bounded as $\sigma_1\|Z\|^2 \leq W \leq \sigma_2\|Z\|^2$ where $Z = \left[z^T, \dot{z}^T\right]^T$, $\sigma_1 = \min\{\frac{1}{2}, \lambda_2 c_{ip}\}$, and $\sigma_2 = \max\{\frac{3}{2}, \left(\nu^2 + \lambda_N c_{ip}\right)\}$. The time derivative of $W(z, \dot{z})$ is given by

$$\begin{aligned}
\dot{W} &= \ddot{z}^T\dot{z} + (\nu\dot{z} + \ddot{z})^T(\nu z + \dot{z}) + 2c_{ip}\dot{z}^TLz \\
&= (-\nu\dot{z} - f_z - c_{ip}Lz)^T\dot{z} + (-f_z - c_{ip}Lz)^T(\nu z + \dot{z}) + 2c_{ip}\dot{z}^TLz \\
&= (-\nu\dot{z} - f_z)^T\dot{z} - f_z^T(\nu z + \dot{z}) - \nu c_{ip}z^TLz \\
&\leq -\nu\dot{z}^T\dot{z} - 2f_z^T\dot{z} - \nu f_z^Tz - \nu c_{ip}z^TLz \\
&\leq -\nu\|\dot{z}\|^2 + 4U_f\sqrt{N}\|\dot{z}\| + 2\nu U_f\sqrt{N}\|z\| - \lambda_2\nu c_{ip}\|z\|^2 \\
&\leq -\mu_1\|Z\|^2 + \mu_2\|Z\|,
\end{aligned}$$

where $\mu_1 = \nu\min\{1, \lambda_2 c_{ip}\}$ and $\mu_2 = 2U_f\sqrt{N}\sqrt{4+\nu^2}$. According to Theorem 4.18 in Khalil (2002), $z(t)$ and $\dot{z}(t)$ are bounded. □

Based on Theorem 1, there exist two positive constant scalars U_z and $U_{\dot{z}}$ so that $|z_i|$ and $|\dot{z}_i|$ are bounded by U_z and $U_{\dot{z}}$, respectively. According to (9), (10), and Theorem 1, $\left|\ddot{z}_j\right|$ can be bounded as

$$\left|\ddot{z}_j\right| \leq \nu U_{\dot{z}} + 2U_{f_s} + \left|F(z_{j+1} - z_j)\right| + \left|F(z_{j-1} - z_j)\right|.$$

Based on Property 1, $\left|F(z_i - z_j)\right|$ can be bounded by $g(2U_z)$. Therefore, the upper bound of $\left|\ddot{z}_j\right|$, denoted by $U_{\ddot{z}}$, can be determined as $U_{\ddot{z}} = \nu U_{\dot{z}} + 2U_{f_s} + 2g(2U_z)$.

Remark 4. As shown in Theorem 1, boundedness of the states $z(t)$ and $\dot{z}(t)$ is analytically proven for the linear inter-particle interactions, but not proven for the Morse type interactions. As illustrated by the example in Section 6, the tracking control objective may still be achieved under the developed control method for the Morse-type inter-particle interactions. It provides some level of confidence that the results are general but not assuredness that they are.

Lemma 2. *The auxiliary error $\tilde{\Psi}(t)$ defined in (27) can be upper bounded as*

$$\left|\tilde{\Psi}(t)\right| \leq \rho(\|w\|)\|w\|, \tag{31}$$

where $\rho(\|w\|)$ is a positive, globally invertible, nondecreasing function and $w(t)$ is defined as $w \triangleq [e_1, e_2, r]^T \in \mathbb{R}^3$.

Proof. Define a function $\Upsilon(x, \dot{x}) \in \mathbb{R}$ as

$$\Upsilon(x, \dot{x}) = \frac{1}{N}\sum_{j=1}^{N}(\dot{z}_j + \dot{x})f_{sj}'(z_j + x).$$

From (18) and (23), $\Psi(t)$ and $\Psi_c(t)$ can be written as

$$\Psi(t) = \Upsilon(x_{cm}, \dot{x}_{cm}), \qquad \Psi_c(t) = \Upsilon(x_d, \dot{x}_d).$$

Based on the mean value theorem (Edwards, 1994), there exists $0 < \epsilon < 1$ such that

$$\begin{aligned}
\tilde{\Psi}(t) &= \Psi(t) - \Psi_c(t) \\
&= \left.\frac{\partial\Upsilon(x, \dot{x})}{\partial x}\right|_{(x,\dot{x})=(q,\dot{q})}(x_{cm} - x_d) + \left.\frac{\partial\Upsilon(x, \dot{x})}{\partial\dot{x}}\right|_{(x,\dot{x})=(q,\dot{q})}(\dot{x}_{cm} - \dot{x}_d)
\end{aligned} \tag{32}$$

where $q = x_d + \epsilon(x_{cm} - x_d)$ and $\dot{q} = \dot{x}_d + \epsilon(\dot{x}_{cm} - \dot{x}_d)$.

In (32), the partial derivatives can be calculated as

$$\frac{\partial\Upsilon(x, \dot{x})}{\partial x} = \frac{1}{N}\sum_{j=1}^{N}(\dot{z}_j + \dot{x})f_{sj}''(z_j + x) \tag{33}$$

$$\frac{\partial\Upsilon(x, \dot{x})}{\partial\dot{x}} = \frac{1}{N}\sum_{j=1}^{N}f_{sj}'(z_j + x).$$

Since $\dot{z}_j(t)$, $f'_{sj}\left(z_j+x\right)$ and $f''_{sj}\left(z_j+x\right)$ are bounded, the partial derivatives in (33) can be upper bounded as

$$\left|\frac{\partial \Upsilon(x,\dot{x})}{\partial x}\right|_{(x,\dot{x})=(q,\dot{q})} \leq (U_{\dot{z}}+|\dot{q}|)U_{f''}$$

$$\left|\frac{\partial \Upsilon(x,\dot{x})}{\partial \dot{x}}\right|_{(x,\dot{x})=(q,\dot{q})} \leq U_{f'}$$

by using Assumption 3 and Theorem 1. Based on (11) and (12) (i.e., $x_{cm}-x_d=e_1$ and $\dot{x}_{cm}-\dot{x}_d=e_2-\alpha_1 e_1$), $|\dot{q}|$ can be upper bounded as

$$|\dot{q}| \leq |\dot{x}_d| + |e_2-\alpha_1 e_1| \leq U_{\dot{x}_d} + |e_2-\alpha_1 e_1|.$$

Consequently, $\left|\tilde{\Psi}(t)\right|$ can be upper bounded as

$$\left|\tilde{\Psi}(t)\right| \leq (U_{\dot{z}}+U_{\dot{x}_d}+|e_2-\alpha_1 e_1|)U_{f''}\,|e_1| + U_{f'}\,|e_2-\alpha_1 e_1|.$$

Therefore, it can be concluded that

$$\left|\tilde{\Psi}(t)\right| \leq \rho\left(\|[e_1,e_2]\|\right)\|[e_1,e_2]\|, \tag{34}$$

where $\rho(\cdot)$ is a positive, globally invertible, nondecreasing function. Furthermore, since $\|[e_1,e_2]\| \leq \|w\|$, it can be concluded from inequality (34) that $\left|\tilde{\Psi}(t)\right| \leq \rho\left(\|w\|\right)\|w\|$. □

Theorem 2. *The controller given in (15) and (20) achieves semiglobal asymptotic position tracking in the sense that*

$$e_1(t) \to 0 \quad \text{as } t \to \infty,$$

provided that k_2 is selected according to the following sufficient condition

$$k_2 > U_{\Psi_c} + \frac{1}{\alpha_2}U_{\dot{\Psi}_c}, \tag{35}$$

where U_{Ψ_c} and $U_{\dot{\Psi}_c}$ are introduced in (25). In addition, all system signals are bounded, and $f(x_{cm},z)$ can be identified in the sense that

$$f(x_{cm},z)-\hat{f}(t) \to 0 \quad \text{as } t \to \infty.$$

Proof. The proof is similar to previous work in Makkar et al. (2007). Let $\mathcal{D} \subset \mathbb{R}^4$ be a domain containing $y(t)=0$, where $y(t) \in \mathbb{R}^4$ is defined as $y(t) \triangleq [w^T(t), \sqrt{P(t)}]^T$ with $w(t)=[e_1,e_2,r]^T$ and $P(t) \in \mathbb{R}$ defined as

$$P(t) \triangleq k_2\,|e_2(0)| - e_2(0)\Psi_c(0) - Q(t), \tag{36}$$

where $Q(t) \in \mathbb{R}$ is generated by

$$\dot{Q} \triangleq r(\Psi_c - k_2\operatorname{sgn}(e_2)). \tag{37}$$

The time derivative of $P(t)$ is

$$\dot{P} = -\dot{Q} = -r(\Psi_c - k_2\operatorname{sgn}(e_2)). \tag{38}$$

Provided that the inequality (35) is satisfied, the following inequality can be obtained (Xian et al., 2004)

$$Q(t) \leq k_2\,|e_2(0)| - e_2(0)\Psi_c(0). \tag{39}$$

Hence, (39) can be used to conclude that $P(t) \geq 0$.

Let $V(y,t) \in \mathbb{R}$ be a continuously differentiable positive definite function defined as

$$V(y,t) \triangleq \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}r^2 + P, \tag{40}$$

which can be bounded as

$$\frac{1}{2}\|y\|^2 \leq V(y,t) \leq \|y\|^2. \tag{41}$$

The time derivative of $V(y,t)$ is

$$\begin{aligned}\dot{V} &= -\alpha_1 e_1^2 - \alpha_2 e_2^2 - r^2 + e_2 e_1 + e_2 r - k_1 r^2 + r\tilde{\Psi}(t)\\ &\leq -\alpha_1 e_1^2 - \alpha_2 e_2^2 - r^2 - k_1 r^2 + r\tilde{\Psi}(t)\\ &\quad + \frac{1}{2}\left(e_1^2+e_2^2\right) + \frac{1}{2}\left(e_2^2+r^2\right)\\ &\leq -\sigma_3\|w\|^2 - \left(k_1|r|^2 - \rho(\|w\|)\,|r|\,\|w\|\right)\end{aligned} \tag{42}$$

by using (12), (13), (22), (31) and (38). In (42), σ_3 is equal to $\min\{\alpha_1-\frac{1}{2}, \alpha_2-1, \frac{1}{2}\}$ where α_1 and α_2 must be selected according to $\alpha_1 > \frac{1}{2}$ and $\alpha_2 > 1$.

Completing the squares for the second and third terms in (42) gives

$$\begin{aligned}\dot{V} &\leq -\sigma_3\|w\|^2 + \frac{\rho^2(\|w\|)\,\|w\|^2}{4k_1}\\ &= -\sigma_4\|w\|^2,\end{aligned} \tag{43}$$

where $\sigma_4 = \sigma_3 - \frac{\rho^2(\|w\|)}{4k_1}$. The function $\sigma_4\|w\|^2$ is continuous and positive semi-definite on the domain $D = \{y \in \mathbb{R}^4 \mid \|y\| \leq \rho^{-1}(2\sqrt{\sigma_3 k_1})\}$.

The inequalities in (41) and (43) can be used to show that $V(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Hence, $e_1(t), e_2(t), r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Furthermore, from (12) and (13), $\dot{e}_1(t), \dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ exist and are bounded, based on (11)–(13), $\dot{x}_{cm}(t), \ddot{x}_{cm}(t) \in \mathcal{L}_\infty$ in D. From $\dot{x}_{cm}(t), z(t), \dot{z}(t), f_{si}(t) \in \mathcal{L}_\infty$, it can be concluded that $\Psi(t) \in \mathcal{L}_\infty$ based on (18). From (15) and (17) to (19), $u(t), \hat{f}(t), \dot{\hat{f}}(t), \dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t), \dot{e}_2(t), \dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, $\sigma_4\|w(t)\|^2$ is uniformly continuous in $\mathcal{D}$. There exists a bounded set $\mathcal{D}_0 \subset \mathcal{D}$, so that Theorem 8.4 of Khalil (2002) can be applied to show that $\sigma_4\|w(t)\|^2 \to 0$ as $t \to \infty$, $\forall y(0) \in \mathcal{D}_0$. Thus, $e_1(t), r(t) \to 0$ as $t \to \infty$. Furthermore, $r(t) \to 0$ indicates that $f(x_{cm},z) - \hat{f}(t) \to 0$ based on (16). By increasing the control gain k_1, the domain of attraction $\mathcal{D}_0$ can be made arbitrarily large so that a semiglobal asymptotic stability result can be obtained. □

In this paper, the single particle dynamics are only enabled to be bounded while the CM is controlled to track a desired trajectory. Physically, even if the CM is stable and the states are bounded, the single particles may behave wildly and the neighboring particles may fall far apart of their physical links. So, it is interesting and important to consider all realistic physical constraints in nanoscale friction control and study the single particle dynamics. Due to the limited number of control inputs and limited measurement of states, the stabilization problem of single particle dynamics is not addressed in this paper.

6. Simulation results

In this section, an example is provided to show the performance of the proposed control design method.

Output tracking control. Consider an array of nanoparticles that is consisted of $N=120$ particles. The Morse inter-particle interaction force is given by $F(x) = \frac{c_{ip}}{\beta}\left(e^{-\beta x} - e^{-2\beta x}\right)$ with $c_{ip}=0.26$ and $\beta=0.1$. The damping coefficient is $\nu=0.6$. The misfit length is $b=\frac{\pi}{6}$ and the substrate force function is $f_{s1}(x)=\sin(x)+\sin^2(2x)$. Position tracking control has been conducted in this simulation. The control gains are selected as $\alpha_1=2$, $\alpha_2=2$, $k_1=10$, and $k_2=5$. As shown in Fig. 2, the position of the CM is controlled to track the desired position trajectory $x_d(t) = 1+\frac{1}{2}\cos(t)$. Fig. 3 shows the identification of $f(x_{cm},z)$. Fig. 4 shows the control input $u(t)$ for position tracking control.

If the Morse inter-particle interaction force is replaced by a linear function (e.g., $F(x)=0.2x$), the asymptotic tracking results

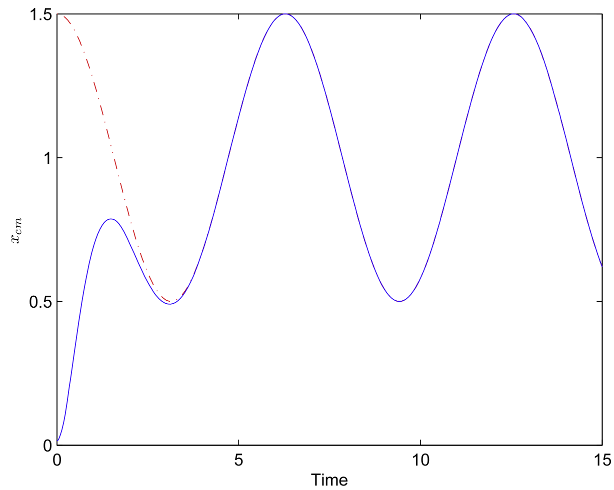

Fig. 2. Position tracking. The solid and dashed curves represent the actual and desired positions of the CM, respectively.

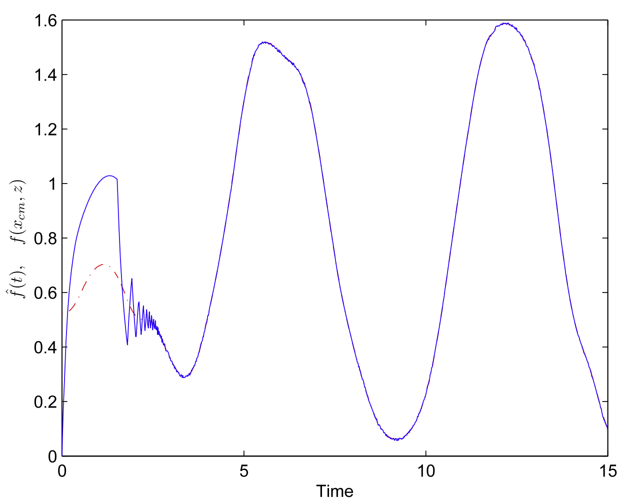

Fig. 3. The identification of $f(x_{cm}, z)$. The dashed curve represents $f(x_{cm}, z)$ while the solid curve represents the identified value of $f(x_{cm}, z)$.

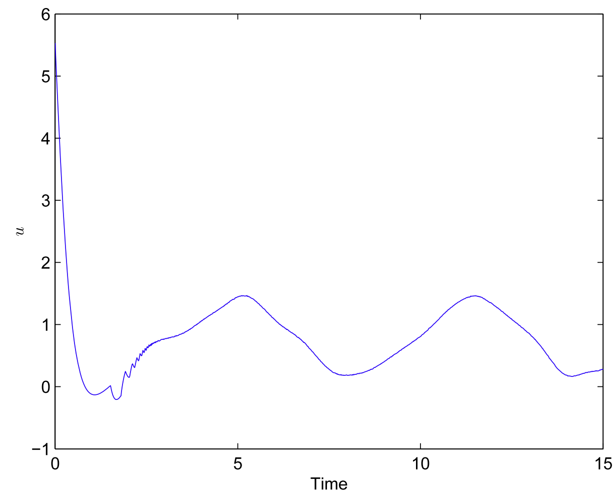

Fig. 4. The control input $u(t)$ for position tracking.

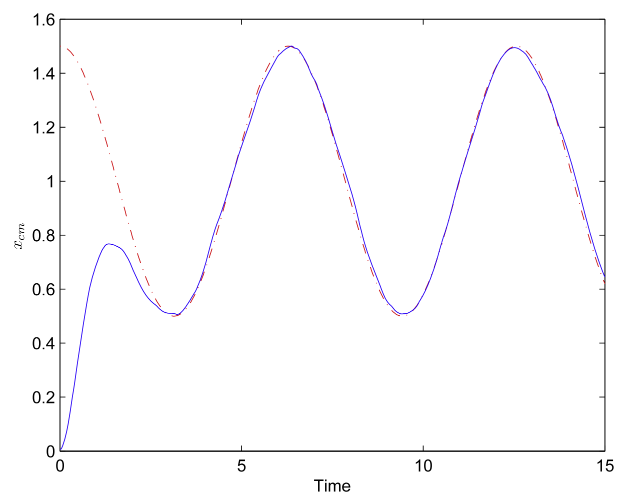

Fig. 5. Position tracking in the presence of measurement noise. The solid and dashed curves represent the actual and desired positions of the CM, respectively.

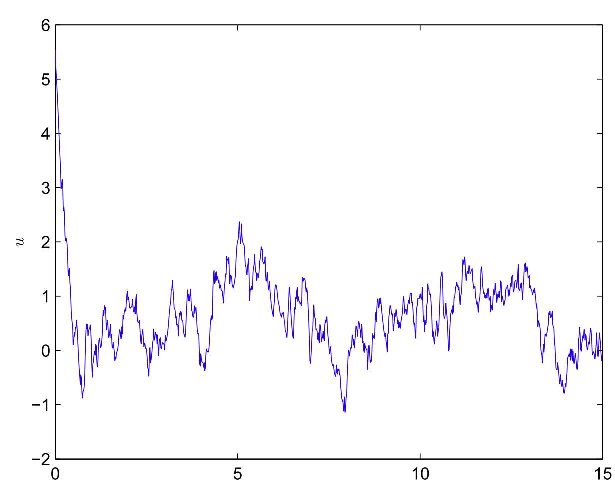

Fig. 6. The control input $u(t)$ for position tracking in the presence of measurement noise.

can be obtained in the numerical simulation and the results are not shown here. What we showed in Figs. 2 and 3 is that even when the inter-particle interaction force is a more general Morse type function, the asymptotic tracking can still be achieved using the same type of controller.

Robustness study. The control development and corresponding stability analysis in this paper are based on Assumption 2. In practice, it is very difficult to obtain accurate measurements at nanoscale and therefore there exist measurement errors. In this example, the robustness of the proposed controller is studied when there exists position measurement error $\delta x_{cm}(t)$ and velocity measurement error $\delta \dot{x}_{cm}(t)$. The measurement errors are modeled as zero mean white noise with the standard deviation equal to 0.1 and the sample period equal to 0.01. The position of the CM is shown in Fig. 5, which demonstrated robustness of the proposed controller in the presence of measurement error. Fig. 6 shows the control input $u(t)$ for position tracking control in the presence of measurement noise.

7. Conclusion

In this paper, the tracking control problem of an array of nanoparticles moving on a substrate is studied. The control goal is to make the center of the mass of the particles track a desired trajectory. A nonlinear identifier is first developed to identify the unmodeled particle dynamics. A feedback controller is then developed based on the identifier to control the center of mass of the array. In the case where the inter-particle interaction is linear, the developed controller is proven to generate semiglobal asymptotic tracking, which is demonstrated by a Lyapunov-based analysis and numerical simulation results. In our future work, we plan to provide rigorous proof for the closed-loop stability analysis for the case where the inter-particle interaction is of the general Morse type. In addition, we intend to extend our current control approach and investigate the control design that compensates for all the realistic physical constraints associated with the single particle dynamics of the nanoscale control system.

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Guoqiang Hu received his B.Eng. degree from the University of Science and Technology of China in 2002, M.Phil. degree from the Chinese University of Hong Kong in 2004, and Ph.D. degree from the University of Florida in 2007. In 2008, he joined Kansas State University as an assistant professor in the Department of Mechanical and Nuclear Engineering. Dr. Hu is a member of IEEE and serves as an associate editor on the conference editorial board of the IEEE Control Systems Society. His research interests include vision-based control, cooperative control, multi-agent robotics, and nonlinear and adaptive control of dynamic systems.

Warren Dixon received his Ph.D. degree in 2000 from the Department of Electrical and Computer Engineering from Clemson University. After completing his doctoral studies he was selected as an Eugene P. Wigner Fellow at Oak Ridge National Laboratory (ORNL). In 2004, Dr. Dixon joined the faculty of the University of Florida in the Mechanical and Aerospace Engineering Department. Dr. Dixon's main research interest has been the development and application of Lyapunov-based control techniques for uncertain nonlinear systems. He has published 3 books, an edited collection, 6 chapters, and over 200 refereed journal and conference papers. His work has been recognized by the 2009 American Automatic Control Council (AACC) O. Hugo Schuck Award, 2006 IEEE Robotics and Automation Society (RAS) Early Academic Career Award, an NSF CAREER Award (2006–2011), 2004 DOE Outstanding Mentor Award, and the 2001 ORNL Early Career Award for Engineering Achievement. Dr. Dixon is a senior member of IEEE. He serves on several IEEE CSS and ASME technical committees, is a member of numerous conference program and organizing committees, and serves on the conference editorial board for the IEEE CSS and RAS and the ASME DSC. He served as an appointed member to the IEEE CSS Board of Governors for 2008. He is currently an associate editor for *ASME Journal of Journal of Dynamic Systems, Measurement and Control*; *Automatica*; *IEEE Transactions on Systems Man and Cybernetics: Part B Cybernetics*; *and the International Journal of Robust and Nonlinear Control*.

Han Ding received his Ph.D. degree from Huazhong University of Science and Technology (HUST), Wuhan, China, in 1989. Supported by the Alexander von Humboldt Foundation, he was with the University of Stuttgart, Germany from 1993 to 1994. He has been a Professor at HUST ever since 1997 and is now Director of State Key Lab of Digital Manufacturing Equipment and Technology there. Dr. Ding acted as an Associate Editor of IEEE Trans. on Automation Science and Engineering from 2004 to 2007. Currently, he is a Technical Editor of IEEE/ASME Trans. on Mechatronics. His research interests include robotics and equipment automation.