Reconstruction of a deformed tumor for treatment planning of interstitial photodynamic therapy: A computational feasibility study

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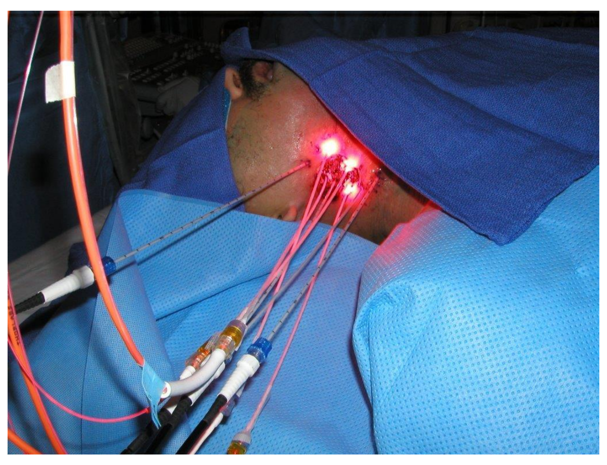
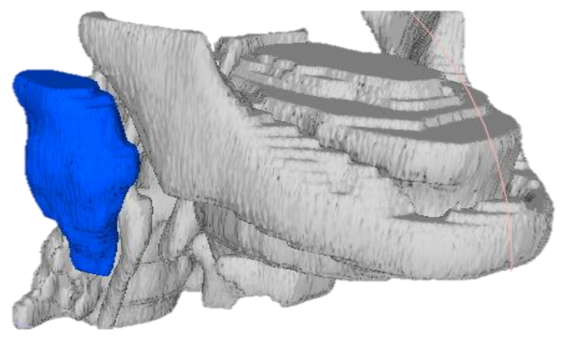
BACKGROUND

- Interstitial Photodynamic Therapy (I-PDT) involves the activation of a photosensitizer by a therapeutic light resulting in tumor cell destruction.
- I-PDT has been applied for the treatment of locally advance d head and neck caner (LAHNC).
- In I-PDT, light is provided via catheter-embedded fiber optics.
- We have developed a finite element model (FEM) for computing the light propagation during I-PDT.
- CT scans of a patient with LAHNC are used to create three-dimensional (3-D) geometries for the FEM.
- While CT is used for the FEM, ultrasound is used for the guidance of fiber insertion.
- For treatment planning, the number and location of source fibers is based on the tumor size and location.
- Tumor size also dictates the simulated light dose volume histogram. However, there is not much research into the impact of tumor deformation during fiber insertion on the light dose delivered.



I-PDT Procedure for LAHNC



3D reconstruction of LAHNC

MOTIVATION

Problem

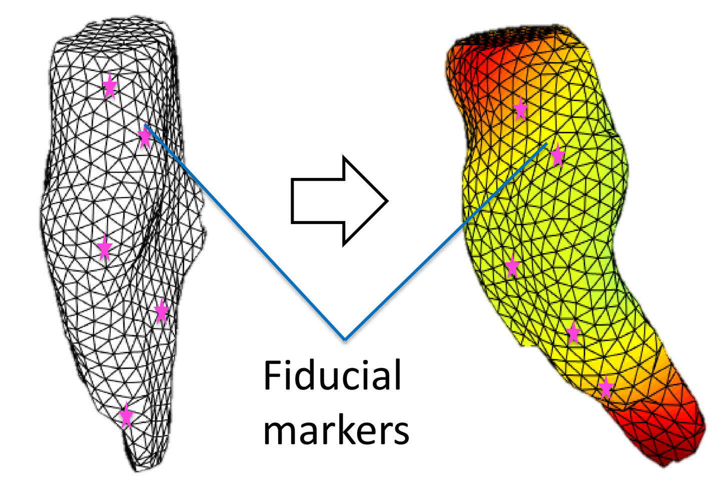
CT is initially used to reconstruct digital LAHNC model for I-PDT preplanning. Treatment failure occurs at marginal region due to the mismatch between the original tumor shape used in the preplan and the actually deformed shape during operation.

Utilization of fiducial markers (FMs)

FMs are gold seeds that are implanted in or around a tumor to help pinpoint the tumor's location in past works. In our context, relative displacement of FMs encodes information about the deformed shape, and thus can be used for capturing deformation

Goal

From a computational perspective, predict the deformed shape of a LAHNC during I-PDT procedure from (i) Initial 3D reconstruction in preplanning, and (ii) Traced FM displacements during two imaging modalities.



Initial tumor shape reconstruction for preplanning

Deformed tumor shape during I-PDT procedure

METHOD

Mathematical model

Standard linear finite element method is used for guiding computation. Assuming that there is no external forces on interior nodes, the nodal displacement vector x can be linearly mapped to surface nodal force vector f_u.

$$\begin{bmatrix} K_u \\ K_l \end{bmatrix} x = \begin{bmatrix} f_u \\ 0 \end{bmatrix}$$

The constraints imposed by FMs are formulated by using a 0-1 indicator matrix D and observed FM displacement vector d.

$$Dx = d$$

By the observation that LAHNCs are generally surrounded by soft tissue, force field smoothness is used for regularizing the above under-determined system. Laplacian energy on f_u is used as the mathematical formulation of this smoothness.

$$||L(f_u)||_2^2 = x^T K^* x$$

Optimization formulation

The shape prediction process is formulated as a problem of finding the smoothest force distribution on tumor surface. Constraints induced by FMs and assumption of no external force on interior nodes are satisfied throughout computation. The objective function is quadratic in nodal displacement vector x, and thus allows for fast optimization process.

Force field Laplacian energy minimization problem:

$$\begin{aligned} &\text{minimize} && ||L(f_u)||_2^2 = x^T K^* x \\ &\text{with respect to} && x \\ &\text{subject to} && Dx = d \\ & && K_l x = 0 \end{aligned}$$

f_u: force vector (surface nodes)
L: Laplacian matrix
K^*: $(MLK_u)^T(MLK_u)$
M: inverse mass matrix
K_u: upper stiffness matrix(surface nodes)
D: binary indicator matrix
d: measured displacements of FMs
K_l: lower stiffness matrix(interior nodes)

Light propagation modeling

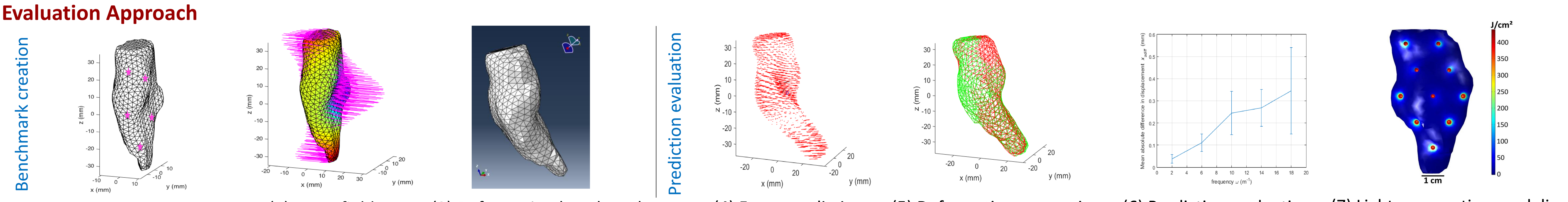
Our finite element model (FEM) for computing the light propagation was described previously in Oakley et al. In this approach, the three-dimensional (3-D) time-dependent diffusion equation as derived from the equation for radiative transfer was applied.

$$\frac{1}{c_n}\left(\frac{\partial}{\partial t}\Phi(x,y,z,t) - \nabla(\alpha^n \nabla\Phi(x,y,z,t))\right) = -\mu_a^n \Phi(x,y,z,t)$$

$$\text{where } \alpha^n = c_n \cdot [3(\mu_a^n + (1-g)\mu_s^n)]^{-1}$$

$\Phi(x,y,z,t)$ is the photon flux (Photons/m²/sec), α^n is the optical diffusion coefficient (m²/sec) of tissue n, μ_a^n and μ_s^n are the linear absorption and scattering coefficients (1/m) of tissue n, g is the optical anisotropy factor, and c_n is the speed of light in tissue.

Evaluation Approach



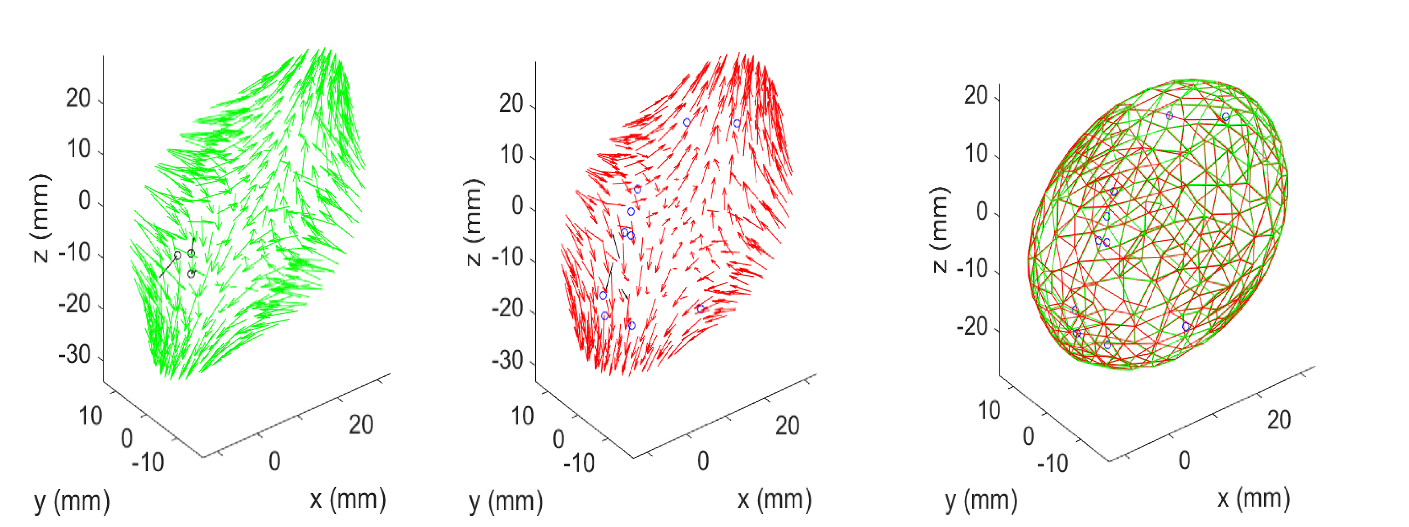
(1) Tumor model & FMs (2) Force field (3) Deformation benchmark (4) Force prediction (5) Deformation comparison (6) Prediction evaluation (7) Light propagation modeling

(1)Given a 3D digital tumor, place FMs randomly on its surface/ inside its volume. (2) Apply discretized force field to the surface nodes. (3)Using a commercial FEA package, solve the solid mechanics deformation problem, and trace the FM displacements. (4) Use the computed FM displacements as input to our algorithm to predict the applied force field and the tumor deformation – prediction model. (5) Compare the prediction against the benchmark. (6) Repeat the above process for a varying number of FMs and their placements, and report outcomes over multiple runs. (7) Use light propagation modeling on undeformed and deformed models for analyzing deformation effects in I-PDT. Quantitatively measure the possible improvement from using our algorithm.

RESULTS

Synthetic (sphere) model

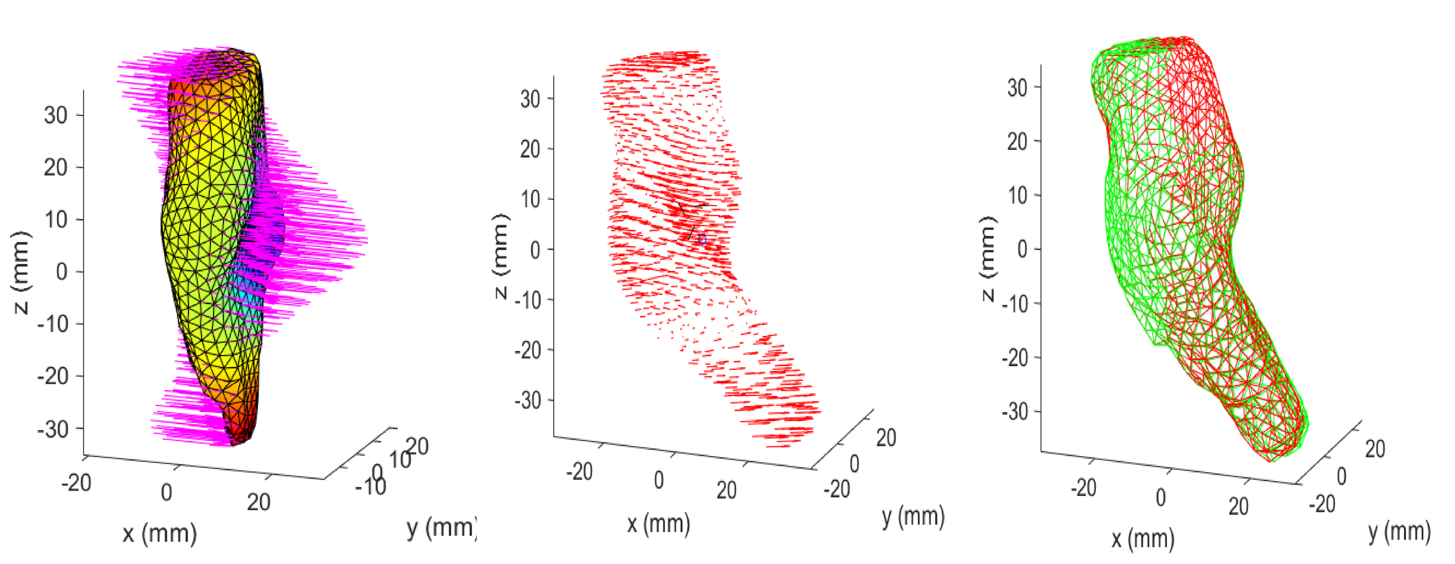
Our algorithm is tested on spheres of 30mm and 80mm diameters (normal range of LAHNC size) with infinitely differentiable sinusoidal force field on their surfaces. The force prediction is very close to the applied force field in benchmark. The maximum surface offset between benchmark and predicted shape is 0.7mm among all cases (normal uncertainty in ultrasound imaging)



(1) Sinusoidal force field (2) Force prediction (3) Deformation comparison

Real tumor model

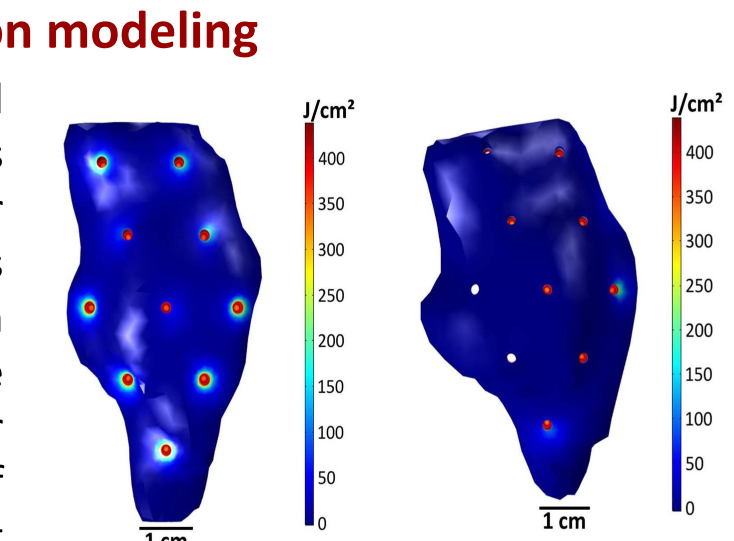
Our algorithm is also tested on a real tumor model reconstructed from CT scanning. Smooth force field is applied on tumor surface for simulating realistic bending behavior. The prediction of the displacement field is quite accurate, with the maximum offset on the surface being within 1mm (normal uncertainty in distance measurement using ultrasound imaging).



(1) Smooth force field (2) Force prediction (3) Deformation comparison

Results from Light propagation modeling

The fluence (J/cm²) was computed when 10 source diffuser fibers were inserted into the tumor volume. The treatment time was 500 seconds. There is a maximum difference of 28% between the percent of the original tumor volume and the percent of deformed tumor model that receives the prescribed light dose



(1) Original tumor (2) Deformed tumor

CONCLUSION

- Developed an optimization algorithm
  – initial tumor shape and fiducial markers → deformed shape
- Demonstrated effect of deformation in I-PDT procedure
  – light propagation modeling
- Mathematical formulation of force field smoothness
  – Laplacian energy
- Fast computation in predicting deformation
  –1.3s in sphere case and 5.5s in real tumor case
- High prediction accuracy
  – within typical ultrasound imaging resolution