Thin Layer Liquid Acoustical Filters

by

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ABSTRACT

The transmission of acoustic waves through two liquid media separated by an intermediate quarter-wave liquid layer was theoretically and experimentally investigated. Plane sound waves incident upon liquid quarter-wave layers exhibit behavior analogous to that of light waves in transparent thin film filters. A transmitting transducer was immersed in an oil medium, and a receiving transducer was immersed in a glycerine medium. The quarter-wave layer of water having an intermediate acoustic impedance separated the oil and glycerine media. It was found that sound transmissivity through the system was significantly increased at the frequencies corresponding to different quarter-wave thicknesses.

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CONTENTS

INTRODUCTION

Wave phenomena are of fundamental importance in many areas of physics. The theory and applications derived for one type of wave can often be extended to another. Thus, the analogy between light and sound has lead to important results in acoustics derived from their optical counterparts.[1] Examples of such results would include acoustical holography, ultrasonic imaging, and defraction theory.

Until recently,[2] however, thin film optical filters have not lead to corresponding acoustical counterparts. While absorption filters are the most common type of optical filter, another type of filter based on interference in thin films is becoming increasingly important in both theoretical studies and practical applications. Physicists have known for nearly 300 years that transparent thin films exhibit unusual optical characteristics.[3] In 1704 Newton described his experiments on the colors of soap bubbles but was unable to explain his observations in terms of interference effects. It was not until 1801, when the principle of interference was reported by Young, that thin film optical phenomena were explainable. In 1817, Fraunhofer produced anti-reflection coatings by treating tarnished glass with sulfuric or nitric acid. He found that the treated side of the glass reflected much less light than the other side and concluded that a new transparent product with a different refractive index must have been deposited on the treated side of the glass. In

1891 Taylor observed that the tarnish film on flint glass telescope lenses increased their transparency. He even got a patent in 1904 for chemically producing the tarnish film. In the decade before World War II various thin film anti-reflection coatings were produced for optical instruments, and in 1936 Strong made the first thin film coatings designed solely for antireflection purposes. In 1939 Geffcken obtained a German patent for narrow bandpass filters using the principle of the Fabry-Perot interferometer. Thus, by the beginning of World War II the stage was set for a rapid expansion of thin film technology. The increasing need for complex optical devices, the development of efficient high vacuum systems in which thin films can be deposited, and the subsequent expansion of optical technology, including lasers, have all resulted in a corresponding increase in the understanding and use of a wide variety of thin film optical filters.

The purpose of this experiment was to extend thin film optical filter theory to thin layered liquid acoustic media. While experimental work has been done involving anti-reflective coatings in liquid media,[4] I have been unable to find any published references to experimental work in which all of the media involved, including the thin layer, are liquids. Accordingly, I attempted to devise a way of producing plane interfaces between experimental liquids having different sound propagation characteristics.

The idea of using a highly compliant membrane (such as mylar) for separating the different liquid media was discarded because such a membrane could introduce its own interference effects and because it would not be able to establish a plane interface between the media. If, however, immiscible liquids of progressively decreasing density are stacked vertically, plane horizontal interfaces are automatically established by the force of gravity. This latter approach was used in the experiment.

THEORY

Before proceeding to a discussion of acoustical interference effects in thin layers, it would be useful to consider the simpler situation in which a plane longitudinal sound wave passes from one homogeneous medium through a plane interface with another such medium. Unless the two media have identical characteristics, a portion of the wave incident upon the interface will be reflected back into the first medium and a portion will be refracted and transmitted into the second medium. In order to evaluate the amplitude of the reflected and refracted components, the concept of acoustic impedance is used.

The characteristic acoustic impedance Z of a given medium is defined as the ratio of the acoustic pressure p (i.e. the excess pressure created by a compressional sound wave above equilibrium pressure) to the particle velocity $d\eta$ (i.e. the velocity of individual particles in the compressional wave).[5] Since acoustic pressure is analogous to voltage and particle velocity is analogous to current, the characteristic acoustic impedance of a medium can be thought of as being analogous to the characteristic impedance of an electrical transmission line.

As pointed out above,

$$Z = \frac{p}{d\eta} \qquad (1)$$

But

$$p = iBk\eta \tag{2}$$

where B is a measure of the compressibility of the fluid defined as its bulk modulus or the ratio of change in pressure to change in volume, k is the wavenumber, and η is the particle displacement caused by the compressional wave. And for a plane wave,

$$d\eta = i\omega\eta \tag{3}$$

Substituting Eq. (2) and Eq. (3) gives

$$Z = \frac{iBk\eta}{i\omega\eta} \tag{4}$$

$$= \frac{Bk}{\omega} \tag{5}$$

Since $c = \omega/k$

$$Z = \frac{B}{c} \tag{6}$$

Since $B = c^2\rho$, where ρ is the mass density, we have

$$Z = \frac{c^2\rho}{c} = \rho c \tag{7}$$

Thus it can be seen that the concept of characteristic acoustic impedance of a medium, which was originally defined in terms of pressure and particle velocity, may also be described in terms of the speed of sound in the fluid and the density of that fluid, or

$$Z = \frac{p}{d\eta} = \rho c \tag{8}$$

We are now in a position to consider a plane sound wave meeting the interface between two liquid media having different acoustic impedances. Since the two media are in complete contact at every point across the plane interface, the two quantities entering into the definition of acoustic impedance, particle velocity $d\eta$ and acoustic pressure p, will both be continuous across the interface. However, the densities ρ and speeds of sound c in the two media will have different values. Figure 1 shows a plane sound wave in a liquid medium having a characteristic acoustic impedance of $Z_A = \rho_A c_A$ contacting, at normal incidence, the plane interface between another medium having a characteristic impedance of $Z_B = \rho_B c_B$.

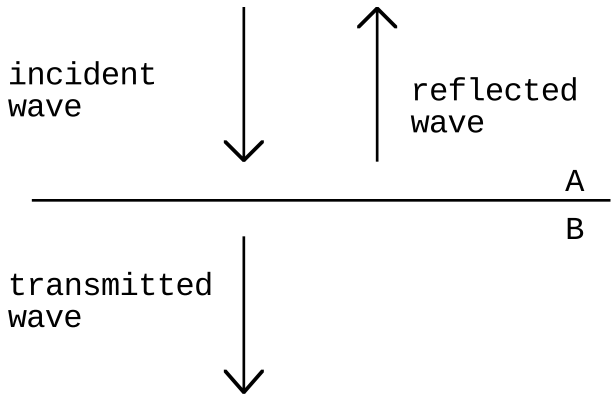

Figure 1.

Since the particle velocity $d\eta$ and the acoustic pressure p are continuous across the interface,

$$d\eta_i + d\eta_r = d\eta_t \tag{9}$$

and

$$p_i + p_r = p_t \tag{10}$$

where i, r, and t refer to the incident, reflected, and transmitted waves. From Eq. (8)

$$p_i = \rho_A c_A d\eta_i \tag{11}$$

$$p_r = -\rho_A c_A d\eta_r \tag{12}$$

$$p_t = \rho_B c_B d\eta_t \tag{13}$$

Thus

$$\rho_A c_A d\eta_i - \rho_A c_A d\eta_r = \rho_B c_B d\eta_t \tag{14}$$

Using Eq. (7)

$$Z_A d\eta_i - Z_A d\eta_r = Z_B d\eta_t \tag{15}$$

In order to eliminate $d\eta_t$, we use Eq. (9) which gives

$$Z_A d\eta_i - Z_A d\eta_r = Z_B(d\eta_i + d\eta_r) \tag{16}$$

$$Z_A d\eta_i - Z_A d\eta_r = Z_B d\eta_i + Z_B d\eta_r \quad (17)$$

$$(Z_A - Z_B) d\eta_i = (Z_A + Z_B) d\eta_r \quad (18)$$

Thus the amplitude reflection coefficient R is given by

$$R = \frac{d\eta_r}{d\eta_i} = \frac{(Z_A - Z_B)}{(Z_A + Z_B)} \quad (19)$$

It can be seen from Eq. (19) how the reflection coefficient depends upon the acoustic impedance of the two media. Where $Z_A > Z_B$ (producing a positive value for R) the incident and reflected particle velocities are in phase but the acoustic pressures are out of phase. This produces a reduction in acoustic pressure in the incident medium. Where $Z_A < Z_B$ (producing a negative value for R) the acoustic pressures are in phase but the particle velocities are out of phase. This produces an increase of pressure in the incident medium.

In order to determine the amplitude transmission coefficient T, we can substitute the value of $d\eta_r$ from Eq. (9) into Eq. (15) which produces

$$Z_A d\eta_i - Z_A(d\eta_t - d\eta_i) = Z_B d\eta_t \quad (20)$$

$$2Z_A d\eta_i = Z_B d\eta_t + Z_A d\eta_t \quad (21)$$

Thus the amplitude transmission coefficient T is given by

$$T = \frac{d\eta_t}{d\eta_i} = \frac{2Z_A}{Z_A + Z_B} \quad (22)$$

Unlike the case of reflection, the particle velocities and acoustic pressures for the transmitted wave are both in phase with the incident velocity and pressure.

Since the intensity of a sound wave is proportional to the square of the particle velocity times the acoustical impedance of the medium, the intensity coefficient of reflection is given by

$$\frac{I_r}{I_i} = \frac{Z_A(d\eta_r^2)}{Z_A(d\eta_i^2)} = \left(\frac{Z_A - Z_B}{Z_A + Z_B}\right)^2 \tag{23}$$

Similarly

$$\frac{I_t}{I_i} = \frac{Z_B(d\eta_t^2)}{Z_A(d\eta_i^2)} = \frac{Z_B}{Z_A}\left(\frac{2Z_A}{Z_A + Z_B}\right)^2 = \frac{4Z_AZ_B}{(Z_A + Z_B)^2} \tag{24}$$

Since the total energy before and after a reflection at a single plane interface is unchanged,

$$\frac{I_r}{I_i} + \frac{I_t}{I_i} = 1 \tag{25}$$

Thus

$$I_i + I_t = I_r \tag{26}$$

Having considered reflection and transmission at the interface between two sound conducting media, we are now in a position to consider the case in which these two media are separated by a thin layer of a third medium. The plane wave that reflects from the thin layer is made up of the superposition of a number of reflected waves: (i) the wave reflected from the upper interface of the thin layer; (ii) the wave which refracts through the upper interface and is then reflected from the lower interface and ultimately leaves the thin layer through the upper interface; (iii) the wave which is refracted into the thin layer and is reflected three times from the upper and lower interfaces leaving the thin layer through the upper interface; and (iv) the additional waves resulting from additional internal reflections within the thin layer. It is the combined effect of the superposition of all of these waves which gives rise to constructive or destructive interference. The coefficient of reflection will, therefore, be a function of *d* (thickness of the thin layer), the acoustic impedances of the media involved, and the wavelength of the incident plane wave. It is important to note that since reflection from a thin layer is dependent upon wavelength, it is fundamentally different from reflection from a single interface which is independent of frequency.

Derivation of the formula for reflection from a thin layer is best approached by the use of matrix methods. We begin by considering figure 2.

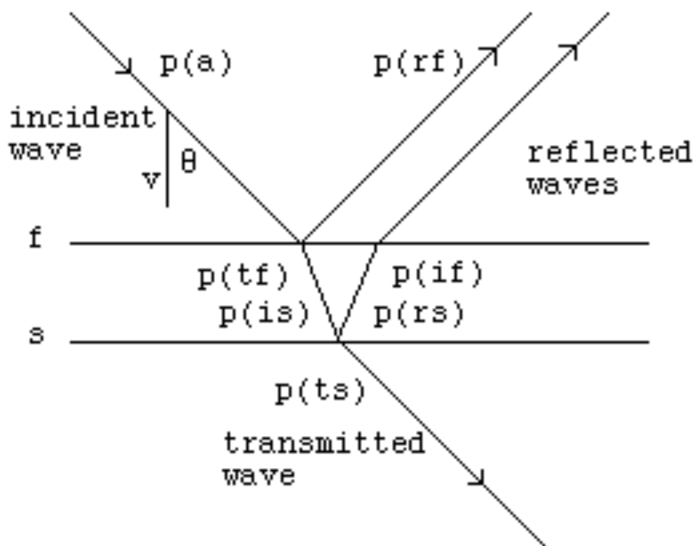

Figure 2.

The acoustic pressure and the vertical component of the particle velocity will have constant magnitudes across each interface f and s. This means that the total *p* field of incident and refracted rays on one side of an interface must equal the total p field of incident and refracted rays on the opposite side of the interface. This means that acoustic pressure at interface f (p_f) is

$$p_f = p_a + p_{rf} = p_{tf} + p_{if} \qquad (27)$$

where p_a is the pressure of the incident beam, and p_{rf} is the total reflected pressure at interface f, p_{tf} is the total transmitted pressure at interface f, p_{if} is the total incident pressure at interface f.

Equivalently, at interface s,

$$p_s = p_{is} + p_{rs} = p_{ts} \qquad (28)$$

The v field is not always perpendicular to the incident plane. Thus the corresponding equations include a $cos\ \theta$ term. The v field equations can therefore be given by

$$v_f = v_a\ cos\ \theta_a - v_{rf}\ cos\theta_a = v_{tf}\ cos\ \theta_{tf} - v_{if}\ cos\ \theta_{tf} \qquad (29)$$

and

$$v_s = v_{is}\ cos\ \theta_{tf} - v_{rs}\ cos\theta_{tf} = v_{ts}\ cos\ \theta_{ts} \qquad (30)$$

Since $p = Zv$, these can be written,

$$v_f = \frac{1}{Z}(p_a - p_{rs}) = \frac{1}{Z}(p_{tf} - p_{if}) \qquad (31)$$

$$v_s = \frac{1}{Z}(p_{is} - p_{rs}) = \frac{1}{Z}p_{st} \qquad (32)$$

It can be shown that the phase difference ϕ between the f and s interfaces is given by,

$$p_{is} = p_{tf}e^{-i\phi} \qquad (33)$$

$$p_{if} = p_{rs}e^{-i\phi} \qquad (34)$$

where $\phi = (2\pi f/v)d = 2\pi d/\lambda$. Substituting these equations into Eq. (28) and Eq. (32) gives

$$p_f = p_{tf}e^{-i\phi} + p_{if}e^{i\phi} \tag{35}$$

and

$$v_f = \frac{1}{Z}\left(p_{tf}e^{-i\phi} - p_{if}e^{i\phi}\right) = \frac{1}{Z}p_{ts} \tag{36}$$

Solving for p_{tf} and p_{if}

$$p_{tf} = \left(\frac{\frac{1}{Z_f}p_s + v_s}{\frac{2}{Z}}\right)e^{i\phi} \tag{37}$$

$$p_{if} = \left(\frac{\frac{1}{Z_f}p_s - v_s}{\frac{2}{Z}}\right)e^{-i\phi} \tag{38}$$

Substituting these equations into Eq. (27) and Eq. (29)

$$p_f = p_s \cos\phi + v_s(iZ_f \sin\phi) \tag{39}$$

$$v_f = p_s\frac{(i \sin\phi)}{Z} + v_s \cos\phi \tag{40}$$

which in matrix form is

$$\begin{pmatrix} p_f \\ v_f \end{pmatrix} = \begin{pmatrix} \cos\phi & iZ\sin\phi \\ \frac{i\sin\phi}{Z} & \cos\phi \end{pmatrix}\begin{pmatrix} p_s \\ v_s \end{pmatrix} \tag{41}$$

Therefore the characteristic transfer matrix for a single layer is

$$M = \begin{pmatrix} \cos\phi & iZ\sin\phi \\ \frac{i\sin\phi}{Z} & \cos\phi \end{pmatrix} \tag{42}$$

Recalling that R and T are the reflection and transmission coefficients respectively,

$$1 + R = m_{11}T + \frac{m_{12}T}{Z_C} \tag{43}$$

$$\frac{(1 - R)}{Z_A} = m_{21}T + \frac{m_{22}T}{Z_C} \tag{44}$$

Solving these for *R* and *T*,

$$R = \frac{Z_A m_{11} + Z_A Z_C m_{12} - m_{21} - Z_C m_{22}}{Z_A m_{11} + Z_A Z_C m_{12} + m_{21} + Z_C m_{22}} \tag{45}$$

$$T = \frac{2Z_A}{Z_A m_{11} + Z_A Z_C m_{12} + m_{21} + Z_C m_{22}} \tag{46}$$

By substituting in the matrix elements we get

$$R = \frac{Z_B(Z_A - Z_C)\ cos\ \phi + i(Z_A Z_C - Z_B^2)\ sin\ \phi}{Z_B(Z_A + Z_C)\ cos\ \phi + i(Z_A Z_C + Z_B^2)\ sin\ \phi} \tag{47}$$

and

$$T = \frac{2Z_A}{Z_B(Z_A + Z_C)\ cos\ \phi + i(Z_A Z_C + Z_B^2)\ sin\ \phi} \tag{48}$$

The last two equations give values for the amplitude coefficients for reflection and transmission. Since intensity is proportional to the square of amplitude, the corresponding intensity coefficients are given by

$$\frac{I_r}{I_i} = \frac{Z_B^2(Z_A - Z_C)^2\ cos^2\ \phi + (Z_A Z_C - Z_B^2)^2\ sin^2\ \phi}{Z_B^2(Z_A + Z_C)^2\ cos^2\ \phi + (Z_A Z_C + Z_B^2)^2\ sin^2\ \phi} \tag{49}$$

and

$$\frac{I_t}{I_i} = \frac{4Z_A^2}{Z_B^2(Z_A + Z_C)^2\ cos^2_{?}\ \phi + i(Z_A Z_C + Z_B^2)^2\ sin^2\ \phi} \tag{50}$$

For the special case in which the thickness of the intermediate layer is equal to a quarter-wave length, or any odd multiple thereof, Eq. (47) produces for normal incidence

$$R = \frac{\left(Z_B^2 - Z_A Z_C\right)}{\left(Z_B^2 + Z_A Z_C\right)} \qquad (51)$$

and

$$T = \frac{\left(Z_B^2 - Z_A Z_C\right)}{\left(Z_B^2 + Z_A Z_C\right)} + 1 \qquad (52)$$

Using Eq. (22) and Eq. (52) together with published[6] acoustic impedances for the oil, water, and glycerine used in this experiment, we get an amplitude transmission coefficient T of .67 for the oil-glycerine case and .93 where the oil and glycerine are separated by a quarter-wave layer of distilled water. Accordingly, the quarter-wave water layer should act as an anti-reflective impedance matching mechanism which significantly increases the coefficient of transmission through the system.

APPARATUS

As mentioned in the introduction, the purpose of this experiment was to extend the analogy between optical and acoustical waves by attempting to demonstrate interference effects in thin liquid layers in much the same way as such effects are displayed in optical thin films. One of the first considerations in designing such an experiment was finding a method in which different liquid layers could be placed in continuous contact at a plane boundary. If the boundaries were to be oriented in any position other than horizontal, it seemed obvious that it would be extremely difficult to maintain the required plane interface. Accordingly, it was decided to use three immiscible liquids of different densities so that the plane interfaces could be established simply by the force of gravity.

Another consideration was whether the experiment should be designed to directly measure the reflection of sound waves from the thin layer or whether it should measure the transmitted wave. In the former case, the transmitting and receiving transducers would be on the same side of the thin layer which could give rise to two problems by: (i) preventing intensity measurements from being made at a normal angle with the interface; and (ii) requiring the two transducers to be mounted relatively close together which would make it more difficult to isolate the reflected wave. Accordingly, it was decided to mount the transducers vertically on opposite sides on the thin layer thereby measuring transmitted sound levels.

Because sound waves, under appropriate conditions, can be propagated through a cylinder as plane waves, consideration was given to conducting this experiment in a relatively narrow cylinder. Such a configuration would have had the advantages of insuring that a true plane wave was being considered and minimizing the effects of reverberation. However, the use of a relatively thin cylinder could create large resonance effects which might mask the interference effects of the thin layer. It was therefore decided to place the three liquid media and the transducers in a much larger tank in an attempt to minimize frequency dependent resonance effects. Even though a true plane wave would not be produced, it was felt that any departure from planarity would not be significant in view of the relatively small angle subtended by the face of the recieving transducer.

Once the basic configuration for the experiment had been determined, it was necessary to select the three liquids to be used in the experiment. The three liquids not only had to be immiscible but had to have successively increasing acoustic impedances and correspondingly increasing densities. Glycerine ($HOCH_2CHOHCH_2OH$) supplied by Spectrum Chemical Co. with a published acoustical impedance of 2.34 and a density of 1.26 was used for the lower medium. Distilled water with an acoustical impedance of 1.55 and a density of 1.00 was used for the quarter wave layer, and Johnsons baby oil with a published acoustical impedance of 1.17 and a density of .821 was used for the upper medium.

The overall height of the tank was 42 cm. It has a hexagonal cross section with an interior width of 26 cm. between faces. A larger rectangular tank was not used because of liquid media cost considerations. The face of the lower (receiving) transducer was 7.4 cm above the bottom of the tank, and the face of the upper (transmitting) transducer was mounted directly above at a distance of 21.6 cm from the bottom of the tank. The tank was filled with glycerin to a depth of 15.9 cm. In order to minimize reflections within the tank, its interior sides and bottom were lined with Sonix foam.

Figure 3 is a schematic diagram of the equipment used in the experiment.

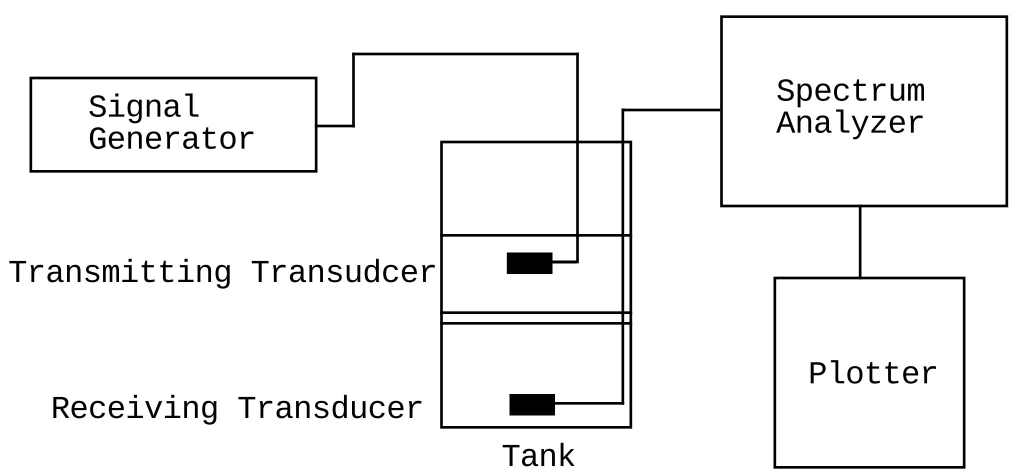

Figure 3

The signal generator was a Wavetek Model 180 Sweep/Function Generator. Each of the transducers was a Channel Products Untrasonic Nebulizer Model CPMT having a natural frequency of 1.35 MHz. The transducers were each contained in an immersable mounting designed for these transducers. The Spectrum Analyzer was a Hewlett Packard Model 3580A, having a range of 0 Hz to 50,000 Hz. The plotter was a Hewlett Packard Model 7044A X-Y Plotter.

PROCEDURE

After the tank was lined with acoustic foam, the lower (receiving) transducer was imbedded vertically in the center of the layer of foam covering the bottom of the tank. The bottom of this transducer was located 4.4 cm above the solid bottom of the tank in order to reduce the direct transmission of sound from the body of the tank to the transducer. Glycerine was then poured into the tank to a depth of 15.9 cm which produced a distance of 8.5 from the face of the the transducer to the top surface of the glycerine. In order to obtian calibration data for sound waves passing through the single oil-glycerine interface, oil was then carefully poured onto the glycerine to an additional depth of 6.6 cm above the top surface of the glycerine. The upper (transmitting) transducer was then immersed into the oil and mounted to the wall and floor of the laboratory without any direct connection to the tank. The distance between the faces of the two transducers was 14.2 cm. After this arrangement, calibration data were obtained.

The signal generator was set to sweep from 10,000 Hz to 50,000 Hz in the sine wave mode. The upper frequency limit was selected at 50,000 Hz as this corresponded with the upper limit of the spectrum analyzer. Both sweep rate and amplitude were set to their maximum values on the signal generator.

The spectrum analyzer was set to a resolution bandwidth of 100 Hz, a sweep time of 10 seconds per division, and a Log 10 dB per division amplitude mode. Calibration graphs were plotted on the X-Y plotter for the oil-glycerine interface at center frequencies of 20,000 Hz, 30,000 Hz, and 40,000 Hz and frequency spans of 5 kHz, 2 kHz, and 1 kHz per division for each such center frequency.

After the calibration graphs were completed, a quarter-wave thick layer of distilled water was inserted between the oil and glycerine by using a large bulb syringe. The syringe was carefully lowered into the oil so that its tip was just above the existing interface. Because water has a density intermediate between those of oil and glycerine, it simply flowed from the syringe and spread out into a smooth layer. The thickness of the water layer was measured with a millimeter scale by viewing the layer through a thin slit in the acoustic foam.

The first experimental quarter-wave layer was fixed at a thickness of .842 cm. which corresponds to a frequency of 46,000 Hz. For this layer thickness, three plots were made at frequency spans 5, 2, and 1 kHz/Div and leaving the setting of the signal generator unchanged from those used in the calibration procedure.

The existing quarter-wave water layer was then successively increased in thickness by adding additional water to the layer with the syringe. Using this procedure, quarter-wave water layers for frequencies corresponding to

40,000 Hz and 30,000 Hz were made, and additional plots were recorded for these frequencies.

RESULTS

Graph 1 shows the calibration curve for transmitted sound intensity through the single oil-glycerine interface for the frequency range from 30,000 Hz to 50,000 Hz. The vertical axis represents relative sound intensities in dB. Graph 2 shows the corresponding data after the quarter-wave water layer for 46,000 Hz was inserted between the oil and glycerine media.

Graph 3 showns the calibration curve for the frequency range from 35,000 Hz to 45,000 Hz. Graph 4 shows transmission data after the quarter-wave water layer was increased in thickness to correspond to a frequency of 40,000 Hz.

Graph 5 showns the calibration curve for the frequency range from 20,000 Hz to 40,000 Hz. Graph 6 shows transmission data after the quarter-wave water layer was increased in thickness to correspond to a frequency of 30,000 Hz.

At first glance these data appear difficult to interpret. All three calibration curves (graphs 1, 3, and 5) show large fluctuations in transmitted intensity between the oil-glycerine interface which, from Eq. (22), should not exhibit frequency dependent effects. Thus, such variations are probably the result of (i) diffraction effects resulting from the small size of the transmitting transducer face in comparison to the wavelengths being observed, (ii) unidentifiable reflections and resonances inherent in the experiment design; (iii) unidentified frequency dependent attenuation characteristics of the liquid media; and (iv) unidentified frequency dependent reflection characteristics of the acoustic foam. In any event, a comparison of graphs 2, 4, and 6 with graphs 1, 3, and 5, respectively, reveals quarter-wave interference effects that are consistent with theory. As pointed out in the theory section, a quarter-wave layer of distilled water between the two oil and glycerine media should have the effect of increasing the amplitude coefficient of transmission from .67 to .93 and increasing the intensity coefficient of transmission from .45 to .86. In other words, the quarter-wave water layer between these two liquids should act in a way analagous to an anti-reflective coating on a lens by increasing the transmissivity at the intended frequency.

In order to compare each calibration curve with its corresponding quarter-wave transmission graph, three additional graphs were prepared. Graphs 7, 8, and 9 show the *difference* in dB between such data as a function of frequency. It is significant that graphs 7, 8, and 9 show a distinct peak in transmitted intensity at, or very close to, the frequencies intended. In fact, these peaks probably represent a more accurate way of determining the quarter-wave thicknesses than the millimeter scale that was actually used. The measurements for d made with the millimeter scale probably do not have a precision better than 4%, and each of the measured maxima on graphs 7, 8, and 9 are well within this range.

In view of the problems associated with this experiment as mentioned above, it would be appropriate to suggest possible future improvements. In the first place, use of a vertical cylinder narrow enough to propagate plane waves should be reconsidered. Any resonances associated with such a column would at least be more predictable than the irregular fluctuations observed in this experiment. A narrow column also would not require any anti-reflective lining and would minimize the problem of reflection from side walls. Secondly, a future experiment should attempt to measure coefficient of reflection instead of transmission inasmuch as the theoretical percentage change in reflection coefficient from a quarter-wave layer is much larger that the percentage change in transmission coefficient. Such an experiment could take the form of three cylinders joined in a Y configuration. Although measurements would not be made at normal incidence, this should not present any real difficulties if the angle of incidence is kept low. Finally, the use of pulses rather than continuous waves would further eliminate unwanted reverberations.

CONCLUSIONS

Despite the fluctuations in the calibration curves, this experiment successfully demonstrated frequency dependent interference effects from a liquid quarter-wave layer having an acoustic impedance intermediate between the two adjoining liquid media. While the optical properties of transparent quarter-wave plates have long been known, this experiment, to my knowledge, is the first to demonstrate that these properties are also applicable to sound waves in liquid media. In short, as has happened so many other times in physics, what was predicted from work in one area has been extended to another.

SYMBOLS

Symbol	Definition
B	bulk modulus
c	acoustic wave velocity
$d\eta$	particle velocity
I	intensity
k	wave number
p	acoustic pressure
R	amplitude reflection coefficient
T	amplitude transmission coefficient
v	vertical component of particle velocity
Z	acoustic impedance
η	displacement
ρ	mass density
ϕ	phase
ω	angular frequency

Calibration Curve: dB Vs. Frequency in Hz

dB

Frequency in Hz

Graph 1

40,000

46,000

34,000

Quarter-Wave Transmission Results: 46,000 Hz

dB Vs. Frequency in Hz

Graph 2

dB

Frequency in Hz

40,000

46,000

34,000

Calibration Curve: dB Vs. Frequency in Hz

dB

Frequency in Hz

Graph 3

40,000

45,000

35,000

Quarter-Wave Transmission Results: 40,000 Hz

dB Vs. Frequency in Hz

Graph 4

dB

Frequency in Hz

40,000

46,000

34,000

Calibration Curve: dB Vs. Frequency in Hz

dB

Frequency in Hz

Graph 5

30,000

40,000

20,000

Quarter-Wave Transmission Results: 30,000 Hz

dB Vs. Frequency in Hz

Graph 6

dB

Frequency in Hz

20,000

40,000

30,000

Graph 7

difference 46,000

Graph 8

difference 40,000

Graph 9

difference 30,000

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