Aren't Dummy Alternatives only Technical Shortcuts?*

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Schlenker (forthcoming) argues that a conjunction $(p \wedge q)$ embedded in a sentence (written: $\varphi(p \wedge q)$) is infelicitous if *the first conjunct is idle, whatever the second conjunct could have been.* The technical visage of this condition is revealed in (1a). Intuitively, it would be more satisfying to express this condition as in (1b) so that we do not need consider second conjuncts which by themselves would make the sentence odd (e.g. tautologies, contradictions). In other words, we claim that there is no need to scrutinize complex considerations about the first conjunct to note that a sentence which later contains a tautology or a contradiction is odd.

(1) a. $\forall \beta \in \mathcal{L}\ , \varphi(p \wedge \beta) \Leftrightarrow \varphi(\beta)$
   b. $\forall \beta \in \mathcal{L}^*, \varphi(p \wedge \beta) \Leftrightarrow \varphi(\beta)$

(2) Notations:
   a. $\mathcal{L}\ $ = the set of propositions in the language
   b. $\mathcal{L}^* = \mathcal{L} \setminus \{\bot, \top\}$

The technical proofs provided in Schlenker (forthcoming) make heavy use of tautologies to show that the system matches the achievement of dynamic semantics (Heim, 1983). In this note we investigate whether this has to be the case and in particular, our main goals are to:

- Exemplify the problem (section 1).
- Propositional logics: Investigate equivalent versions of the theory (theorem 7); show in particular that tautologies and contradictions do not play any fundamental role (version 9b); extend this result to the incremental approach of the theory (section 2.3).
- Discuss examples showing that tautologies do play a crucial role when modality or quantification is introduced (sections 3.1 and 3.2).
- Suggest that a more general solution to the problem might arise in more powerful frameworks (section 4).

1 Using Tautologies

Schlenker predicts that a sentence like *John knows that it is raining* is felicitous only if (3a) holds (p denotes the proposition true iff *it is raining*). The result in (3) prefigures the equivalence of this system with the achievement of dynamic semantics (e.g. presuppositions project above negation).

(3) Theorem: the following conditions are equivalent:
   a. $\forall \beta, \neg(p \wedge \beta) \Leftrightarrow \neg\beta$
   b. p

This result is proved as follows:

(4) Proof of (3)
   a. (3a) $\Leftarrow$ (3b): If p is true, $(p \wedge \beta)$ is equivalent to β; thus, $(p \wedge \beta)$ is false iff β is false.

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b. (3a) $\Rightarrow$ (3b): If we choose β to be a tautology in (3a), we obtain that the context is such that $\neg(p \wedge \top) \Leftrightarrow \neg\top$. The right-hand side of this equivalence is false so the left-hand side must be false too: $\neg(p \wedge \top)$ is false, i.e. $p \wedge \top$ is true, i.e. p is true.

In (4b), we replaced β by a tautology and this seems to conflict with the general motivations for the theory. Indeed, the use of a tautology in a conjunction is always redundant and sentences including redundant material should be disdain.

In the following section, we show that the tautology only plays a technical role. For instance, would we instantiate β with any proposition and its negation that we would obtain the same effect.

2 Propositional Logics

2.1 Main intuition underlying the technical proofs

We will repeatedly make use of the following simple idea: a tautology might be replaced by several non tautologous propositions which disjunction ends up being a tautology, again. This way, we prove that conditions involving tautologies may be replaced by several conditions, none of which involving tautologies.

Notations

We always consider a sentence where an atomic proposition with presupposition p is embedded (written: $\varphi(S_p)$, if S_p is the presuppositional element). We also consider variants of this sentence where the presuppositional element has been replaced by any β or $p \wedge \beta$ in particular (i.e. $\varphi(\beta)$ and $\varphi(p \wedge \beta)$). Finally, $\Phi(\beta)$ stands for the important: $[\varphi(p \wedge \beta) \Leftrightarrow \varphi(\beta)]$.

2.2 Equivalent theories

In this section, we present general conditions which allow a version of the theory to match the predictions of dynamic semantics with a restricted set of potential conjuncts "β". Let us start with a set of technical results:

(5) Lemma.

a. $\Phi(\bot)$ and $\Phi(p)$ are true.

b. If $\beta_1 \Rightarrow \beta_2$, then $\Phi(\beta_2) \Rightarrow \Phi(\beta_1)$.

c. $\forall \beta_1, \beta_2,\ \Phi(\beta_1) \wedge \Phi(\beta_2) \Leftrightarrow \Phi(\beta_1 \vee \beta_2)$

(6) Proof of lemma (5).

a. – $p \wedge \bot$ is equivalent to $\bot$ and therefore, $\varphi(p \wedge \bot) \Leftrightarrow \varphi(\bot)$, i.e. $\Phi(\bot)$.
– Similarly, $p \wedge p$ is equivalent to p and therefore, $\varphi(p \wedge p) \Leftrightarrow \varphi(p)$, i.e. $\Phi(p)$.

b. We distinguish two possible cases:
1) If β_1 and β_2 have the same truth-value, $\Phi(\beta_1)$ and $\Phi(\beta_2)$ are also equivalent.
2) If they have different truth-values, then β_1 is false (otherwise it also entails the truth of β_2). So, $\Phi(\beta_1)$ is true ($\Phi(\bot)$ is always true) and $\Phi(\beta_2) \Rightarrow \Phi(\beta_1)$.

c. $\Leftarrow$: this direction of the equivalence is a direct consequence of the previous result (5b): a disjunction is entailed by each of its disjuncts.
$\Rightarrow$: $\beta_1 \vee \beta_2$ is equivalent to at least one of its disjuncts. If, for instance, it is equivalent

to its first disjunct β_1, $\Phi(\beta_1)$ and $\Phi(\beta_1 \vee \beta_2)$ are also equivalent.
Overall, $\Phi(\beta_1) \Leftrightarrow \Phi(\beta_1 \vee \beta_2)$ or $\Phi(\beta_2) \Leftrightarrow \Phi(\beta_1 \vee \beta_2)$ (or both) and the result follows.

We can now obtain the conditions under which a set of potential conjuncts is big enough to match the result of dynamic semantics:

(7) General equivalence criterion.

a. $\forall\beta \in \mathcal{L}$, $\Phi(\beta)$

b. $\forall\beta \in \mathcal{L}'$, $\Phi(\beta)$ (with $\mathcal{L}' \subset \mathcal{L}$)
If not trivial, the conditions above are equivalent if and only if:

c. $\exists\beta_1, \ldots, \beta_n$ in $\mathcal{L}'$ s.t. $\beta_1 \vee \cdots \vee \beta_n \vee p \equiv \top$

(8) Proof of (7).
(7a) always entails (7b), we need to show that the entailment in the other directions holds if and only if the criterion (7c) is satisfied.

a. Let us first clarify what we mean by *if not trivial.* If the sentence is such that the place where p appears does not play any role (i.e. $\varphi(\top)$ is not equivalent to $\varphi(\bot)$), we do not want to appeal to any criterion because both (7a) and (7b) are trivially satisfied.

b. If the criterion holds, the equivalence holds.
Assuming that (7c) is satisfied and that (7b) also holds, we show that (7a) also holds: Let us choose a set of β_i satisfying (7c). From (5a) we induce that $\Phi(p)$ and from (7b) that $\forall i$, $\Phi(\beta_i)$. Thus, $\Phi(\top)$ also holds (inductively applying (5c) to the fact that $\Phi(\beta_1) \wedge \Phi(\beta_n) \wedge \Phi(p)$). Furthermore, for every $\beta \in \mathcal{L}$, $\beta \Rightarrow \top$ and, applying (5b), we obtain the result: $\Phi(\beta)$ (since it is entailed by $\Phi(\top)$).

c. If the criterion does not hold, the equivalence does not hold.
Let us assume that the criterion does not hold and that (7b) holds.
– If $\Phi(\top)$ is not true: the equivalence between (7a) and (7b) indeed does not hold (the former is false while the latter is true, by hypothesis).
– If $\Phi(\top)$ is true: i.e. $\varphi(p)$ is equivalent to $\varphi(\top)$. Since the criterion (7c) does not hold, p is not true. This leads us back to the degenerated case we excluded right away where $\varphi(\top)$ and $\varphi(\bot)$ are equivalent.

Thanks to theorem (7), we can now present different "versions" of the theory which would yield identical predictions. Most importantly for our purpose, the versions in (9b), (9c) and (9e) do not make use of redundant elements (as tautologies always are).

(9) Equivalent variants of the theory.
The following conditions are equivalent:

a. $\forall\beta \in \mathcal{L}$, $\Phi(\beta)$

b. $\forall\beta \in \mathcal{L}^*, \Phi(\beta)$

c. $\exists\beta \in \mathcal{L}$, $\Phi(\beta) \wedge \Phi(\neg\beta)$

d. $\Phi(\top)$

e. $\Phi(\neg p)$

2.3 Extension to the incremental version of the theory

The results we described so far apply to the *global* version of the Transparency theory (cf. Schlenker, 2006). The original version of the theory which proved to match the predictions of dynamic semantics is *incremental*: the condition not only quantifies over potential second conjuncts but also over potential sentence completions.

We show in this subsection that 1) the previous result extends to the incremental version of the theory and 2) tautologies and contradictions can be removed from the sentence completions we need to consider. (We do not provide a general statement such as (7) to avoid intricate syntactic considerations).

Notations

For the purpose of this section, we extend our previous notations as follows: $\Phi(\beta, \gamma)$ now represents the equivalence between the following variants of the original sentence: 1) the original sentence where $p \wedge \beta$ replaces an atomic proposition with presupposition p and γ replaces whatever follows it; 2) the original sentence where β replaces an atomic proposition with presupposition p and γ replaces whatever follows it.

In other words, if the original sentence starts with α followed by a presuppositional element (which would be followed by further linguistic material), then $\Phi(\beta, \gamma)$ stands for $[\alpha(p \wedge \beta)\gamma \Leftrightarrow \alpha\beta\gamma]$.

$\mathcal{S}$ stands for the set of sentence completions γ which would lead to a well-formed sentence; $\mathcal{S}^*$ is the subset of these sentence completions which do not contain any tautologies or contradictions.

Results

(10) Lemma
For all β, for all γ, we can find γ_1 and γ_2 without any tautologies or contradictions such that: $\Phi(\beta, \gamma_1) \wedge \Phi(\beta, \gamma_2) \Rightarrow \Phi(\beta, \gamma)$.

(11) Proof of (10).
Let β be an arbitrary element of $\mathcal{L}$ and γ an element of $\mathcal{S}(\alpha)$. If γ contains a tautology, we can write: $\gamma = \delta\top\delta'$. Let β' be a non trivial element of $\mathcal{L}$ and define $\gamma_1 = \delta\beta'\delta'$ and $\gamma_2 = \delta\neg\beta'\delta'$. Then:
– If β' is true: γ is equivalent to γ_1 and therefore $\Phi(\beta, \gamma) \Leftrightarrow \Phi(\beta, \gamma_1)$.
– Similarly, if $\neg\beta'$ is true: γ is equivalent to γ_2 and $\Phi(\beta, \gamma) \Leftrightarrow \Phi(\beta, \gamma_2)$.
Since β' or $\neg\beta'$ must be true: $\Phi(\beta, \gamma_1) \wedge \Phi(\beta, \gamma_2) \Rightarrow \Phi(\beta, \gamma)$.
To extend the proof to cases where γ contains a contradiction, we may just replace this contradiction by the negation of a tautology and apply the same mechanism. Cases where γ contains several tautologies or contradictions follow straightforwardly by induction.

The following theorem is an immediate consequence of this result, it states that tautologies and contradictions are not necessary. (This combines without any difficulty with the previous restrictions on the set of potentials β one need to quantify over, e.g. theorem 9).

(12) Theorem
The following conditions are equivalent:
a. $\forall\beta \in \mathcal{L}, \forall\gamma \in \mathcal{S}\ , \Phi(\beta, \gamma)$
b. $\forall\beta \in \mathcal{L}, \forall\gamma \in \mathcal{S}^*, \Phi(\beta, \gamma)$

2.4 Summary of the results in propositional logics

The main result of this section is that the achievements of dynamic semantics can be derived from the Transparency theory (be it *global* or *incremental*) without making any disturbing use of tautologies or contradictions.

3 Modality and Quantification

3.1 Modality

Unfortunately, the previous results do not extend to modal logic. An instance of a problematic sentence is *The king is necessary bald*[1]. Indeed, we need to consider the tautology has a respectable conjunct in this case:

(13) Problem in modal logics.
The following conditions are *not* equivalent (provided that p is not trivial):
a. $\forall \beta \in \mathcal{L}\ , \Box (p \wedge \beta) \Leftrightarrow \Box \beta$
b. $\forall \beta \in \mathcal{L}^*, \Box (p \wedge \beta) \Leftrightarrow \Box \beta$

On the one hand, it is straightforward that (13a) entails $\Box p$ (instantiating β with a tautology). On the other hand, (13b) does not entail $\Box p$. Indeed, this latter condition tells us that for any $\beta \in \mathcal{L}^*$, $\Box \beta \Rightarrow \Box p$. But a conjunction of n such conditionals would give us the following kind of result: $[\Box \beta_1 \vee \Box \beta_2 \vee \cdots \vee \Box \beta_n] \Rightarrow \Box p$. This will only be equivalent to $\Box p$ if we can be sure that one of the elements β_1, β_2, ..., β_n is a tautology. It is possible than in certain contexts, we do not know which proposition is a tautology among a set of propositions, but such a situation does not hold in full generality.

3.2 Quantification

A similar counter-example can be given for quantificational cases: *Everybody knows that he is incompetent*[1]. The formal result is that (without full explanation of the notations):

(14) Problem in the quantificational case.
The following conditions are *not* equivalent (provided that P is not trivial):
a. for all predicates β, $[\forall x,\ (P \cap \beta)x] \Leftrightarrow [\forall x,\ \beta x]$
b. for all non trivial predicates β, $[\forall x,\ (P \cap \beta)x] \Leftrightarrow [\forall x,\ \beta x]$

4 Extending the Logics

Our bet would be that the partial problem we sketched might find its solution within a more powerful semantics system. Imagine for instance that our language is such that we can express a sequence of propositions which are as close as possible to being tautologies but for which there always remains a little doubt (e.g. *John will live at least 1 more year*, *John will live at least 1 more month*, *John will live at least 1 more day*, etc.). Let us note such propositions r_θ (e.g. r_θ is true

[1]As below/above, this can be viewed as unproblematic since this particular sentence also entails its presupposition. However, the negation of this sentence does not entail its presupposition anymore although the derivation should be strictly similar.

in each possible world with at least this probability θ). It would require some work to introduce a semantics for these expressions but we might certainly expect that if every $\Phi(r_\theta)$ holds, $\Phi(\top)$ must hold too.

The underlying intuition is simple: we might not need to use tautologies if we can find propositions as close to being tautologous as possible. I leave this suggestion as it is because a proper investigation would involve nonessential machinery.

5 Conclusion

We prove in this technical note that the use of tautologies is necessary for the Transparency theory to match the achievements of dynamic semantics at least in quantificational cases (or in modal cases which are not discussed in full length in Schlenker (2006) but should work similarly).

This result might inform us about the fundamental nature of the Transparency principle and the competition principles it involves. On the other hand, we suggested that these limitations would not hold in any more powerful logical framework.

References

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