Truth approximation via abductive belief change

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Abstract

We investigate the logical and conceptual connections between abductive reasoning construed as a process of belief change, on the one hand, and truth approximation, construed as increasing (estimated) verisimilitude, on the other. We introduce the notion of '(verisimilitude-guided) abductive belief change' and discuss under what conditions abductively changing our theories or beliefs does lead them closer to the truth, and hence tracks truth approximation conceived as the main aim of inquiry. The consequences of our analysis for some recent discussions concerning belief revision aiming at truth approximation and inference to the best explanation are also highlighted.

Keywords: Verisimilitude, truthlikeness, abduction, belief revision, theory change, truth approximation, inference to the best explanation.

1 Introduction

This article investigates some of the logical and conceptual connections among three fields of research which have attracted much attention in the last few decades: abduction, belief change and truth approximation. In Section 2, we briefly survey the recent philosophical discussion on these three topics, whose origins can be traced back to the work of Charles Sanders Peirce. Then, we proceed as follows. In Section 3, we introduce the notion of truth approximation as increasing (estimated) verisimilitude, and specify what it means for a theory to be closer than another to 'the truth'. The connection between truth approximation, on the one hand, and abduction and belief change, on the other, is best studied once abductive reasoning is interpreted as the dynamic process of revising one's belief in response to incoming, possibly conflicting, information. This interpretation is explored in Section 4, where we present the so-called AGM theory of belief change (Section 4.1) and a simple model of 'abductive belief change', that specifies how a given theory should change in order to incorporate some explanation of the incoming evidence (Section 4.2). Finally, in Section 5, we analyse the relations between abduction and verisimilitude, asking whether, and under what conditions, abductive belief change effectively tracks truth approximation. More specifically, we show how assessments of verisimilitude can be used as a guide for comparing the competing explanations for a given piece of evidence and choose among them. The resulting notion of 'verisimilitude-guided' abductive belief change provides a 'verisimilitudinarian' version of inference to the best explanation, according to which the abductive revision of theories by incoming evidence increases, or even maximizes, their estimated verisimilitude.

2 Abduction, belief change and truth approximation

Peirce coined the term 'abduction' to denote the pattern of reasoning—that he also called 'retroduction' (CP 1.68) or 'hypothesis' (CP 2.623)—involved in 'the operation of adopting an explanatory

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hypothesis' for a given piece of evidence (CP 5.189).[1] Like induction, and contrary to deduction, abduction is a form of ampliative and uncertain reasoning, in the sense that even if the truth of the premises is taken for granted, the conclusion of an abductive argument may be false. The logical form of an abductive inference, according to Peirce (CP 5.189), is the following:

> The surprising fact, C, is observed;
> But if A were true, C would be a matter of course,
> Hence, there is reason to suspect that A is true.

From the point of view of deductive logic, the invalid inference 'if A then C; but C; therefore, A' is an instance of the well-known fallacy of 'affirming the consequence'; however, Peirce noted that an argument does not need to be valid in order to be strong, i.e., to make its conclusions worth of further consideration (CP 5.192). Indeed, Peirce insisted that, although the conclusions of abductive arguments are always tentative and conjectural, abduction is a fundamental form of inference both in scientific and everyday reasoning.[2]

Abduction started attracting the attention of philosophers of science in the 1960s, when Hanson [19] suggested that Peirce's schema provides 'a logic of scientific discovery' and Harman [23] argued that 'inference to the best explanation' (IBE, for short), as he called abduction, is the core of any ampliative or non-deductive inference. These pioneering contributions made clear that there are at least two different ways—respectively, a 'weak' and a 'strong' one—of assessing the proper role of abductive inference. According to the first, weak interpretation, abduction has a primary *discovery* (or 'strategic' or 'heuristic' [70, p. 203]) function, that of suggesting or finding promising or 'testworthy' hypotheses which are then set out to further inquiry or empirical testing. According to the second, strong (or *justification*) reading, abduction can be formulated as a rule of acceptance, since it gives reasons to tentatively accept its conclusion as the 'best' explanatory hypothesis among the available ones.[3]

There has been a lot of discussion about whether Peirce endorsed the discovery or the justification view of abductive inference, or both. Neglecting this exegetical point, we shall rest content with the remark that abduction plays a crucial role in his analysis of belief formation and change [39, p. 42]. Indeed, as Atocha Aliseda made clear [2, 4], one can view abduction not as a kind of logical inference, but as a process of belief change in which an explanation for a given belief or observation is assimilated within the *corpus* of one's currently accepted beliefs. In this connection, it is interesting to note that Peirce's so-called [40, p. 12] 'belief-doubt' model of inquiry can be considered as one of the main philosophical sources of inspiration of the contemporary theories of belief change as developed in philosophical logic and AI. In fact, the most prominent account in this area, the so-called 'AGM theory' of belief change (named after Carlos Alchourrón *et al.*, see [1]), was inspired by the epistemological views of Levi, that are in turn explicitly influenced by

[1] When quoting from Peirce's *Collected Papers* [24], we follow the convention of citing the number of the volume followed by the number of the relevant paragraph.

[2] Peirce's examples are the following: (CP 2.625) 'Fossils are found; say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we suppose the sea once washed over this land. […] Numberless documents and monuments refer to a conqueror called Napoleon Bonaparte. Though we have not seen the man, yet we cannot explain what we have seen, namely, all these documents and monuments, without supposing that he really existed.'

[3] Schurz [70] and Niiniluoto [52] defend, respectively, the discovery and the justification view of abduction. Both views are still discussed in current philosophical literature (see, e.g., [41], [39, 40], [52–55], [70, 72], [30, 32], [26], and, for a survey, [15]). Abduction is also a major theme in logic and AI (see [17], [2, 4], and [30, section 6] for a survey of some of the main computational approaches) and in cognitive science, diagnostic and legal reasoning, argumentation theory and model-based reasoning [42, 77].

Peirce [74, section 1].[4] Thus, Peirce appears as 'the crucial, logical and historical link' between Nineteenth- and Twentieth-century analyses of belief change no less than, as emphasized by Laudan [38, pp. 275–276], between the corresponding discussions of the so-called 'approach-to-the-truth view' of scientific progress.

The approach-to-the-truth view portrays science as a self-corrective enterprise which, in the long run, leads our hypotheses and theories closer to the truth. Although its origins are older, a mature formulation of this idea is to be found in the works of such Nineteenth-century thinkers as William Whewell, Pierre Duhem and Peirce himself (cf. [38] and [48, pp. 45–46]). In particular, Peirce explicitly argued that truth is the ideal aim of scientific inquiry and that the development of science can be construed in terms of convergence to the truth. Later pragmatists interpreted Peirce's position as supporting the so-called 'consensus theory of truth', which *defines* truth as the limit of inquiry. This would amount to the 'self-trivialization' [38] of the thesis that science approaches the truth, which would then become a mere tautology. In contrast with this interpretation, Niiniluoto has convincingly argued that Peirce was 'an epistemological realist and optimist' [49, p. 137] who believed that, as a matter of fact and with high probability, scientific progress would ultimately bring the beliefs of the scientific community towards the truth.[5]

These ideas were revived and further clarified, starting in the 1960s, within the modern 'verisimilitudinarian' account of scientific progress [13]. The notion of verisimilitude or truthlikeness—construed as closeness or similarity or approximation to the whole truth about a given target domain—was introduced by Popper in order to defend the idea that progress can be explained in terms of the increasing verisimilitude of scientific theories [63, 64].[6] According to this approach, such theory changes as that from Newton's to Einstein's theory are progressive because, although the new theory is, strictly speaking, presumably false, we have good reasons to believe that it is closer to the truth than the superseded one: increasing verisimilitude is the key ingredient for progress.[7] In the next section, this idea of truth approximation as increasing verisimilitude is briefly presented in more details.

3 Truth approximation as increasing (estimated) verisimilitude

Intuitively, a statement, hypothesis, or theory T is highly verisimilar if it says many things about the target domain, and if many of these things are (almost exactly) true. Thus, the (degree of)

[4]The AGM theory of belief change (note that 'belief dynamics', 'belief revision' and 'theory change' are also commonly used labels) is only one of the formal approaches to the analysis of theory change developed, starting in the mid-1970s, by AI scholars, logicians, epistemologists and philosophers of science (cf. [10] and [55]). Interestingly, this theory emerged from the confluence of two, initially completely separated, research projects, that of Gärdenfors devoted to the analysis of counterfactual conditionals in scientific reasoning, and that of Alchourrón and Makinson focusing on the derogation and amendment of legal codes [43]. Standard introductions to the AGM theory can be found in the first dedicated monograph [18] and in the first textbook presentation [20].

[5]Niiniluoto offers an illuminating reconstruction of the approach-to-the-truth tradition, and of Peirce's contribution in particular, in his [48, in particular chapters 3, 5, and 7, 9], [49, section 5.3], [55] and [57].

[6]It is perhaps worth noting, in order to avoid possible confusions, that Peirce (CP 2.662–663, 8.222–224, and 8.237) also used the term 'verisimilitude', to denote a special kind of support given to a theory by some evidence [49, p. 162]. As Niiniluoto showed [47, p. 250], this Peircean notion can be explicated in terms of the modern notion of verisimilitude, a point overlooked in more recent discussion [5].

[7]After Miller [44] and Tichý [75] independently proved that, on the basis of Popper's explication of this notion, a false theory can never be closer to the truth than another (true or false) theory, such authors as Miller [45, 46], Niiniluoto [48, 49, 52], Oddie [58, 60], Kuipers [28, 29, 31] and Schurz and Weingartner [69, 71] developed a number of post-Popperian theories of verisimilitude that succeed in avoiding the problems encountered by Popper's definition. A survey of the history of theories of verisimilitude is provided by Niiniluoto [50]; see also [59] and, for a technical comparison of the main positions, [78]. For an introduction to the verisimilitudinarian approach to scientific progress see [57] and [13].

verisimilitude $Vs(T)$ of T must depend on both its content, i.e. how much T says, and its accuracy, i.e. how much of what T says is in fact true. In Popper's words, verisimilitude 'represents the idea of approaching comprehensive truth. It thus combines truth and content' [63, p. 237]. A simple way of making these intuitions precise is offered by the so-called 'basic feature approach'—for short, 'BF-approach'—to verisimilitude [8–10, 12].[8]

Suppose that the domain under inquiry ('the world') is described by a propositional language $\mathcal{L}_n$ with n logically independent atomic sentences $p_1,\ldots,p_n$. The 'basic features' of the world are then described by the 'basic sentences' or 'literals' of $\mathcal{L}_n$, i.e. by its atoms and their negations. Accordingly, 'the whole truth'—or simply 'the truth'—about the world, as described within the language, can be construed as the most complete true description of the basic features of world, i.e. as the conjunction of the true basic sentences in $\mathcal{L}_n$. Then, focusing on the information conveyed by T about the basic features of the world, the verisimilitude of theory T is interpreted in terms of the balance of true and false basic sentences entailed by T.

More precisely, let us consider a constituent C of $\mathcal{L}_n$, that is a conjunction of n basic sentences, one for each atomic sentence. Note that there are exactly $q=2^n$ constituents and that only one of them, denoted by '$C_\star$', is true; thus, $C_\star$ can be identified with 'the (whole) truth' in $\mathcal{L}_n$. Given theory T, let the 'basic content' of T be the set $b(T)$ of the k basic sentences ($k \leq n$) entailed by T, and assume that $b(T)$ is consistent, i.e. does not contain both p_i and $\neg p_i$ for the same atomic sentence p_i. Each member of $b(T)$ will be called a 'basic claim' of T, i.e. a claim about the basic features of the world. According to the BF-approach, to assess the verisimilitude of T one needs only to consider what T tells about the basic features of the world, i.e. its basic content $b(T)$.

Given constituent C and theory T, let $t(T,C)$ and $f(T,C)$ denote, respectively, the true and false basic content of T with respect to C, i.e. the set of basic claims of T which are true and false in C, respectively (note that $t(T,C)\cup f(T,C)=b(T)$). Calling each element of $t(T,C)$ a *match*, and each element of $f(T,C)$ a *mistake* of T about C, we may say that T is close or similar to C if T makes many matches and few mistakes about C. The degree of true basic content $\mathrm{cont}_t(T,C)$ and the degree of false basic content $\mathrm{cont}_f(T,C)$ of T with respect to C are defined as follows:

$$\mathrm{cont}_t(T,C) \overset{\mathrm{df}}{=} \frac{|t(T,C)|}{n} \quad \text{and} \quad \mathrm{cont}_f(T,C) \overset{\mathrm{df}}{=} \frac{|f(T,C)|}{n}. \tag{3.1}$$

In words, $\mathrm{cont}_t(T,C)$ is the normalized number of matches, and $\mathrm{cont}_f(T,C)$ the normalized number of mistakes, that T makes with respect to C. In order to assess the similarity of T to C, $\mathrm{cont}_t(T,C)$ may be construed as the overall *reward* attributed to the matches of T and $-\mathrm{cont}_f(T,C)$ as the overall *penalty* attributed to the mistakes of T. A 'contrast measure' of similarity between T and C is a weighted average of the prize due to T's matches and of the penalty due to T's mistakes [9, p. 188]:[9]

$$s_\phi(T,C) \overset{\mathrm{df}}{=} \mathrm{cont}_t(T,C) - \phi\,\mathrm{cont}_f(T,C), \tag{3.2}$$

where $\phi > 0$. Intuitively, different values of ϕ reflect the relative weight assigned to truths and falsehoods, i.e. to the matches and mistakes of T with respect to C. If $\phi = 1$, then the similarity of T to C simply equals the difference between the normalized number of matches and mistakes of T

[8]For early motivation see [16]; Kuipers anticipated the key ideas underlying the BF-approach in his [27]; for further refinements along these lines, see also [33–35].

[9]The name 'contrast measures' refers to the fact that s_ϕ can be construed as an application of the contrast model of similarity introduced by Tversky [76] in his study of the similarity between psychological stimuli.

with respect to C. In such case ($\phi=1$), if T is itself a constituent (i.e. if T makes n basic claims), then $s_\phi(T,C)$ reduces to the normalized Hamming (or Dalal) distance between propositional constituents, or the so-called Clifford distance between the monadic constituents of a first-order language [49, p. 313].[10]

The verisimilitude $Vs_\phi(T)$ of T can be construed as the similarity $s(T,C_\star)$ of T to the truth, i.e. to the true constituent $C_\star$ of $\mathcal{L}_n$. Hence,[11]

$$Vs_\phi(T) \stackrel{\text{df}}{=} s_\phi(T,C_\star) = \text{cont}_t(T,C_\star) - \phi\,\text{cont}_f(T,C_\star). \tag{3.3}$$

Some examples will show that Vs_ϕ delivers assessments of closeness to the truth in agreement with the intuition that verisimilitude is a 'mixture' of truth and content (cf. [13, section 2]).

Example 3.1

Without loss of generality, let us assume that $C_\star$ is the conjunction $p_1 \wedge \ldots \wedge p_n$ of the unnegated literals of $\mathcal{L}_n$. Then, theories p_1 and $\neg p_2$ are equally informative, in that both make a single basic claim about the world—however only the former is true, hence it is more verisimilar than the latter: indeed, $Vs_\phi(p_1) = \frac{1}{n} > -\frac{\phi}{n} = Vs_\phi(\neg p_2)$. On the other hand, p_1 and $p_1 \wedge p_2$ are equally accurate, since both are true—however the latter is more informative, hence it is more verisimilar than the former: accordingly, $Vs_\phi(p_1) = \frac{1}{n} < \frac{2}{n} = Vs_\phi(p_1 \wedge p_2)$.

In this connection, one may note that a widely shared adequacy condition for any verisimilitude measure Vs is the Popperian requirement that verisimilitude co-varies with logical strength among true theories (for discussion, see [49, pp. 186–187, 233, 235–236]):

$$\text{If } T_1 \text{ and } T_2 \text{ are true and } T_1 \text{ entails } T_2, \text{ then } Vs(T_1) \geq Vs(T_2). \tag{3.4}$$

One can easily check that Vs_ϕ satisfies this requirement with respect to the basic contents of T_1 and T_2: if T_1 and T_2 are true and $b(T_1) \supseteq b(T_2)$ then $Vs_\phi(T_1) \geq Vs_\phi(T_2)$.[12]

Condition (3.4), however, does not hold among false theories, since logically stronger falsities may well lead us farther from the truth: if T_1 and T_2 are both false, the more verisimilar theory will be the one making more matches and less mistakes. Finally, an important consequence of the fact that verisimilitude is a combination of truth and content is that a false theory may be more verisimilar than a true one. In fact, a true theory may be too weak or 'cautious' to be highly verisimilar, whereas a very informative or 'bold' theory may be highly verisimilar, although false.

Example 3.2

Assume again that $C_\star \equiv p_1 \wedge \ldots \wedge p_n$. Then, in most cases, the false theory $T_1 \equiv p_1 \wedge \ldots \wedge p_{n-1} \wedge \neg p_n$ will be more verisimilar than the true theory $T_2 \equiv p_1$ since it provides us with much more (true) information about the world. Indeed, one can check that $Vs_\phi(T_1) > Vs_\phi(T_2)$ if and only if $\phi < n-2$ (for instance for $\phi=1$ and $n=4$).

[10] It is worth noting that, if T is not a constituent, it will typically commit, besides proper mistakes, also omissions or 'errors of ignorance' [49, p. 159] about C, corresponding to the atomic sentences p_i such that neither p_i nor $\neg p_i$ are basic claims of T. Such errors of ignorance may also be called the *lacunae* of T (cf. [30, p. 310], [3, p. 142] and [4, p. 157]). In (3.2), the lacunae of T are implicitly assigned a weight of 0, which is greater than the weight ($=-\phi$) for mistakes and smaller than the weight ($=1$) of matches of T; in other words, each lacuna is better than each mistake and worse than each match of T, as far as the assessment of T's similarity to C is concerned.

[11] One may note that measure Vs_ϕ is not normalized, and varies between $-\phi$ and 1. To obtain a normalized measure of the verisimilitude of T it is sufficient to define $Vs_\phi(T)$ as $(s_\phi(T,C_\star)+\phi)/(1+\phi)$, which varies between 0 and 1.

[12] Indeed, Vs_ϕ satisfies also the stronger requirement that among true theories, the one with the greater degree of (true) basic content is more verisimilar than the other; i.e. if T_1 and T_2 are true and $\text{cont}_t(T_1,C) > \text{cont}_t(T_2,C)$ then $Vs_\phi(T_1) > Vs_\phi(T_2)$.

Verisimilitude theorists did not fail to notice that in most interesting cases 'the truth' is simply unknown, so that the estimated verisimilitude of competing theories, not their verisimilitude, is the crucial point of interest. Accordingly, the theory of verisimilitude has traditionally been seen as addressing both a logical and an epistemic problem.[13] The logical problem of verisimilitude amounts to the preliminary definition of an appropriate measure of verisimilitude, like Vs_ϕ, allowing for a comparison of any two theories with regard to their closeness to the, supposedly known, truth. The epistemic problem of verisimilitude, on the other hand, amounts to the definition of an appropriate notion of estimated verisimilitude by which the estimated closeness to the unknown truth of any two theories could be compared on the basis of the available evidence. Niiniluoto has proposed a solution to the latter problem by defining a measure of 'expected verisimilitude' based on an underlying epistemic probability distribution P defined over the set of the constituents $C_1,\ldots,C_q$ of $\mathcal{L}_n$ and expressing the rational degrees of belief in the truth of each alternative C_i given the available evidence e. The expected degree of verisimilitude of T is then calculated by summing up the verisimilitude of T in each state multiplied by the corresponding probability given e (see [49, p. 269] and [58, p. 180]):

$$EVs_\phi(T|e) \overset{\mathrm{df}}{=} \sum_{C_i} s_\phi(T,C_i)P(C_i|e). \tag{3.5}$$

This definition provides a measure $EVs_\phi(T|e)$ of the degree of estimated closeness to the truth of theory T, given the evidence. It also allows us to compare two different theories T_1 and T_2 with regard to their estimated closeness to the truth, and to say, e.g., that theory T_1 seems in view of e more verisimilar than theory T_2, i.e. that it is reasonable to claim, given e, that T_1 is more verisimilar than T_1, when $EVs_\phi(T_1|e) > EVs_\phi(T_2|e)$.

4 Abductive belief change

The notion of verisimilitude introduced in the previous section allows one to analyse both abductive inference and belief change from the perspective of truth approximation. Niiniluoto [52, 53] and Kuipers [32], for instance, have studied the relations between abduction and verisimilitude under the assumption that abductive inference provides reasons to accept its conclusion as a truthlike explanatory hypothesis (cf. also [15, Sec. 2]). In this article, we shall follow a related but different route, focusing on the relations between truth approximation and abductive reasoning construed as a process of belief change [2, 4, 30, 62]. This amounts to study whether abductively changing one's beliefs does lead these beliefs closer to the truth. To this purpose, we shall first of all briefly survey the standard AGM theory of belief change (Section 4.1) and then introduce the notion of 'abductive belief change' as developed within that tradition (Section 4.2). In Section 5 we will then analyse the relations between abductive belief change, on the one hand, and truth approximation, on the other.

4.1 The AGM theory of belief change

The AGM theory studies how the beliefs of an ideally rational agent $\mathcal{A}$ should change in response to certain inputs coming from some information source. The beliefs accepted by $\mathcal{A}$ at any given time are represented by (the elements of) a *belief set* or theory T, which is a consistent and logically closed set of sentences of $\mathcal{L}_n$. A '(doxastic) input' is a sentence p of $\mathcal{L}_n$ that has to be added to, or removed

[13] See, e.g., [49, p. 263] and [50, p. 20]; for a critical discussion of this distinction, see [46, Chs. 10–11].

from, T, in such a way that the updated belief set is still consistent and as close as possible to T. The latter requirement expresses a basic methodological principle underlying the AGM approach, known as the principle of minimal change, according to which when T is updated in response to a given doxastic input, a 'minimal change' of T is accomplished. This means that $\mathcal{A}$ will keep believing as many of the old beliefs as possible and start to believe as few new beliefs as possible.[14]

Within the AGM theory, three change operations are defined by which a theory T is updated in response to an input p, called the 'expansion', the 'revision' and the 'contraction' of T by p. Suppose first that $\mathcal{A}$ has to add input p to T. If $\mathcal{A}$ already accepts p before receiving it—i.e. if $p \in T$—then $\mathcal{A}$'s appropriate response is keeping T unchanged. Otherwise, two possibilities arise, depending on whether p is logically compatible with T. If the input is compatible with the beliefs of $\mathcal{A}$—i.e. if $\neg p \notin T$—the operation by which $\mathcal{A}$ should update T by the addition of p is called expansion and the expanded belief set is denoted by '$T+p$'. Alternatively, if p is incompatible with T—i.e. if $\neg p \in T$—$\mathcal{A}$ has to *revise* T by p, and the revised belief set is denoted by '$T*p$'. In both expansion and revision, input p is added to T. The third AGM operation, contraction, is instead performed when agent $\mathcal{A}$ has to remove input p from T. In this case, if p does not belong to the beliefs of $\mathcal{A}$—i.e. if $p \notin T$—then $\mathcal{A}$ will keep T unchanged. Otherwise, $\mathcal{A}$ will contract T by p, and the resulting belief set will be denoted by '$T-p$'.

As far as expansion is concerned, there is no special difficulty in defining the corresponding operation in purely logical (or set-theoretical) terms, as follows:

Definition 4.1
Given a belief set T and an input p, the expansion of T by p is $T+p \stackrel{\text{df}}{=} \text{Cn}(T \cup \{p\})$,

where Cn is the operation of classical logical consequence.

The definition of revision and contraction is instead more problematic, since both operations require that some elements of T are withdrawn. Consequently, logic alone cannot fully determine the result of these operations, since there are in general many alternative ways to remove a given sentence from T. As Schurz [73, p. 218] notes, this is an instance of the well-known 'Duhem problem' that philosophers of science are familiar with. As an example, suppose that $T = \text{Cn}(q, q \rightarrow p)$ and that $\mathcal{A}$ has to remove p from T, i.e. to perform a contraction of T by p. There are at least *two* ways of performing the required change—i.e. $\mathcal{A}$ may remove either q or $q \rightarrow p$ (but not both, if the requirement of minimal change has to be fulfilled). The choice between these alternatives will typically depend on the relative 'importance' that $\mathcal{A}$ attaches to the sentences in T—i.e. on extra-logical considerations.

Note that this problem of choice does not arise only in the case of contraction, but also in that of revision; in fact, in order to revise T by a contradictory input p, $\mathcal{A}$ will first need to remove $\neg p$ from T before accepting p. This intuition, which goes back to [39], leads to the so-called Levi identity, i.e. to the definition of revision in terms of expansion and contraction:

$$T*p = (T - \neg p) + p. \tag{4.1}$$

In fact, if p is incompatible with T, i.e. if $\neg p \in T$, contracting T by $\neg p$ leads to the belief set $T - \neg p$ which is compatible with p, and can consistently be expanded by p: the result of these two-steps process is the required revision of T by p.

[14] This principle is variously known as the principle of 'informational economy' [18], of 'conservativity' [65], and of 'minimum mutilation' [66]. For a detailed critical examination of the underlying ideas and their application to the AGM theory, see [67].

In the last 30 years, AGM theorists have developed a number of formal definitions of revision and contraction, that we do not need to consider here. What is instead worth noting is a general assumption underlying the AGM account of belief change. When agent $\mathcal{A}$, with belief set T, receives the input p, it is assumed that $\mathcal{A}$ will always change T with p. In other words, $\mathcal{A}$ will never weigh p against the 'old' information in T in order to asses whether p is more or less valuable than that information. This is apparent, for instance, from Definition 4.1 of expansion, which implies that $p \in T+p$ (the so-called *Success* postulate). This means that the input has always priority over the belief set; for this reason, AGM belief change is known as 'prioritized' belief change. By 'non-prioritized' belief change is meant, instead, a process in which no special priority is assigned to the new information due to its novelty [22]. In the next section, non-prioritized change operations will be introduced.

4.2 A simple model of abductive belief change

The summary in the previous section should have made clear that, as Sven O. Hansson puts it, '[t]he standard [AGM] framework [...] is not suitable for analyzing the mechanisms of change in science' [21, p. 43], and needs important modifications in order to serve as a realistic account of belief change in scientific or even ordinary contexts [11, pp. 464–465]. To mention just one central problem, emphasized by Schurz [72, p. 80], AGM lacks 'learning ability' or 'epistemic creativity' in the sense that when a new piece of information is acquired, it is simply accommodated within the agent's belief set, under the only constraint of maintaining consistency. However, what actually happens in science seems to be very different. When a black swan is observed, the scientific corpus is not simply updated by removing the hypothesis that all swans are white, but scientists also try to figure out an alternative hypothesis explaining the evidence. Accordingly, studying abductive, or explanatory, operations of belief change appears as an important task in order to develop more adequate and realistic models of belief change.

The first model of this kind was introduced by Maurice Pagnucco in his (still unpublished) dissertation [62] and by Aliseda [2, 4] within AI. The basic idea is that, when agent $\mathcal{A}$ has to expand or revise the belief set T by input p, $\mathcal{A}$ does not simply add p to T (while maintaining overall consistency) but tries to *explain* p by adding to T also some hypothesis abduced from p. This amounts to defining two new kinds of belief change operations—i.e. 'abductive expansion' and 'abductive revision'—depending on whether p is or is not logically compatible with T.[15] Aliseda [4, pp. 183 ff.] discusses these operations in the following terms: an abductive expansion of T is triggered by an 'abductive *novelty*' (i.e. an input p such that neither p nor $\neg p$ belong to T), while an abductive revision is required in the presence of an 'abductive *anomaly*' (i.e. an input p such that $\neg p$ belongs to T). This terminology is close to Peirce's remark that the abductive process begins with the observation of a 'surprising' fact: indeed, p is surprising both when it is simply unknown to the agent (it is novel) and when it contradicts the beliefs of the agent (it is anomalous).

Before defining abductive expansion and revision, the notion of 'abduced hypothesis' (simply called 'abduction' and 'explanation' by Pagnucco and Aliseda, respectively) must be specified [62, p. 79]:

Definition 4.2
An *abduced hypothesis* for p (with respect to T) is any sentence h such that (i) $T \cup \{h\}$ entails p, and (ii) $T \cup \{h\}$ is consistent.

[15] Of course, since abduction is an *ampliative* pattern of reasoning, the notion of 'abductive contraction' does not seem to make any sense.

Any abduced hypothesis h is a potential explanation of p (with respect to T) in the (very weak) sense that, by adding h to T, $\mathcal{A}$ is then able to deduce p from his updated beliefs. This is the idea underlying the definition of the operation $\oplus$ of abductive expansion ([62, p. 102] and [4, p. 184]):

DEFINITION 4.3
Given a belief set T and an input p, the abductive expansion of T by p is

$$T \oplus p \stackrel{\text{df}}{=} \begin{cases} T+h & \text{if there is an abductive hypothesis } h \text{ for } p \\ T & \text{otherwise.} \end{cases}$$

Note that, if $\mathcal{A}$ is able to find an abductive hypothesis h for p, the abductive expansion of T *by* p is identical to the (standard AGM) expansion of T *by* h. In the opposite case, i.e. if no explanation for p in available given his beliefs T, $\mathcal{A}$ will simply refuse to perform the change and will keep believing T alone.[16] In this sense, abductive expansion is a non-prioritized belief change operation, since it may happen that $p \notin T \oplus p$ (and hence that the Success postulate fails). The same is true for the operation $\otimes$ of abductive revision, which is defined by the Levi identity (4.1) in a suitably modified form ([62, p. 161] and [4, p. 185]):

DEFINITION 4.4
Given a belief set T and an input p, the abductive revision of T by p is $T \otimes p \stackrel{\text{df}}{=} (T - \neg p) \oplus p$.

Note that if agent $\mathcal{A}$ does not believe $\neg p$, then $T \otimes p$ is just $T \oplus p$. If, however, p is a genuine anomaly, then $\mathcal{A}$ will first contract T by $\neg p$, in order to remove any obstacle to the abductive process, and then will search for some abductive hypothesis h explaining p, and add it to T.

Schurz has rightly noted [72, section 4.2.2] that the operations of abductive belief change just defined are no more adequate, as realistic models of scientific change, than the corresponding AGM operations of standard (non-abductive) expansion and revision. One reason for this is that all the requirements highlighted by the philosophical discussion, from Hempel [25] onward, on what does constitute a ‘good’ explanation of p with respect to T are usually neglected in the logical and AI-research ([72, p. 88]; for an exception, see [4, Ch. 5]). As an example, nothing in Definition 4.2 prevents one to choose, as an abductive hypothesis for p, p itself: but this would amount to trivially self-explaining p, since in this case $T \otimes p = T \oplus p = T + p$. Accordingly, additional conditions are needed in order to narrow down the range of possible abductive hypotheses for p (cf. also [26]). However, as for the Duhem problem of contraction and revision discussed in Section 4.1, also in this case purely logical considerations are insufficient to specify adequate conditions of this kind (cf. [72], [62, pp. 57 ff.] and [4, pp. 72–74].) In the next section, we will show how the notion of (expected) verisimilitude introduced in Section 3 may help in tackling this problem.

5 Belief change and truth approximation

Recently, the AGM theory has started attracting the attention of philosophers of science, as testified, for instance, by the publication of an entire collection on *Belief Revision Meets Philosophy of Science* [61]. An issue that remained unexplored in this work is whether the operations of belief change defined within the AGM theory are effective means to approach the truth about the underlying

[16] As Jesús Zamora Bonilla has noted in correspondence, identifying $T \oplus p$ with T in case no abductive hypothesis h for p is available amounts to the radical, and unrealistic, recommendation that $\mathcal{A}$ should simply ‘ignore the facts’ if $\mathcal{A}$ is unable to explain them. A different move would be to define $T \oplus p$ as $T + p$ if no abductive hypothesis h for p is available; to my knowledge, an operation of this kind has not yet been discussed in the literature.

domain. This question was first raised by Niiniluoto [51] and has been further explored by a number of scholars, mainly among the contributors to a special issue of *Erkenntnis* [36] entirely devoted to the topic of *Belief Revision Aiming at Truth Approximation*.[17] After presenting the main problems arising in this connection (Section 5.1), we shall analyse the relationships between abductive belief change and verisimilitude (Section 5.2) and suggest how to define operations of abductive expansion and revision aiming at truth approximation (Section 5.3).

5.1 Truth approximation via AGM belief change

Both the theory of verisimilitude and the AGM theory can be viewed as accounts of theory change in science [21, 51, 55]. Nonetheless, they stem from very different perspectives on the goals of rational inquiry. On one side, the notion of truth clearly retains a central role in the analysis of truth approximation. AGM theorists, by contrast, have traditionally regarded 'the concepts of truth and falsity' as 'irrelevant for the analysis of belief systems' ([18, p. 20]; see also [43]).[18] Niiniluoto was the first to ask whether and to what extent AGM belief change can track truth approximation conceived as a central cognitive goal of inquiry. This amounts to ask under what conditions AGM change operations lead agent $\mathcal{A}$'s beliefs closer to the truth than they were before the change.

At first sight, one may think that, when a belief set T is expanded or revised by a *true* input p, the resulting belief set should be closer to the truth than T. The surprising result of Niiniluoto [51] was exactly to prove that this does not hold in general, nor can be required as an adequacy condition for belief change operations or verisimilitude measures. Indeed, a closer look reveals that the addition of true inputs may well lead the agent's beliefs farther from the truth, as the following example shows (cf. [73, p. 210, example 4] and [37, p. 154]).

EXAMPLE 5.1
Assume, as before, that $C_\star \equiv p_1 \wedge \ldots \wedge p_n$ is the truth about a given domain of inquiry. Moreover, suppose that the belief set of agent $\mathcal{A}$ is $T = \mathrm{Cn}(p_1, p_2 \rightarrow \neg p_3 \wedge \neg p_4 \wedge \neg p_5)$: note that T is false, due to the false conditional accepted by $\mathcal{A}$. When $\mathcal{A}$ receives the input p_2, which does not contradict his beliefs, $\mathcal{A}$ will expand T by p_2, obtaining $T + p_2 = \mathrm{Cn}(p_1, p_2, \neg p_3, \neg p_4, \neg p_5)$. It follows that now $\mathcal{A}$ accepts one true belief (p_2) and three false beliefs ($\neg p_3, \neg p_4, \neg p_5$) on which he previously suspended the judgment. Most verisimilitude theorists would agree that, in this case, $T + p$ is likely *less* verisimilar than T, i.e. that the expansion by a true input led $\mathcal{A}$'s beliefs farther from the truth. In fact, while both T and $T + p$ are false, the latter contains more, and more 'serious', falsehoods than the former. Indeed, one can check that, as far as the contrast measure Vs_ϕ is concerned (see Section 3), since the basic contents of T and $T + p_2$ are, respectively, $b(T) = \{p_1\}$ and $b(T + p_2) = \{p_1, p_2, \neg p_3, \neg p_4, \neg p_5\}$, $Vs_\phi(T + p_2) = 2 - 3\phi/n < \frac{1}{n} = Vs_\phi(T)$ just in case $\phi > 1/3$.

Similar examples can be used to show that also the revision of a false T by a true p may lead $T * p$ farther from the truth than T. Moreover, they can be easily generalized: indeed, in all cases where $\mathcal{A}$ accepts a 'true-to-false implication' [73, p. 210] of the form $p \rightarrow q_1 \wedge \ldots \wedge q_k$, where p is true and the q_i are false, adding the new true belief p will force $\mathcal{A}$ to accept a possibly enormous number of new false beliefs.

[17] See in particular [9, 33, 56, 73]; further references include [6, 7, 10, 12, 14, 34, 55]. A survey of the main results obtained so far can be found in the introductory essay to the mentioned issue by Kuipers and Schurz [37].

[18] With some exceptions: Hans Rott, for instance, has warned that AGM theorists 'should worry more about truth' meant as one of the basic aims of scientific inquiry; see [67, pp. 513, 518 and ff., and in particular note 38]. More recently, Rott has however remarked that, within AGM, 'it is difficult to have truth as the goal of inquiry' [68, p. 60, note 2]. Cf. also [40, pp. 29–30].

TABLE 1. Truth tables for abductive expansion (left) and for abductive revision (right)

	T	p	h	$T \oplus p$		T	p	h	$T \otimes p$
1.	t	t	t	t	1.	*t*	*t*	*t*	*t*
2.	t	t	f	f	2.	*t*	*t*	*f*	*f*
3.	*t*	*f*	*t*	–	3.	*t*	*f*	*t*	–
4.	t	f	f	f	4.	t	f	f	f
5.	f	t	t	f	5.	f	t	t	f/t
6.	f	t	f	f	6.	f	t	f	f
7.	f	f	t	f	7.	f	f	t	f
8.	f	f	f	f	8.	f	f	f	f

Besides this general negative result, Niiniluoto further proved that the only 'safe case' in which belief change effectively tracks truth approximation is when a true input is added to a *true* belief set; more precisely [51, eq. 11]:[19]

$$\text{If both } T \text{ and } p \text{ are true, then } T+p \text{ is more verisimilar than } T. \qquad (5.1)$$

In this case, in fact, both T and $T+p$ are true, but the latter is more informative (in the sense that $T \subseteq T+p$), and hence more verisimilar, than the former. Thus, result (5.1) holds for all verisimilitude measure which—as Niiniluoto's favoured measure and the Vs_ϕ measure defined in Section 3—satisfy the Popperian requirement (3.4) according to which, among truths, verisimilitude increases with logical strength.

Unfortunately, expanding true theories by true inputs is a relatively uninteresting case of cognitive progress, corresponding to the very naïve view according to which $\mathcal{A}$ approaches the truth simply by accumulating more and more truths about the world—a view which is unacceptable as an adequate account of belief change aiming at truth approximation (cf. [13, section 5]). Indeed, a real agent (like a scientist) will normally entertain also false beliefs; and one of the reasons of revising these beliefs lies exactly in the hope to correct them and getting closer to the truth. In this connection, we will now study whether and under what conditions abductive expansion and revision effectively track truth approximation.

5.2 Truth approximation via abductive belief change

In abductive expansion and revision, agent $\mathcal{A}$ looks for an abductive hypothesis h explaining input p and, if such an h is found, adds it to T. Usually, the issue of whether the input or the abductive hypothesis are true is completely neglected in the logical and AI discussion. Table 1 summarizes the eight logical possibilities which arise when a true/false belief set T is, respectively, abductively expanded or revised by a true/false input p and a true/false abductive hypothesis h.[20]

As far as abductive expansion is concerned, it is easy to check that $T \oplus p$ is true if and only if both T and h (and hence p) are true (case 1), and false in all other cases. If T is true, $T \oplus p$ is false if either h or p is false. Note that even if both T and p are true, $\mathcal{A}$ may fail to find a true abductive

[19] Note that, if both T and p are true, then p cannot contradict T, since T cannot entail $\neg p$, which is false. It follows that the revision $T * p$ is identical to the expansion $T+p$, and so that (trivially) also $T * p$ is more verisimilar than T.

[20] Note that, in both the expansion and the revision table, the third row is excluded for logical reasons, since true T and h cannot entail a false p. Recall that, by Definition 4.2, h is such that $T \cup \{h\}$ entails p.

hypothesis for p, and so $T \oplus p$ may be false (case 2). Moreover, if T is false (cases 5–8), then $T \oplus p$ is also false, even if both p and h are true (case 5). This is obvious, since abductive expansion is a purely ampliative operation, that cannot remove any falsehoods in T.

The case of abductive revision is slightly more interesting under this respect. First of all, if both T and p are true, then p does not contradict T, and $T \otimes p$ collapses in the abductive *expansion* of T by p; thus cases 1 and 2 of the revision table are reduced to the corresponding two cases of the expansion table. For the remaining cases, the main difference with respect to abductive expansion concerns case 5, when a false T is abductively revised by a true p with a true h. In this case, it may happen that the contraction of T by a false $\neg p$ removes *all* falsehoods in T, leading to a true belief set $T - \neg p$; by expanding this belief set with a true h, one then obtains $T \otimes p$ which is true.[21] In sum, if T is false and both p and h are true, $T \otimes p$ may be either true or false.

The previous considerations already suggest that abductive belief change, like standard AGM belief change, is in general not effective for truth approximation. Indeed, the only positive result that can be proved in this connection is the 'abductive' version of Niiniluoto's theorem (5.1) for standard AGM expansion:[22]

$$\text{If both } T \text{ and } h \text{ are true, then } T \oplus p \text{ is more verisimilar than } T. \tag{5.2}$$

This result holds for all accounts of verisimilitude satisfying the Popperian requirement (3.4) according to which, among truths, verisimilitude co-varies with logical strength. If, however, T is false, then $T \oplus p$ and $T \otimes p$ may be less verisimilar than T *even if p is true*. This may depend on one of the following circumstances. First, the abductive hypothesis h for p may be false (case 6 of Table 1); second, even if h is true, h may be far from the truth. In both cases, the abductive change may lead $T \oplus p$ and $T \otimes p$ farther from the truth than T.

5.3 Abductive belief change, (expected) verisimilitude and IBE

In the previous section, we have discussed the relationships between abductive belief change and truth approximation as if the truth values of theory T and input p were known. Although this assumption is useful for the purposes of conceptual analysis, as empashized by Niiniluoto [56, p. 177] practical rules of belief change cannot be directly based upon absolute truth values, since in typical situations these are simply unknown. In other words, in real-life situations, including everyday reasoning and scientific inference, one has to operate under uncertainty and risk.

When approximation to the truth is construed as the aim of inquiry, expected verisimilitude (as defined in Section 3) appears as a suitable candidate notion in order to define operations of (abductive) belief change in the service of truth approximation [56, Sec. 7, in particular p. 178]. In fact, suppose that agent $\mathcal{A}$ accepts theory T and receives input p. If $\mathcal{A}$ aims at having highly verisimilar beliefs about the world, $\mathcal{A}$ will try to find an explanation for p which will increase the verisimilitude of his beliefs, as assessed on the basis of his current beliefs in T (representing the

[21] A simple example is the following. Suppose that the language contains only two atomic sentences, p and q, and that they are both true. Moreover, assume that $T = \text{Cn}(\neg p)$ is a false belief set which is abductively revised by the true input p. Then, T must be first contracted by $\neg p$: one possible result is $T - \neg p = \text{Cn}(\neg p \vee q)$, which is true. As one can check, an abductive hypothesis h for p is $h = p \wedge q$; it follows that $T \otimes p = (T - \neg p) + p \wedge q = \text{Cn}(p \wedge q)$, which is, again, true (actually, it is *the truth* in the language).

[22] The proof is straightforward. If T and h are true, then also p is true, since $T \cup \{h\}$ entails p by Definition 4.2. It follows that $T \otimes p = T \oplus p = T + h$ is also true. Since $T \subseteq T + h$ by definition, $T + h$ is more verisimilar than T for all measures satisfying the Popperian requirement (3.4) discussed in Section 3.

'evidence' available to $\mathcal{A}$ up to this moment). This idea underlies the following definition of the operation $\oplus_{vs}$ of *verisimilitude-guided (vs-guided) abductive expansion*:

DEFINITION 5.2
Given a belief set T and an input p, the vs-guided abductive expansion of T by p is

$$T \oplus_{vs} p \stackrel{\text{df}}{=} \begin{cases} T+h & \text{if there is an abductive hypothesis } h \text{ for } p \text{ such that} \\ & EVs(T+h|T) > EVs(T|T) \\ T & \text{otherwise.} \end{cases}$$

In words, when agent $\mathcal{A}$ receives input p, $\mathcal{A}$ looks for an abductive hypothesis h for p such that the expected verisimilitude of his 'future' beliefs, given his current beliefs T, will be increased once h (and hence p) will be added to T. If such a h exists, then $\mathcal{A}$ will expand T to $T \oplus_{vs} p$ which, by definition, is expected to be closer to the truth than T.

Interestingly, the operation $\otimes_{vs}$ of *vs-guided abductive revision* cannot be defined using the Levi identity and the corresponding operation of vs-guided abductive expansion. Consider, in fact, the following definition:

$$T \otimes_{vs} p \stackrel{\text{df}}{=} (T - \neg p) \oplus_{vs} p. \tag{5.3}$$

This definition would have the following, unwelcome consequence. Suppose that $\mathcal{A}$ accepts $\neg p$ and receives input p. Then, $\mathcal{A}$ will contract T by $\neg p$, obtaining as a result belief set $T - \neg p$. Now, $\mathcal{A}$ will try to perform a vs-guided abductive expansion of $T - \neg p$ by p. Here, two problems may arise. First, even if successful in finding an abductive hypothesis h such that $EVs((T - \neg p) + h|T) > EVs(T - \neg p|T)$, it may well happen that $EVs((T - \neg p) + h|T) < EVs(T|T)$, i.e. that the expected verisimilitude of $\mathcal{A}$'s beliefs decreases, instead of increasing, after the change. Second, if such an h does not exist, then the result of the change will be $T - \neg p$ instead of the original T: in other words, $\mathcal{A}$ will loose some beliefs only because $\mathcal{A}$ is unable to find a satisfactory explanation for the input. In both cases, the result of the operation defined in (5.3) will be unacceptable. This suggests that, more generally, all adequate operations of vs-guided abductive expansion and revision cannot satisfy the Levi identity.[23] The following definition avoids the above problems:

DEFINITION 5.3
Given a belief set T and an input p, the vs-guided abductive revision of T by p is

$$T \otimes_{vs} p \stackrel{\text{df}}{=} \begin{cases} (T - \neg p) + h & \text{if there is an abductive hypothesis } h \text{ for } p \\ & \text{such that } EVs((T - \neg p) + h|T) > EVs(T|T) \\ T & \text{otherwise.} \end{cases}$$

In other words, a vs-guided abductive revision of T by p amounts to the abductive revision $T \otimes p$ of T by p if this is expected to increase the verisimilitude of T, and to T otherwise.

In his study of IBE, Peter Lipton [41, in particular pp. 59 and 148 ff.] discusses abductive reasoning as a two-stage process: first, a (short) list of abductive hypotheses is generated; second, one is selected as the most promising one. This raises two different problems concerning abduction, which may be called the generation and the selection problem.[24] Definitions 5.2 and 5.3 can be viewed as answers

[23] Schurz [72, section 4.3.2] discusses at some length the reasons why the Levi identity fails for abductive expansion and revision.

[24] Cf. also Aliseda [4, pp. 72–74] and Schurz [72, p. 93]. Discussing Aliseda's analysis of abductive reasoning, Valeriano Iranzo [26, p. 340] calls these two problems the 'logical' and 'epistemological' problems of abduction, respectively.

to the first, generation problem. Indeed, in those definitions assessments of verisimilitude are used as a 'filter' to narrow down the number of promising explanatory hypotheses in the light of the purpose of truth approximation. Not all abductive hypotheses in the sense of Definition 4.2 are good candidate explanations, but only those that, once accepted in the current theory, increase the expected verisimilitude of the final, revised theory. Note that, in general, many abductive hypotheses may fulfill this condition; thus, given input p and theory T, one will normally be confronted with a multiplicity of competing vs-guided abductive expansions and revisions of T by p. In other words, these operations represent a kind of inference, as it were, to the *good* explanations.

To model inference to the *best* explanation in our framework, one needs to tackle the second, selection problem of abduction: i.e. how to compare the competing abductive hypotheses and select one of them as the most promising one. A solution to this problem, again based on assessments of the relative expected verisimilitude of competing explanations, suggests itself. Let us consider only the case of abductive expansion, that of abductive revision being similar under all relevant respects. Suppose that both h and h' are 'good' abductive hypotheses for p with respect to T in the sense of Definition 5.2, i.e. that $EVs(T+h|T) > EVs(T|T)$ and $EVs(T+h'|T) > EVs(T|T)$. Then one can say that h is *better* than h' (as an abductive hypothesis for p with respect to T) if and only if $EVs(T+h|T) > EVs(T+h'|T)$. In other words, good abductive hypotheses are naturally ordered with respect to their 'goodness', depending on the relative expected verisimilitude of the different expansions of T by p that they license. In turn, h is the best abductive hypothesis if and only if $EVs(T+h|T)$ is not only greater than $EVs(T|T)$ but also greater than $EVs(T+h'|T)$ for each alternative good abductive hypothesis h'. This idea provides what may me called a 'verisimilitudinarian' version of IBE.

To conclude, when abduction is construed as a process of belief change, the question of whether explanatory reasoning effectively tracks truth approximation can be rigorously analysed. In particular, we have shown that (vs-guided) abductive expansion and revision can lead the beliefs of agent $\mathcal{A}$ closer to the truth, when appropriate conditions are placed on the choice of the relevant explanatory hypotheses. It must be noted, however, that Definitions 5.2 and 5.3 immediately implies that the expected verisimilitude of $\mathcal{A}$'s beliefs is increased by those operations. Thus, one might repeat Laudan's charge of 'trivialization' against the very idea of truth approximation via (abductive) belief change. To answer this charge, an account is needed of the ideal conditions under which abductive belief change actually increases the verisimilitude of our theories. As an example, Niiniluoto has shown that convergence to the truth is guaranteed when available evidence is progressively revised with new, incoming data, and, in the limit, is eventually true and fully informative about the various kinds of individuals of the domain under inquiry [56, p. 179]. Further work is needed to explore whether, and to what extent, results of this kind are available as far as verisimilitude-guided abductive belief change is concerned.

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