Strongly Semantic Information as Information About the Truth

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Abstract Some authors, most notably Luciano Floridi, have recently argued for a notion of "strongly" semantic information, according to which information "encapsulates" truth (the so-called "veridicality thesis"). We propose a simple framework to compare different formal explications of this concept and assess their relative merits. It turns out that the most adequate proposal is that based on the notion of "partial truth", which measures the amount of "information about the truth" conveyed by a given statement. We conclude with some critical remarks concerning the veridicality thesis in connection with the role played by truth and information as relevant cognitive goals of inquiry.

Keywords (Strongly) Semantic information · Truth · Misinformation · Veridicality thesis · Verisimilitude · Truthlikeness · Partial truth · Informative truth · Cognitive decision theory

1 Introduction

In recent years, philosophical interest in the concept of information and its logical analysis has been growing steadily (cf., e.g., [1, 2, 9, 13, 14, 22]). Philosophers and logicians have proposed various definitions of (semantic) information, and tried to elucidate the connections between this notion and related concepts like truth, probability, confirmation, and truthlikeness. While classical accounts, both in philosophy [4] and in (mathematical) information theory [32], define information in terms of (im)probability, more recent proposals try to link together information and truth (see, in particular, [10]). According to these proposals, the classical notion of

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information should be replaced, or at least supplemented with, a notion of "strongly semantic" information (henceforth, SSI), construed as well-formed, meaningful and "veridical" or "truthful" data about a given domain. This so-called "veridicality thesis" would imply "that 'true information' is simply redundant and 'false information', i.e., misinformation, is merely pseudo-information" [13, p. 82].[1] In this paper, we shall survey different formal explications of SSI, explore their conceptual relationships, and highlight their implications for the debate about the veridicality thesis triggered by Floridi's definition of SSI.

In Sect. 2, we review the "classical" definition of semantic information due to Carnap and Bar-Hillel [4] in the light of the critiques that it has received. The notion of SSI, and the related veridicality thesis, is discussed in Sect. 3. In Sect. 4, we survey three recent proposals, including Floridi's one, that define SSI in terms of different combinations of truth and information. We introduce a simple framework which allows us to compare these proposals, and argue in favor of one of them, which identifies SSI with the amount of information about "the truth" conveyed by a given statement. On this basis, in Sect. 5 we conclude that, in order to define sound notions of (true) information and misinformation, one can safely dispense with the veridicality thesis, that can be however accepted as a thesis concerning the epistemic goals guiding rational inquiry and cognitive decision making.

2 Information and Truth

The classical theory of (semantic) information [4, 32] is based on what Jon Barwise [1, p. 491] has called the "Inverse Relationship Principle" (IRP), i.e., the intuition that "eliminating possibilities corresponds to increasing information" [1, p. 488].[2] If A is a statement in a given language, IRP amounts to say that the information content of A can be represented as the set of (the linguistic descriptions of) all the possible state of affairs or "possible worlds" which are excluded by, or incompatible with, A. Accordingly, the amount of information conveyed by A will be proportional to the cardinality of that set. For the sake of simplicity, let us consider a finite propositional language $\mathscr{L}_n$ with n logically independent atomic sentences $p_1, \ldots, p_n$.[3] An atomic sentence p_i and its negation $\neg p_i$ are called "basic sentences" or "literals" of $\mathscr{L}_n$. Within $\mathscr{L}_n$, possible worlds are described by the so-called constituents of $\mathscr{L}_n$, which are conjunctions of n literals, one for each atomic sentence. Note that the set $\mathscr{C}$ of the constituents of $\mathscr{L}_n$ includes $q = 2^n$ elements and that only one of them, denoted by "$C_\star$", is true; thus, $C_\star$ can be construed as "the (whole) truth" in $\mathscr{L}_n$, i.e., as the complete true description of the actual world w.

[1] As a terminological remark, note that while "misinformation" simply denotes false or incorrect information, "disinformation" is false information deliberately intended to deceive or mislead.

[2] This idea can be traced back at least to Karl Popper [27, in particular Sects. 34 and 35, and Appendix IX, p. 411, footnote 8]; cf. also [3, p. 406].

[3] Nothing substantial, in what follows, depends on such assumption.

Given an arbitrary statement A of $\mathscr{L}_n$, let $R(A)$ be the the "range" of A, i.e., the set of constituents which entail A, corresponding to the set of possible worlds in which A is true (cf. [3, Sect. 18]). Then, the (semantic) information content of A is defined as:

$$Cont(A) \stackrel{\text{df}}{=} \mathscr{C} \setminus R(A) = R(\neg A). \tag{1}$$

A definition of the amount of information content of A can be given assuming that a probability distribution p is defined over the sentences of $\mathscr{L}_n$ [4, p. 15]:

$$cont(A) \stackrel{\text{df}}{=} 1 - p(A) = p(\neg A). \tag{2}$$

In agreement with IRP, the information conveyed by A is thus inversely related to the probability of A.[4]

Two immediate consequences of definitions in 1 and 2 are here worth noting. First, if $\top$ is an arbitrary logical truth of $\mathscr{L}_n$, then

$$Cont(\top) = \varnothing \text{ and } cont(\top) = 0 \tag{3}$$

since $\top$ is true in all possible worlds ($R(\top) = \mathscr{C}$) and hence $p(\top) = 1$. Second, if $\bot$ is an arbitrary logically false statement of $\mathscr{L}_n$, then

$$Cont(\bot) = \mathscr{C} \text{ and } cont(\bot) = 1 \tag{4}$$

since $\bot$ is false in all possible worlds ($R(\bot) = \varnothing$) and hence $p(\bot) = 0$. In short, tautologies are the least informative, and contradictions the most informative, statements of $\mathscr{L}_n$.

As D'Agostino and Floridi [7, p. 272] note, results 3 and 4 point to "two main difficulties" of the classical theory of semantic information as based on IRP. The first one is what Hintikka [20, p. 222] called "the scandal of deduction": since in classical deductive logic conclusion C is deducible from premises $P_1, \ldots, P_n$ if and only if the conditional $P_1 \wedge \cdots \wedge P_n \rightarrow C$ is a logical truth, to say that tautologies are completely uninformative is to say that logical inferences never yield an increase of information. This is another way of saying that deductive reasoning is "non-ampliative", i.e., that conclusion C conveys no information besides that contained in the premises P_i.[5] In this paper, we shall be concerned only with the second difficulty, called "the Bar-Hillel-Carnap semantic Paradox (BCP)" by Floridi [10, p. 198].

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[4] In the literature, it is usual to say that Eqs. (1) and (2) define the (amount of) "substantive information" or "information content" of A, as opposed to the "unexpectedness" or "surprise value" of A, which is defined as $\inf(A) = -\log p(A)$ [4, p. 20]. On this distinction, see for instance Hintikka [18, p. 313] and Kuipers [22, p. 865].

[5] To hush up this "scandal", Hintikka [19] developed a distinction (for polyadic first-order languages) between "depth" and "surface" information, according to which logical truths may contain a positive amount of (surface) information (cf. also [31]).

Already Carnap and Bar-Hillel [4, pp. 7–8], while defending result 4 as a perfectly acceptable consequence of their theory, pointed out that it is *prima facie* counterintuitive:

> It might perhaps, at first, seems strange that a self-contradictory sentence, hence one which no ideal receiver would accept, is regarded as carrying with it the most inclusive information. It should, however, be emphasized that semantic information is here not meant as implying truth. A false sentence which happens to say much is thereby highly informative in our sense. Whether the information it carries is true or false, scientifically valuable or not, and so forth, does not concern us. A self-contradictory sentence asserts too much; it is too informative to be true.

As made clear in the above quote, BCP follows from the assumption that truth and information are independent concepts, in the sense that A doesn't need to be true in order to be informative—the so-called assumption of "alethic neutrality" (AN) [11, p. 359].[6]

Many philosophers (e.g. [9, pp. 41 ff]) have noted that AN is at variance with the ordinary use of the term "information", which is often employed as a synonym of "true information". In fact, we are used to say that "[a] person is 'well-informed' when he or she knows much—and thereby is aware of many truths" [24, p. 155]. On the other hand, AN appears more acceptable as far as other common uses of this term are concerned, for instance when we speak of the "information" processed by a computer [24]. Thus, linguistic intuitions are insufficient to clarify the question whether information and truth are or not independent. Some scholars, most notably Luciano Floridi [10–13], have forcefully argued that AN should be rejected in favor of the so-called "veridicality thesis" (VT), according to which genuine information has to be (at least approximately) true. According to VT, BCP would be solved since contradictions, being a paradigmatic case of false statements, are not informative at all. What exactly VT implies for the classical definition of semantic information is however unclear, and will be discussed in the next section.

3 What is "Strongly" Semantic Information?

Under its weakest reading, VT simply says that truth and information are not independent concepts, as the classical theory of semantic information assumes, and that an adequate theory of strongly semantic information (SSI) has to take both concepts into account. According to Floridi, VT says, more precisely, that "information encapsulates truth" [10, p. 198] in the sense that A has to be "truthful" [11, p. 366] or "veridical" [13, p. 105] in order to be informative at all. The underlying intuition is expressed by Dretske [9, p. 44–45] in these terms:

[6] This does not mean that this assumption is the only culprit. As noted during discussion at the *Trends in Logic XI* conference, a way of avoiding BCP would be to adopt a non-classical logic according to which contradictions do not entail everything and hence are not maximally informative. Systems of this kind are provided by those "connexive logics" that reject the classical principle *ex contradictione quodlibet* (for all A, $\perp$ entails A) in favor of the (Aristotelian) intuition that *ex contradictione nihil sequitur* (cf. [33, Sect. 1.3]).

> If everything I say to you is false, then I have given you no information. At least I have given you no information of the kind I purported to be giving. [...] In this sense of the term, *false* information and *mis*-information are not kinds of information—any more than decoy ducks and rubber ducks are kinds of ducks.

Taking this idea at face value, VT would imply that only true statements are informative. In turn, this would amount to define SSI as follows (cf. [12, p. 40]):

$$A \text{ is a piece of SSI iff } A \text{ is true.} \tag{5}$$

By definition, A is true iff $C_\star \in R(A)$. The (amount of) SSI conveyed by A would be still defined by $Cont(A)$ and $cont(A)$, but only true statements would be allowed to occur within (1) and (2).

Condition (5) is a straightforward formulation of the thesis that "information encapsulates truth", but it is doubtful that supporters of VT would be ready to subscribe to it. In fact, it implies that *all* false statements are plainly uninformative. As a consequence, (5) provides a solution of BCP, but too a strong one, which is at least as counterintuitive as BCP itself. In fact, both in science and in ordinary contexts any piece of information at disposal is arguably at best approximate and, strictly speaking, false. For example, if one says that "Rudolf Carnap was an influential philosopher of science born in Germany in 1890", while the correct date of birth is 1891, it seems strange to say that this false statement conveys the same amount of information as "Carnap was German or not German" and "Carnap was German and not German", i.e., no information at all. Examples of this kind seems sufficient to exclude (5) as a possible definition of SSI.

Sequoiah-Grayson [30] has argued that the crucial intuition underlying the notion of SSI is that A has to provide some "factual" or "contingent" information to count as a piece of information at all. This would amount to define SSI in terms of the following "contingency requirement" [30, p. 338]:

$$A \text{ is a piece of SSI iff } A \text{ is factual.} \tag{6}$$

Note that A is factual or contingent iff $\varnothing \neq R(A) \subset \mathscr{C}$. According to this view, only tautologies and contradictions are completely uninformative. Thus, BCP is solved by defining SSI not as true, but as factual information. It follows that both false and true contingent statements are informative after all. In particular, as Floridi [10, p. 206] notes[7]:

> two [statements] can both be false and yet significantly more or less distant from the event or state of affairs w about which they purport to be informative, e.g. "there are ten people in the library" and "there are fifty people in the library", when in fact there are nine people in the library. Likewise, two [statements] can both be true and yet deviate more or less significantly from w, e.g. "there is someone in the library" versus "there are 9 or 10 people in the library".

[7] In the following, we replace, without any significant loss of generality, Floridi's talk of "infons"—"discrete items of factual information qualifiable in principle as either true or false, irrespective of their semiotic code and physical implementation" [10, p. 199]—by talk of sentences or statements in the given language $\mathscr{L}_n$.

> This implies that a falsehood with a very low degree of discrepancy may be pragmatically preferable to a truth with a very high degree of discrepancy [28].

The above quotation highlights the strict link between the notion of SSI and that of verisimilitude or truthlikeness, construed, in Popperian terms, as similarity or closeness to "the whole truth" about a given domain (cf. also [2, pp. 90–91]) . The idea of defining SSI in terms of verisimilitude has been indeed proposed (*ante litteram*) by Frické [15] and independently by D'Alfonso [8]; indeed, as Frické [15, p. 882] notes, this proposal explains how true and false statements can be both informative:

> With true statements, verisimilitude increases with specificity and comprehensiveness, so that a highly specific and comprehensive statement will have high verisimilitude; such statements also seem to be very informative. With false statements, verisimilitude is intended to capture what truth they contain; if false statements can convey information, and the view taken here is that they can, it might be about those aspects of reality to which they approximate. Verisimilitude and a concept of information appear to be co-extensive.

Thus, the amount of SSI that a (true or false) contingent statement A conveys will depend on how good an approximation A is to the actual world w (or to the true constituent $C_\star$). To make this idea precise, Floridi [10, Sect. 5, pp. 205–206 in particular] has proposed five conditions that an adequate notion of SSI should fulfil. Departing a little from Floridi's original formulation, they can be phrased as follows:

(SSI1) the true constituent $C_\star$ is maximally informative, since it is the complete true description of the actual world w
(SSI2) tautologies are minimally informative, since they do not convey factual information about w
(SSI3) contradictions are minimally informative, since they do not convey, so to speak, valuable information about w
(SSI4) false factual statements are more informative than contradictions
(SSI5) true factual statements are more informative than tautologies.

Note that SSI3 is required in order to avoid BCP. From SSI1 and SSI5 it follows that $C_\star$ is the most informative statement among all factual truths. Another requirement that can be defended as an adequacy condition for a notion of SSI is the following:

(SSI6) some false factual statements may be more informative than some true factual statements.

In fact, as made clear by Floridi's quotation above, a false statement may be a better approximation to the truth about w than a true one. Characterizing SSI by means of requirements SSI1–6 still leaves open the problem of how to define a rigorous counterpart of this notion, and in particular of how to quantify the amount of SSI conveyed by different statements. In the next section, we shall review and compare different measures of SSI proposed in the literature to address this issue.

4 A Basic Feature Approach to Strongly Semantic Information

Some authors have recently proposed different formal explications of the notion of SSI, in the form of measures of the degree or amount of SSI conveyed by statements of $\mathscr{L}_n$ [5, 8, 10, 13]. According to all these proposals, the degree of SSI of A is high, roughly, when A conveys much true information about w. A simple way of clarifying and comparing these measures is given by the so-called "basic feature" approach to verisimilitude [6] or "BF-approach" for short.

4.1 The Basic Feature Approach to Verisimilitude

According to the BF-approach, the "basic features" of the actual world w are described by the basic sentences or literals of $\mathscr{L}_n$. A conjunctive statement, or c-statement for short, is a consistent conjunction of k literals of $\mathscr{L}_n$, with $k \leq n$.[8] The "basic content" of a c-statement A is the set $b(A)$ of the conjuncts of A: each member of this set will be called a "(basic) claim" of A. One can check that $\mathscr{L}_n$ has exactly 3^n c-statements, including the "tautological" c-statement with $k = 0$ and the 2^n constituents with $k = n$. Indeed, note that $C_\star$ itself is a c-statement, being the conjunction of the true basic sentences in $\mathscr{L}_n$, i.e., the most complete true description of the basic features of w.

When A is compared to $C_\star$, $b(A)$ is partitioned into two subsets: the set $t(A, C_\star)$ of the true claims of A and the set $f(A, C_\star)$ of the false claims of A. Let us call each element of $t(A, C_\star)$ a *match*, and each element of $f(A, C_\star)$ a *mistake* of A. Note that A is true when $f(A, C_\star) = \varnothing$, i.e., when A doesn't make mistakes, and false otherwise. Moreover, A is "completely false" when $t(A, C_\star) = \varnothing$, i.e., when A makes *only* mistakes. For the sake of notational simplicity, let us introduce the symbols k_A, t_A, and f_A to denote, respectively, the number of claims, of matches, and of mistakes, of A—i.e., the cardinalities of $b(A)$, $t(A, C_\star)$, and $f(A, C_\star)$, respectively. The *degree* of basic content $cont_b(A)$, of true basic content $cont_t(A, C)$, and of false basic content $cont_f(A, C)$, of A is defined as follows:

$$cont_b(A) \stackrel{\text{df}}{=} \frac{k_A}{n} \quad \text{and} \quad cont_t(A, C_\star) \stackrel{\text{df}}{=} \frac{t_A}{n} \quad \text{and} \quad cont_f(A, C_\star) \stackrel{\text{df}}{=} \frac{f_A}{n} \tag{7}$$

i.e., as the normalized number of claims, of matches, and of mistakes, made by A.

The number of matches of A divided by the total number of its claims represents an adequate measure for the (degree of) "accuracy" $acc(A)$ of a c-statement A:

$$acc(A) \stackrel{\text{df}}{=} \frac{t_A}{k_A} = \frac{cont_t(A, C_\star)}{cont_b(A)}. \tag{8}$$

[8] In logical parlance, a c-statement is a statement in conjunctive normal form such that each of its clauses is a single literal. Following Oddie [26, p. 86], a c-statement may be also called a "quasi-constituent", since it can be conceived as a "fragment" of a constituent.

Conversely, the (degree of) "inaccuracy" of A can be defined as

$$inacc(A) \stackrel{\text{df}}{=} \frac{f_A}{k_A} = \frac{cont_f(A, C_\star)}{cont_b(A)}. \tag{9}$$

As one can check, $inacc(A) = 1 - acc(A)$. Moreover, if A is true, then $t_A = k_A$ and $acc(A)$ receives its maximum value, i.e., 1; conversely, $inacc(A)$ is 0. When A is completely false, $acc(A) = 0$ and $inacc(A) = 1$. In sum, all true c-statements are maximally accurate, while all completely false c-statements are maximally inaccurate.

As Popper notes, the notion of verisimilitude "represents the idea of *approaching comprehensive truth*. It thus combines truth and content" [28, p. 237, emphasis added].[9] Thus, accuracy is only one "ingredient" of verisimilitude, the other being (information) content. In other words, we may say that a c-statement A is highly verisimilar if it says many things about the target domain, and if many of these things are true; in short, if A makes many matches and few mistakes about w. This intuition is captured by the following "contrast measure" of the verisimilitude of c-statements A [6, p. 188]:

$$vs_\phi(A) \stackrel{\text{df}}{=} cont_t(A, C_\star) - \phi cont_f(A, C_\star) \tag{10}$$

where $\phi > 0$.[10] Intuitively, different values of ϕ reflect the relative weight assigned to truths and falsehoods, i.e., to the matches and mistakes of A. Some interesting feature of this definition are the following. First, while all true A are equally accurate, since $acc(A) = 1$, they may well vary in their relative degree of verisimilitude. More precisely, vs_ϕ satisfies the Popperian requirement that verisimilitude co-varies with logical strength among truths[11]

$$\text{If } A \text{ and } B \text{ are true and } A \text{ entails } B, \text{ then } vs_\phi(A) \geq vs_\phi(B). \tag{11}$$

Thus, logically stronger truths are more verisimilar than weaker ones. This condition, however, doesn't hold amongst false statements, since logically stronger falsehoods may well lead us farther from the truth. In particular[12]

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9 For different accounts of verisimilitude, see [21, 24, 26, 29].

10 One may note that measure vs_ϕ is not normalized, and varies between $-\phi$ and 1. A normalized measure of the verisimilitude of A is $(vs_\phi(A) + \phi)/(1 + \phi)$, which varies between 0 and 1.

11 *Proof* note that, among c-statements, A entails B iff $b(A) \supseteq b(B)$. If both are true, this implies $t(A, C_\star) \supseteq t(B, C_\star)$ and hence $vs_\phi(A) = cont_t(A, C_\star) \geq cont_t(B, C_\star) = vs_\phi(B)$. For discussion of this Popperian requirement, see [24, pp. 186–187, 233, 235–236]. Note also that vs_ϕ satisfies the stronger requirement that among true theories, the one with the greater degree of (true) basic content is more verisimilar than the other; i.e., if A and B are true and $cont_t(A, C) > cont_t(B, C)$ then $vs_\phi(A) > vs_\phi(B)$. □

12 *Proof* if A entails B and both are completely false, then $f(A, C_\star) \supseteq f(B, C_\star)$ and hence $cont_f(A, C_\star) \geq cont_f(B, C_\star)$. Since ϕ is positive, it follows that $vs_\phi(A) = -\phi cont_f(A, C_\star) \leq -\phi cont_f(B, C_\star) = vs_\phi(B)$. □

$$\text{If } A \text{ and } B \text{ are completely false and } A \text{ entails } B, \text{ then } vs_\phi(A) \leq vs_\phi(B). \quad (12)$$

In words, logically *weaker* complete falsehoods are better than stronger ones. Finally, note that a false c-statement may well be more verisimilar than a true one; however, completely false c-statements are always less verisimilar than true ones.[13]

4.2 Quantifying Strongly Semantic Information

Measures of true (basic) content, of (in)accuracy, and of verisimilitude, or combinations of them, have all been proposed as formal explicata of the notion of SSI. For instance, according to Floridi [10], SSI may be construed as a combination of content and accuracy. More precisely, among truths, SSI increases with content, or, better, it decreases with the degree of "vacuity" of A, defined as the normalized cardinality of the range of A:

$$vac(A) = \frac{|R(A)|}{|\mathscr{C}|}. \quad (13)$$

Among falsehoods, SSI increases with accuracy, and decreases with inaccuracy. Moreover, $C_\star$ is assigned the highest degree of SSI, and contradictions the lowest. In sum, Floridi's measure of SSI is defined as follows [10, pp. 208–210]:

$$cont_S(A) \overset{\text{df}}{=} \begin{cases} 1 - vac(A)^2 & \text{if } A \text{ is true and } A \not\equiv C_\star \\ 1 - inacc(A)^2 & \text{if } A \text{ is factually false} \\ 1 & \text{if } A \equiv C_\star \\ 0 & \text{if } A \text{ is contradictory.} \end{cases} \quad (14)$$

One can easily check that Floridi's measure satisfies all requirements SSI1–6 (SSI1 and SSI3 are fulfilled by stipulation). In particular, since tautologies have maximal degree of vacuity, their degree of SSI is 0 (cf. SSI2). Moreover, $cont_S$ satisfies the Popperian requirement (11): if A and B are true and A entails B, then $cont_S(A) \geq cont_S(B)$.

A simpler formulation of (14) is given by the following measure:

$$cont^*_S(A) \overset{\text{df}}{=} \begin{cases} cont_b(A) = cont_t(A, C_\star) & \text{if } A \text{ is true} \\ acc(A) & \text{if } A \text{ is false.} \end{cases} \quad (15)$$

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[13] *Proof* If A is true, then $vs_\phi(A) = cont_t(A, C_\star)$; if B is completely false, then $vs_\phi(B) = -\phi cont_f(B, C_\star)$; since ϕ is positive, it follows that $vs_\phi(A) > vs_\phi(B)$. □

One can check that $cont^*_S$ and $cont_S$ are ordinally equivalent in the sense that, given any two c-statements A and B, $cont^*_S(A) \gtreqless cont^*_S(B)$ iff $cont_S(A) \gtreqless cont_S(B)$.[14] Thus, also the $cont^*_S$ measure satisfies requirements SSI1–6.[15]

According to (15), $cont^*_S$ increases with the degree of (true) basic content among truths, and with accuracy among falsehoods. In this connection, it may be worth noting that $acc(A)$ is a straightforward measure of the "approximate truth" of A, construed as the closeness of *A to being true* [24, pp. 177, 218]. In fact, if A is true, then $acc(A) = 1$; while if A is false, then $acc(A)$ is smaller than 1 and increases the closer A is to being true. Recalling that, if A is completely false, $inacc(A) = 1$, *inacc* can be construed as a measure of the closeness of *A to being completely false*. Since all completely false c-statements are equally (and maximally) inaccurate, if A is completely false then $cont_S(A) = 0$, i.e., A conveys no SSI about w.

Following the idea that SSI is a combination of content and accuracy, D'Alfonso [8] has proposed to use measures of verisimilitude for quantifying SSI. An immediate advantage is that a unique measure vs_ϕ is used to assess the degree of SSI of both true and false statements. One can check that vs_ϕ satisfies requirements SSI1—$vs_\phi(C_\star)$ is maximal, since "nothing is as close as the truth as the whole truth itself" [26, p. 11]—, and SSI4–6. Moreover, since vs_ϕ is undefined for contradictions, one can just stipulate that their degree of SSI is 0, in agreement with SSI3 (but see [8, p. 77]) . However, as D'Alfonso acknowledges [8, p. 73], vs_ϕ violates SSI2, since tautologies are more verisimilar than some false statements. In particular, all completely false statements are less verisimilar than tautologies: in fact, when conceived as an answer to a cognitive problem, a tautology corresponds to suspending the judgment, which is better than accepting "serious" falsehoods. Another problem with vs_ϕ as a measure of SSI is that, among completely false c-statements, vs_ϕ decreases with content. This is perfectly natural as far as verisimilitude is concerned, but it seems at variance with the idea that "SSI encapsulates truth". In fact, a completely false c-statement is not "veridical" at all, in the sense that it conveys no true factual information about the world; accordingly, its degree of SSI should be 0.

In order to overcome these difficulties, Cevolani [5] suggested $cont_t$, the degree of true basic content, as a measure of SSI. Note that this amounts to ignore, in (15), the second half of Floridi's (rephrased) measure $cont^*_S$ and to use $cont_t$ as a measure of SSI for *both* true and false statements. The latter measure was proposed by Hilpinen [17] as an explication of the notion of "partial truth", measuring the amount of *information about the truth* conveyed by a (true or false) statement A (see also [24, Sects. 5.4 and 6.1]). Since all requirements SSI1–6 are satisfied by $cont_t$,

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[14] *Proof sketch* When A is a c-statement, the constituents in its range are 2^{n-k_A}; it follows that $vac(A) = 2^{n-k_A}/2n$, i.e., $1/2^{k_A}$. Thus, among true c-statements, $cont_S(A) = 1 - 1/2^{k_A}$ co-varies with the degree of basic content of A, $b(A) = k_A/n$. As far as false c-statements are concerned, since $inacc(A) = 1 - acc(A)$, $cont_S$ co-varies with the accuracy of A. □

[15] Note that, by definition, a c-statement can not be contradictory; hence, $cont^*_S$ is undefined for contradictions. Of course, it is always possible to stipulate, as Floridi does, that contradictions have a minimum degree of SSI.

this is in fact an adequate measure of SSI.[16] In particular, it delivers a minimum degree of SSI for both tautologies and completely false statements, in agreement with Floridi's measure $cont_S$. Moreover, $cont_t$ increases with content among truths, whereas for false statements it depends on how much true information they convey (cf. [24, p. 176]).

While $cont_S$ and $cont_t$ are ordinally equivalent as far as true statements are concerned, they differ in assessing the degree of SSI conveyed by false statements. If A is a false c-statement, $cont_S(A)$ measures the accuracy or closeness to be true of A, i.e., increases with t_A/k_A, whereas $cont_t(A, C_\star)$ increases with the amount of information about the truth conveyed by A, i.e., increases with t_A/n. As an example, assume that $C_\star \equiv p_1 \wedge \cdots \wedge p_n$ and let A and B be, respectively, the c-statements $p_1 \wedge \neg p_2$ and $p_1 \wedge p_2 \wedge \neg p_3 \wedge \neg p_4$. Then, $cont_S(A) = cont_S(B) = \frac{1}{2}$: according to Floridi, the degree of SSI of A and B is the same, since they are equally accurate. In this sense, $cont_S$ is, so to speak, insensitive to content as far as false statements are concerned (cf. [24, p. 219]). On the other hand, $cont_t(A, C_\star) = \frac{1}{n}$ is smaller than $cont_t(B, C_\star) = \frac{2}{n}$, since B makes two matches instead of one, i.e., conveys more information about the truth than A. Thus, $cont_t$ appears as a more adequate *information* measure than $cont_S$.

In this connection, one may note that $cont_t$ is insensitive to the number of mistakes contained in a false c-statement.[17] For instance, assume again that $C_\star \equiv p_1 \wedge \cdots \wedge p_n$ is the truth and that A is the false c-statement $p_1 \wedge \neg p_2$. If B is obtained from A by adding to it a false claim, for instance if $B \equiv p_1 \wedge \neg p_2 \wedge \neg p_3$, its degree of partial truth does not change, since $cont_t(A, C_\star) = cont_t(B, C_\star) = \frac{1}{n}$. However, B is now less accurate than A: accordingly, $cont_S(B) = \frac{1}{3} < \frac{1}{2} = cont_S(A)$. While it may appear counterintuitive that $cont_t(A, C_\star)$ does not decrease when the number of mistakes made by A increases, this is just another way of saying that $cont_t$ measures informativeness about the truth and not accuracy (nor verisimilitude). In other words, in the example above it is only relevant that both A and B make one match, independently from the number of their mistakes. At a deeper level, this depends on the following feature of $cont_t$:

$$\text{If } A \text{ entails } B, \text{ then } cont_t(A, C_\star) \geq cont_t(B, C_\star). \tag{16}$$

This result says that strengthening a c-statement (i.e., adding to it true or false new claims) never yields a decrease of its degree of partial truth (cf. [24, p. 220]). In turn, comparing result (16) with the Popperian requirement (11) explains the difference between a measure of information about the truth, like $cont_t$, and a measure of closeness to the whole truth like vs_ϕ (for which, of course, (16) does not hold).

To sum up, we considered three ways of quantifying SSI: Floridi's $cont_S$ measure, the verisimilitude measure vs_ϕ, and the partial truth measure $cont_t$. We argued that

[16] Note again that $cont_t$ is undefined for contradictions, which can be assigned a minimum degree of SSI by stipulation. Interestingly, an argument to this effect was already proposed by Hilpinen [17, p. 30].

[17] I thank Gerhard Schurz for raising this point in discussion during the *Trends in Logic XI* conference.

both $cont_S$ and vs_ϕ are inadequate, for different reasons, in evaluating the SSI of false c-statements. Indeed, $cont_S$ cannot discriminate among differently informative but equally (in)accurate c-statements, while vs_ϕ favors less informative completely false c-statements over more informative ones, despite their being on a par relative to their true basic content (none). Thus, we submit that the notion of partial truth, as defined by the $cont_t$ measure, provides the most adequate explication of the concept of SSI, which should be conceived as the amount of information about the truth conveyed by a given statement.

4.3 Generalizing the Basic Feature Approach

One may complain that the approach presented in this section is unsatisfactory since it is restricted to a very special kind of sentences in a formal language, i.e., "conjunctive" statements. Indeed, all the notions considered in the previous discussion, including the $cont_t$ measure, are undefined for statements which can not be expressed as conjunctions of literals. However, our approach can be easily generalized to any language characterized by a suitable notion of constituent—or state description in the sense of Carnap [3]—including first-order monadic and polyadic languages [24] and second-order languages [26]. In such "languages with constituents", any non-contradictory sentence A can be expressed as the disjunction of the constituents entailing A (describing the possible worlds where A is true), i.e., in its so-called normal disjunctive form:

$$A \equiv \bigvee_{C_i \in R(A)} C_i. \tag{17}$$

If one assumes that a "distance function" $\Delta(C_i, C_j)$ is defined between any two constituents C_i and C_j,[18] one can (re)define the notion of partial truth for arbitrary statements as follows [24, pp. 217 ff.]:

$$pt(A) \overset{\text{df}}{=} 1 - \Delta_{max}(A, C_\star) = 1 - \max_{C_i \in R(A)} \Delta(C_i, C_\star). \tag{18}$$

Intuitively, $pt(A)$ is high when A excludes possible worlds which are far from the truth. Note that, if A is a c-statement, $pt(A) = cont_t(A, C_\star)$: i.e., partial truth as defined above generalizes the notion of degree of true basic content as defined in (7).[19]

[18] Usually, $\Delta(C_i, C_j)$ is identified with the so-called normalized Hamming distance (or Dalal distance, as it is also known in the field of AI), i.e., with the number of literals on which C_i and C_j *disagree*, divided by the total number n of atomic sentences.

[19] *Proof* Note that, if A is a c-statement, all constituents C_i in the range of A (which are c-statements themselves) are "completions" of A in the sense that $b(A) \subset b(C_i)$. The constituent in $R(A)$ farthest from $C_\star$ will be the one which makes all possible additional mistakes besides the mistakes already made by A: this means that $\Delta_{max}(A, C_\star) = 1 - \frac{t_A}{n}$. It follows by (7) that $pt(A) = 1 - (1 - \frac{t_A}{n}) = cont_t(A, C_\star)$. □

5 Conclusions: Information and Truth Revisited

Should only true contingent statements count as pieces of information after all? The recent debate on the veridicality thesis, triggered by Floridi's definition of SSI, has not reached a consensus on this point (see [14, Sect.3.2.3], for a survey of the main contributions).

The supporters of VT argue "that 'true information' is simply redundant and 'false information', i.e. misinformation, is merely pseudo-information" [11, p. 352]. According to this view, to come back to our example in Sect. 3, the statement "Rudolf Carnap was an influential philosopher of science born in Germany in 1890" would not be a piece of information at all, but may perhaps be split in two parts: an informative one—i.e.,"Rudolf Carnap was an influential philosopher of science born in Germany"—and a non-informative one—i.e., "Rudolf Carnap was born in 1890" (cf. [11, p. 361]). In short, this would amount to introduce a distinction between "information" and (say) "semantic content", the latter being the alethically neutral concept analized by Carnap and Bar-Hillel [4]. Accordingly, "information" would denote true semantic content and "misinformation" false semantic content (with tautologies and contradictions construed as extreme special cases of this twofold classification). Such a strategy would be perhaps in line with some analyses in the pragmatics of natural languages, like Paul Grice's study of "conversational implicatures" (cf. [11, p. 366]). In particular, the so-called "maxim of quality" of effective communication—"Do not say what you believe to be false" [16, p. 26]—apparently implies that "false information is not an inferior kind of information; it just is not information" [16, p. 371].

In this paper, we followed the opposite strategy of rejecting VT, and treat information and truth as independent notions, in agreement with the classical view of Carnap and Bar-Hillel [4]. The guiding idea of our discussion has been that SSI should not be construed as a new, more adequate explication of the notion of semantic information, but expresses the amount of (classical) semantic information *about the truth* conveyed by a given statement. According to this view, contingent statements are always informative but may be more or less successful in conveying true information about the world. This becomes especially clear as far as c-statements are concerned. In fact, (true and false) c-statements are the more informative about the truth the more true claims, or matches, they make about the world. In particular, completely false c-statements do not convey any information about the truth, and, in this sense, are plainly uninformative since they are not even "partially" true. This is a way of making sense of Dretske's remark that "If *everything* I say to you is false, then I have given you no information" [9, p. 44, emphasis added]. Finally, tautologies are also uninformative about the truth, since they do not convey any amount of factual information.

In this connection, one should note that the present approach also provides a straightforward quantitative definition of misinformation, a task that Floridi [10, p. 217] left for subsequent research. In fact, an adequate measure $misinf(A)$ of the misinformation conveyed by a c-statement A is given by its degree of false basic content defined in (7):

$$misinf(A) \stackrel{\text{df}}{=} cont_f(A, C_\star) = \frac{f_A}{n} \tag{19}$$

i.e., by the normalized number of the mistakes made by A. Since $cont_t(A, C_\star) + cont_f(A, C_\star) = cont_b(A)$, misinformation and partial truth (or information about the truth) are, so to speak, "complementary" notions.[20]

Finally, the fact that truth and information are here treated as independent concepts should not obscure an important point. As emphasized by philosophers of science and cognitive decision theorists—like Popper [28], Levi [23], Hintikka [18, 20], Kuipers [21], and Niiniluoto [24, 25], among others—both truth *and* information are important goals of rational (scientific) inquiry [25, Sect. 3.4]. In other words, at least according to any minimally realist view of science and ordinary knowledge, among the "epistemic utilities" guiding inquiry both truth and information have to play a prominent role. As Niiniluoto [25] notes, if "truth and nothing but the truth" were the only relevant aim of inquiry, then one should accept, as the best hypotheses at disposal, those statements which are more likely to be true—i.e., more probable—given the available evidence. This recommendation would lead to the "extremely conservative policy" of preferring tautologies, as well as statements logically implied by the evidence, over any other available hypothesis. On the other hand, if information were the only relevant epistemic utility, then, due to BCP (cf. Eq. 4), one should always accept contradictory hypotheses. Thus, as Levi [23] made clear, an adequate account of the cognitive goals of inquiry requires some notion of *informative truth*—i.e., some combination of both the truth value and the information content of alternative hypotheses.

The different measures considered in Sect. 4 can all be construed as different explications of this notion of informative truth. Accordingly, SSI can be conceived as a particular kind of epistemic utility combining truth and information, i.e., partial truth. In turn, one can interpret VT as a thesis concerning not information itself, but a corresponding appropriate notion of epistemic utility. In other words, while VT can be safely rejected as far as the *definition* of semantic information is concerned, the idea that "information encapsulates truth" can be accepted as a thesis about the ultimate cognitive goals guiding rational inquiry.

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[20] This is still clearer if one consider the generalization of the definition above to arbitrary (non-conjunctive) statements A. Given (18), the misinformation conveyed by A is given by $1 - pt(A) = \Delta_{max}(A, C_\star)$, that reduces to $misinf(A)$ as far as c-statements are concerned (the proof is straightforward, see footnote 19).

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References

1. Barwise, J. (1997). Information and impossibilities. *Notre Dame Journal of Formal Logic*, *38*(4), 488–515.
2. Bremer, M., & Cohnitz, D. (2004). *Information and information flow: An introduction*. Frankfurt: Ontos Verlag.
3. Carnap, R. (1950). *Logical foundations of probability*. Chicago: University of Chicago Press.
4. Carnap, R., & Bar-Hillel, Y. (1952). *An outline of a theory of semantic information*. Technical Report 247, MIT Research Laboratory of Electronics.
5. Cevolani, G. (2011). Strongly semantic information and verisimilitude. *Ethics & Politics*, *2*, 159–179. http://www2.units.it/etica/
6. Cevolani, G., Crupi, V., & Festa, R. (2011). Verisimilitude and belief change for conjunctive theories. *Erkenntnis*, *75*(2), 183–202.
7. D'Agostino, M., & Floridi, L. (2009). The enduring scandal of deduction. *Synthese*, *167*(2), 271–315.
8. D'Alfonso, S. (2011). On quantifying semantic information. *Information*, *2*(1), 61–101.
9. Dretske, F. (1981). *Knowledge and the flow of information*. Cambridge: MIT Press.
10. Floridi, L. (2004). Outline of a theory of strongly semantic information. *Minds and Machines*, *14*(2), 197–221.
11. Floridi, L. (2005). Is semantic information meaningful data? *Philosophy and Phenomenological Research*, *70*(2), 351–70.
12. Floridi, L. (2007). In defence of the veridical nature of semantic information. *European Journal of Analytic Philosophy*, *3*(1), 31–1.
13. Floridi, L. (2011a). *The philosophy of information*. Oxford: Oxford University Press.
14. Floridi, L. (2011b). Semantic conceptions of information. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Spring 2011 ed.). http://plato.stanford.edu/archives/spr2011/entries/information-semantic/
15. Frické, M. (1997). Information using likeness measures. *Journal of the American Society for Information Science*, *48*(10), 882–892.
16. Grice, H. P. (1989). *Studies in the way of words*. Cambridge: Harvard University Press.
17. Hilpinen, R. (1976). Approximate truth and truthlikeness. In M. Przełecki, K. Szaniawski & R. Wójcicki (Eds.), *Formal methods in the methodology of the empirical sciences* (pp. 19–42). Dordrecht: Reidel.
18. Hintikka, J. (1968). The varieties of information and scientific explanation. In B. V. Rootselaar & J. Staal (Eds.), *Logic, methodology and philosophy of science* III (Vol. 52, pp. 311–331). Amsterdam: Elsevier.
19. Hintikka, J. (1970). Surface information and depth information. In J. Hintikka & P. Suppes (Eds.), *Information and inference*, (pp. 263–297). Dordrecht: Reidel.
20. Hintikka, J. (1973). *Logic, language-games and information*. Oxford: Oxford University Press.
21. Kuipers, T. A. F. (2000). *From instrumentalism to constructive realism*. Dordrecht: Kluwer Academic Publishers.
22. Kuipers, T. A. F. (2006). Inductive aspects of confirmation, information and content. In R. E. Auxier & L. E. Hahn (Eds.), *The philosophy of Jaakko Hintikka* (pp. 855–883). Chicago and La Salle: Open Courts.
23. Levi, I. (1967). *Gambling with truth*. New York: Alfred A. Knopf.
24. Niiniluoto, I. (1987). *Truthlikeness*. Dordrecht: Reidel.
25. Niiniluoto, I. (2011). Scientific progress. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Summer 2011 ed.). http://plato.stanford.edu/entries/scientific-progress/

26. Oddie, G. (1986). *Likeness to truth*. Dordrecht: Reidel.
27. Popper, K. R. (1934). *Logik der Forschung*. Vienna: Julius Springer [revised edition: *The logic of scientific discovery*. London: Routledge, 2002 (Hutchinson, London, 1959)].
28. Popper, K. R. (1963). *Conjectures and refutations* (3rd ed.). London: Routledge and Kegan Paul.
29. Schurz, G., & Weingartner, P. (2010). Zwart and franssen's impossibility theorem holds for possible-world-accounts but not for consequence-accounts to verisimilitude. *Synthese*, *172*, 415–436.
30. Sequoiah-Grayson, S. (2007). The metaphilosophy of information. *Minds and Machines*, *17*(3), 331–44.
31. Sequoiah-Grayson, S. (2008). The scandal of deduction. *Journal of Philosophical Logic*, *37*(1), 67–94.
32. Shannon, C. (1948). A mathematical theory of communication. *Bell System Technical Journal*, *27*, 379–423, 623–656.
33. Wansing, H. (2010). Connexive logic. In E. N. Zalta (Ed.), *The stanford encyclopedia of philosophy* (Fall 2010 ed.). http://plato.stanford.edu/entries/logic-connexive/