Carnapian truthlikeness

GUSTAVO CEVOLANI*, *Center for Logic, Language and Cognition and Department of Philosophy and Education, University of Turin, 10124, Italy.*

Abstract

Theories of truthlikeness (or verisimilitude) are currently being classified according to two independent distinctions: that between 'content' and 'likeness' accounts, and that between 'conjunctive' and 'disjunctive' ones. In this article, I present and discuss a new definition of truthlikeness, which employs Carnap's notion of the content elements entailed by a theory or proposition, and is then labelled 'Carnapian'. After studying in detail the properties and shortcomings of this definition, I argue that it occupies a unique position in the landscape of different approaches to truthlikeness. More precisely, I show that it provides the only explication of truthlikeness which is both 'conjunctive' and 'content-based' in a suitably defined sense.

Keywords: Truthlikeness, verisimilitude, truth approximation, information, content, accuracy, cognitive values, conjunctive approach, Carnap, Popper.

1 Introduction

Current work on 'cognitive values' in formal epistemology and philosophy of science aims at clarifying the role of theoretical virtues like truth, coherence, content, etc., in shaping beliefs and inferences in ordinary and scientific reasoning. The general assumption of so-called accuracy-first epistemology [34, 35] is that cognitive progress is feasible through a sequence of theories or propositions that can approximate, more or less accurately, the target of the inquiry. In most contexts, the truth about a given matter of interest is the relevant target, so that progress is a matter of truth approximation. This view meshes well with those realist accounts of scientific inquiry that embrace the idea that science progresses by devising hypotheses theories, and models which are closer and closer to the truth (e.g. [9, 18, 28, 37, 41]). Starting with Popper [36], a number of scholars have tried to spell out in greater detail the above intuition, developing different formal accounts of verisimilitude or truthlikeness, construed as closeness or similarity to 'the whole truth' concerning a given domain (e.g. [20, 30, 33, 43]).

My starting point in this article is the ongoing work on different approaches to explicating verisimilitude or truthlikeness (I will use the two terms interchangeably), and on whether or not such different accounts may be made compatible [32, 43, 45, 46]. Here, I do not aim at providing an answer to this question, but rather at further clarifying the conceptual landscape of current explications of truthlikeness. To this purpose, I undertake an exploration in the alternate history of last century's philosophy of science, going back to the Popper-Carnap controversy on the goals of science and asking what explication of truthlikeness Carnap would have defended, had he ever proposed one (he did not). As I argue, a defensible answer to this counterfactual question leads to a new, 'Carnapian' definition of verisimilitude which occupies a unique position in the landscape of so-called conjunctive accounts of truthlikeness [6, 40, 43]. It turns out that this definition is not equivalent to Popper's one, but both fail, although for different reasons, to provide an adequate explication of truthlikeness. Of course, it is only possible to speculate that the failure of the 'Carnapian' definition to deliver intuitively sound assessments of truthlikeness has indeed been the reason why Carnap never proposed it.

*E-mail: g.cevolani@gmail.com

Vol. 24 No. 4,
doi:10.1093/jigpal/jzw021 Advance Access published 21 May 2016

I proceed as follows. In Section 2, I briefly survey Popper's discussion of the goals of science and present his definition of truthlikeness. I then focus on Carnap's earlier work on semantic information and show that the latter contained all the ingredients needed to define truthlikeness within Carnap's own approach to the the analysis of scientific reasoning. This Carnapian account of truthlikeness is explored in detail in Section 3 (in qualitative terms) and Section 4 (in quantitative terms). Finally, in Section 5 I highlight the connections between the Carnapian definition and other accounts of truthlikeness currently under investigation.

2 Content, truth and truthlikeness

In the early sixties of the past century, the controversy between Popper's and Carnap's followers concerning the goals of science and the growth of knowledge urged Popper [36, ch. 10] to introduce the first formal explication of truthlikeness. Popper believed that science aims neither at highly probable nor at inductively well-confirmed theories, but at hypotheses with a high degree of verisimilitude, a notion which 'represents the idea of approaching comprehensive truth [and] thus combines truth and content' as the two fundamental cognitive goals of inquiry [36, p. 237].

In Popper's intentions, the idea of verisimilitude should have played at least two related roles. First, it should have supported his realist and falsificationist views about science, by showing how it is possible to keep together two apparently opposite tenets: i.e. that our best theories are bold conjectures which are likely false (and will be quite surely falsified in the future) and that still science progresses toward truth. If a false theory or hypothesis can be closer to the truth than another false theory, Popper argued, then one can coherently maintain a falsificationist attitude in methodology and a realist view of the main aim of science, i.e. truth approximation (see, e.g., [8, 30, 38] for most recent critical discussion about this view of scientific progress).

Popper's second reason to introduce the idea of truthlikeness in the debate was to provide further ammunition in his battle against Carnap's (and other logical empiricists') view of scientific inquiry. Sharply distinguishing the notion of truthlikeness from that of probability, Popper argued that the aim of science is not high (or increasing) probability but instead increasing verisimilitude. Truthlike theories, as Popper was eager to emphasize, are typically highly informative in the sense of providing much (approximately) true information about the relevant domain. As far as informative content is negatively related with probability, this entails that truthlike hypotheses tend to have low probability; hence, increasing probability cannot be the only valuable goal of science.

In order to defend the ideas outlined above, Popper introduced a definition of verisimilitude as based on an apparently very sound intuition: the more true consequences and the less false consequences a theory or proposition h has, the greater its verisimilitude. More precisely, let Cn denote the operation of classical logical consequence as defined on the sentences of some formal language, so that $\mathrm{Cn}(h)$ is the class of sentences entailed by h. Moreover, suppose that $\mathrm{Cn}_T(h)$ denotes the class of true consequences of h, and $\mathrm{Cn}_F(h)$ the class of false consequences of h, to the effect that $\mathrm{Cn}_T(h) \cup \mathrm{Cn}_F(h) = \mathrm{Cn}(h)$. Then, according to Popper [36, p. 233], h is closer to the truth than g if and only if (iff) h has all the true consequences of g (and possibly more) and no more false consequences (and possibly less):

Definition 1 (Popperian truthlikeness [36])
h is at least as close to the truth as g—in symbols, $h \succeq_P g$, where 'P' is for Popper—iff:

$$\mathrm{Cn}_T(h) \supseteq \mathrm{Cn}_T(g) \text{ and } \mathrm{Cn}_F(h) \subseteq \mathrm{Cn}_F(g)$$

Moreover, h is closer to the truth than g ($h \succ_P g$) if at least one of the two above inclusion relations is strict.

When Definition 1 first appeared in the 10th chapter of *Conjecture and refutations*, it did not attract much attention, perhaps because most readers found the definition exactly as it should be (cf. [18], 139). Popper's definition became famous about ten years later, when Miller [22] and Tichý [44] independently proved that it was completely inadequate. More precisely, Miller and Tichý showed that no false theory h can be closer to the truth than another (true or false) theory g according to Popper's definition 1. The so-called Tichý-Miller theorem proved fatal for Popper's explication of verisimilitude, since it showed that definition 1 is worthless for the very purpose Popper proposed it—i.e. ordering false theories according to their closeness to the truth (cf. Figure 3).

The surprising failure of Popper's definition urged a number of scholars to trying to develop more adequate definitions of verisimilitude (for surveys of the main competing accounts currently on the market, see [17, 27, 33]). Interestingly, the most notable participants in the early debate on cognitive values and the goals of science—including Hempel and Levi [14, 21], Carnap himself, and, later, Hintikka [16] and his co-authors—largely ignored Popper's definition of truthlikeness until after the publication of the Tichý-Miller theorem.[1] In particular, to the best of my knowledge, Carnap never discussed the notion of verisimilitude in his writings. In his paper on 'Probability and content measure' [2], Carnap took issue with Popper's 1963 essay, but without mentioning the idea of verisimilitude at all. There, he limited himself to discuss the relative role of probability and content in assessing competing hypotheses in the light of his work on confirmation. Moreover, Carnap died too soon to witness the breakdown of Poppers definition due to the Tichý-Miller theorem. In any case, Carnap never contributed to the debate on verisimilitude—a contribution that one might have expected in view of his previous work on the notions of content and semantic information [1, 3]. This motivates the new, 'Carnapian' account of truthlikeness which is presented in the next two sections.

3 A Carnapian account of truthlikeness

In his now classical work on semantic information, Carnap developed two formal explications of the (amount of) content of a proposition h. The first is sometimes said to express the 'surprise value' associated with h; it is formally identical to Shannon's measure of information in mathematical communication theory—i.e. $\mathrm{inf}(h) = \log_2 \frac{1}{P(h)}$—and will not bother us here.[2] The second measure of information proposed by Carnap is

$$\mathrm{cont}(h) = 1 - P(h) \tag{1}$$

where $P(h)$ is the (logical) probability of h. This definition embodies the Popperian intuition that a statement is the more informative the more possibilities it excludes, a principle later called by Barwise 'the inverse relationship principle' [12, p. 130]. Interestingly, in Carnap and Bar-Hillel's paper [3], definition 1 is introduced as the quantitative counterpart of a more fundamental comparative definition of the content of h, which I now present.

While Carnap's framework is that of first-order logic, for the sake of simplicity, and without loss of generality, I will frame here the main definitions with respect to a finite propositional language

[1] An exception being Hempel, who is reported to have proved a restricted version of the theorem already in 1970 [26, p. 480, fn. 8].

[2] For this distinction, see [16]; see also [19] and [10] for relevant discussion.

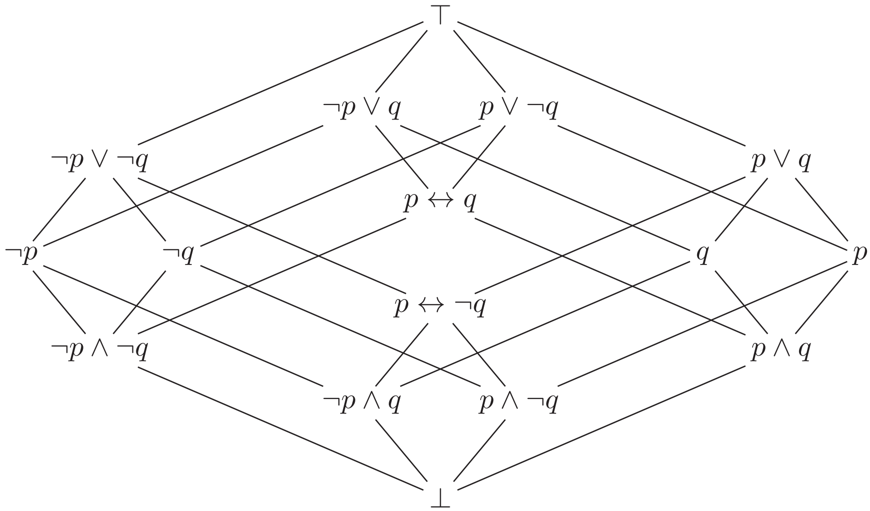

FIG. 1. The 16 propositions of $\mathcal{L}_2$ in increasing order (from top to bottom) of logical strength. Downward lines denote increasing logical strength: if two propositions are (directly or indirectly) connected, the upper one is a consequence of the lower one. The four disjunctions near the top are the content elements of $\mathcal{L}_2$; the four conjunctions near the bottom are its constituents.

$\mathcal{L}_n$ with n atomic sentences. This strategy proves fruitful, especially when the comparison between different explications of truthlikeness is at issue (e.g. [29, 43, 45]); moreover, the extension to richer frameworks is straightforward. Within $\mathcal{L}_n$, one can express 2^{2^n} logically distinct propositions (classes of logically equivalent sentences), including the tautological and the contradictory proposition, denoted $\top$ and $\perp$, respectively. Given two propositions h and g, h is said to be logically stronger than g when h entails g but g does not entail h (in symbols: $h \vDash g$ but $g \nvDash h$); of course, $\perp$ is the logically strongest proposition, and $\top$ the weakest one. In Figure 1, the $2^{2^2} = 16$ propositions of $\mathcal{L}_2$—with p and q as atoms—are represented in increasing order of logical strength, from the top to the bottom of the diagram. This toy example will be used in the following to discuss and compare different notions of truthlikeness.

Among the contingent (i.e. neither tautological nor contradictory), or factual, propositions of $\mathcal{L}_n$, some play a special role and deserve special mention. A 'basic' proposition is an atomic proposition or its negation (e.g. $p, \neg p, q$, and $\neg q$ are the basic propositions of $\mathcal{L}_2$ in Figure 1). A consistent conjunction of n basic sentences (one for each atomic sentence) will be called a constituent (or state description) of $\mathcal{L}_n$.There are $k = 2^n$ constituents, denoted $z_1, \ldots, z_k$, which are the logically strongest factual propositions of $\mathcal{L}_n$; as Fig. 1 shows, they are weaker than a contradiction but stronger than any other proposition (the constituents of $\mathcal{L}_2$ are the four conjunctions $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$ and $\neg p \wedge \neg q$). Note that each constituent is logically incompatible with any other, and that only one of them is true; this is denoted t and is the strongest true statement expressible in $\mathcal{L}_n$. Intuitively, a constituent completely describes a possible state of affairs of the relevant domain (a 'possible world'); thus, t can be construed as 'the (whole) truth' in $\mathcal{L}_n$, i.e. as the complete true description of the actual world. When one of the constituents of $\mathcal{L}_n$ is identified with the truth t, it partitions the set of propositions of $\mathcal{L}_n$ into the class $T = \mathrm{Cn}(t)$ of the true ones and its complement $F = \mathrm{Cn}(\perp) \setminus \mathrm{Cn}(t)$ containing the false ones ($X \setminus Y$ denotes the set-theoretical difference between sets X and Y). For illustrative purposes, I will assume in the following that $p \wedge q$ is the truth of our toy language $\mathcal{L}_2$.

The negations of the constituents of $\mathcal{L}_n$ are the logically weakest factual statements of the language; they are disjunctions of n basic sentences (one for each atomic sentence). These are called by Carnap [1, § 73, p. 406], the content elements of $\mathcal{L}_n$, denoted $e_1,\ldots,e_k$—where each $e_i \equiv \neg z_i$ (for instance, $p \vee \neg q \equiv \neg(\neg p \wedge q)$). As constituents are maximally informative in that they completely describe an entire possible world within $\mathcal{L}_n$, content elements are 'content atoms', as Salmon [39, p. 191] puts it, in the sense that they convey a minimal amount of factual information about the world. In other words, '[j]ust as a state-description says the most that can be said in the given universe of discourse, short of self-contradition, so a content-element says the least, beyond a tautology' [3, p. 10]. Accordingly, in Figure 1 the content elements of $\mathcal{L}_2$—i.e. the four disjunctions $p \vee q$, $p \vee \neg q$, $\neg p \vee q$, and $\neg p \vee \neg q$—are immediately below the tautological proposition $\top$; any other proposition entails at least a content element, i.e. it is lower in the diagram. A trivial observation, which however will prove crucial, is the following:

Remark 1
There is only one false content element in $\mathcal{L}_n$, which is the negation of the truth $\neg t$ and the weakest factual falsehood of $\mathcal{L}_n$.

As Carnap [1, p. 405] shows, any proposition h of $\mathcal{L}_n$ can be expressed (in so-called conjunctive normal form) as the conjunction of the content elements that h entails; thus, the class Cont(h) of all content elements entailed by h is an adequate explication of the content of h:

$$\mathrm{Cont}(h) = \{e_i : h \vDash e_i\} \tag{2}$$

It follows that a tautology has no content at all ($\mathrm{Cont}(\top) = \emptyset$), since it has no factual consequences, while a contradiction has the greatest content possible, since it entails all content elements of $\mathcal{L}_n$.[3] The reader can easily check that the following equivalences hold:

$$\begin{array}{rll} h \text{ entails } g & \text{iff} & \mathrm{Cont}(h) \supseteq \mathrm{Cont}(g) \\ h \text{ is true} & \text{iff} & t \vDash h \\ & \text{iff} & \mathrm{Cont}(h) \subseteq \mathrm{Cont}(t) \\ & \text{iff} & \neg t \notin \mathrm{Cont}(h) \\ h \text{ is false} & \text{iff} & h \vDash \neg t \\ & \text{iff} & \mathrm{Cont}(\neg t) \subseteq \mathrm{Cont}(h) \\ & \text{iff} & \neg t \in \mathrm{Cont}(h) \end{array} \tag{3}$$

Finally, the content of h can be partitioned in two classes:

Definition 2 (True and false content)
Given a proposition h, the *true content* of h is the set $\mathrm{Cont}_T(h) = \mathrm{Cont}(h) \cap \mathrm{Cont}(t)$ of the true content elements of h; the *false content* of h is the set $\mathrm{Cont}_F(h) = \mathrm{Cont}(h) \setminus \mathrm{Cont}_T(t)$ of the false content elements of h.

It immediately follows from Remark 1 and Definition 2 that $\mathrm{Cont}_F(h)$ is either empty or only contains $\neg t$:

$$\begin{array}{l} \text{If } h \text{ is true then } \mathrm{Cont}_T(h) = \mathrm{Cont}(h) \text{ and } \mathrm{Cont}_F(h) = \emptyset \\ \text{If } h \text{ is false then } \mathrm{Cont}_T(h) = \mathrm{Cont}(h) \setminus \{\neg t\} \text{ and } \mathrm{Cont}_F(h) = \{\neg t\} \end{array} \tag{4}$$

[3] The latter result, according to which contradictions are maximally informative in Carnap's theory, has been sometimes felt as problematic [12, 109 ff.]; however, it is a simple consequence of the fact that here information and truth are treated as independent cognitive values (for discussion, see [4]).

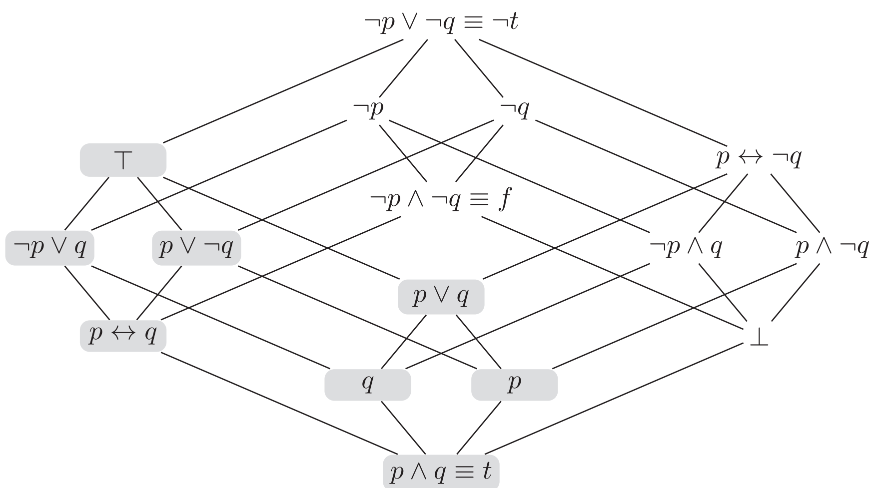

FIG. 2. The 16 propositions of $\mathcal{L}_2$ in increasing order (from top to bottom) of Carnapian truthlikeness, assuming that the truth t is $p \wedge q$, and hence $\neg p \vee \neg q$ is the false content element and $f \equiv \neg p \wedge \neg q$ the worst constituent (true propositions are shadowed). Downward lines denote increasing truthlikeness: if two propositions are (directly or indirectly) connected, the lower one is more verisimilar than the upper one.

As we already noted in Section 1, Carnap never discussed verisimilitude in his writings. Still, his notion of content element suggests a very intuitive definition of verisimilitude, as based on Definition 2. In fact, following Popper's intuition that truthlikeness is a combination of truth and content—i.e. that h is highly truthlike when h tells many things and many of these things are true—it is only natural to propose the following definition, using Definition 1 as a benchmark:

DEFINITION 3 ('Carnapian' truthlikeness)
h is at least as close to the truth as g—in symbols, $h \succeq g$—iff:

$$\mathrm{Cont}_T(h) \supseteq \mathrm{Cont}_T(g) \text{ and } \mathrm{Cont}_F(h) \subseteq \mathrm{Cont}_F(g)$$

Moreover, h is closer to the truth than g ($h \succ g$) if at least one of the two above inequalities is strict.

According to such definition, h is closer to the truth than g iff h entails more true content elements and less false content elements than g. Figure 2 illustrates the $\succeq$-ordering of the propositions of our toy language $\mathcal{L}_2$ according to their Carnapian truthlikeness. Note that, as in the case of Popperian truthlikeness, many propositions turn out to be incomparable in terms of verisimilitude (they correspond to the propositions which are not joined, neither directly nor indirectly, by a line in the figure; for instance, p and q, or their negations). For the sake of comparison, Fig. 3 depicts the truthlikeness ordering $\succ_P$ induced by Popper's verisimilitude Definition 1 (cf. [45], fig. 1, p. 10).

Definition 3 has some attractive consequences. First, the whole truth is the most verisimilar proposition, since it entails all and only the true content elements of the language:

$$\text{for any } h \neq t,\ t \succ h \tag{5}$$

This is easily seen since $\mathrm{Cont}_T(t) \supseteq \mathrm{Cont}_T(h)$ and $\mathrm{Cont}_F(t) = \emptyset \subseteq \mathrm{Cont}_F(h)$, with at least one inclusion relation strict, for any h different from t itself. Moreover, Carnapian truthlikeness meets what

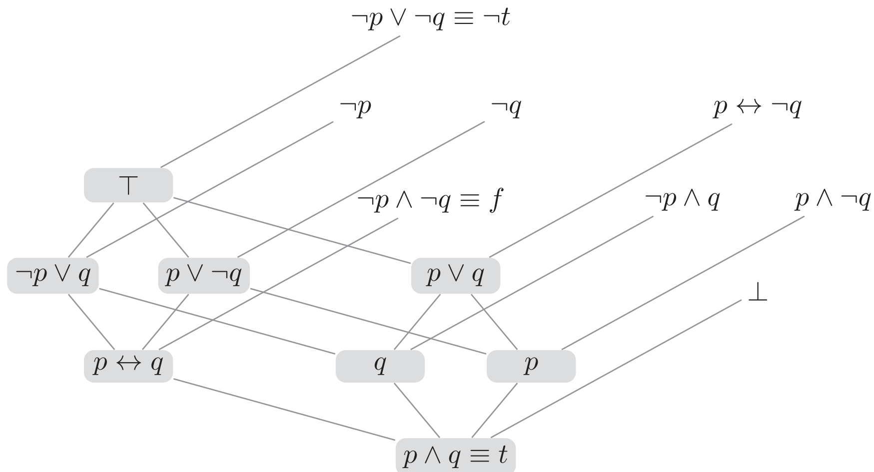

FIG. 3. Popper's truthlikeness ordering $\succ_P$ for $\mathcal{L}_2$, assuming $t \equiv p \wedge q$; note that all false propositions are mutually incomparable, i.e. Condition (10) is violated.

Oddie [32, p. 1651] calls the 'strong value of content for truths', i.e. the requirement that, among truths, verisimilitude increases with content:

$$\text{if } h \text{ and } g \text{ are true, then } h \succ g \text{ iff } \mathrm{Cont}(h) \supset \mathrm{Cont}(g) \tag{6}$$

It follows that the tautology is the less verisimilar true proposition:

$$\text{if } h \text{ is factually true, then } h \succ \top \tag{7}$$

However, the tautology is not the proposition which is farther from the truth than any other: this distinctive position is occupied by the negation of the truth itself, which is the weakest falsehood of $\mathcal{L}_n$, i.e. its only false content element:

$$\text{for any } h \neq \neg t,\ h \succ \neg t \tag{8}$$

This is simply because $\mathrm{Cont}_T(h) \supseteq \emptyset = \mathrm{Cont}_T(\neg t)$ and $\mathrm{Cont}_F(h) \subseteq \{\neg t\} = \mathrm{Cont}_F(\neg t)$ hold, with at least one inclusion relation strict, for any h different from $\neg t$.

Another desirable result (cf. [26], p. 233, condition M7) is that if h is false, then its strongest true consequence $h \vee t$ is more truthlike than h itself:

$$\text{if } h \text{ is factually false, then } h \vee t \succ h \tag{9}$$

Note that $h \vee t$ is the same as the true content of h, since $\mathrm{Cont}(h \vee t) = \mathrm{Cont}(h) \cap \mathrm{Cont}(t) = \mathrm{Cont}_T(h)$ by Definition 2. Finally, the Carnapian account of truthlikeness avoids the crucial shortcoming of Popper's account, i.e. what Oddie [32, p. 1652] calls 'the relative trivialization of truthlikeness for falsehoods', according to which false propositions are incomparable in verisimilitude. In short, the Carnapian account does not fall prey of the Tichý-Miller theorem, as the following result shows:

$$\text{there are } h \text{ and } g \text{ such that } h \text{ and } g \text{ are false and } h \succ g \tag{10}$$

Thus, it is possible to compare the Carnapian truthlikeness of two different false propositions, as Popper desired but was unable to achieve with his definition (since $\succ_P$ fails to deliver the above result). As an example (cf. Fig. 1), $h \equiv \neg p \wedge q$ is closer to the truth $p \wedge q$ than $g \equiv \neg p$, since $\mathrm{Cont}_F(h) = \mathrm{Cont}_F(g)$ but $\mathrm{Cont}_T(h) = \{p \vee q, \neg p \vee q, \neg p \vee \neg q\} \supset \{\neg p \vee q, \neg p \vee \neg q\} = \mathrm{Cont}_T(g)$. This shows that Definition 3 is an adequate definition of truthlikeness at least in the minimal sense of satisfying the 'non-triviality principle' [27, p. 3] according to which a false theory may be more truthlike than another false theory. (All post-Popperian accounts of truthlikeness meets this adequacy requirement.)

Unfortunately, the Carnapian account avoids the relative trivialization of truthlikeness for falsehoods (i.e. the negation of 10) only by entailing the following highly contested principle, that Oddie [32, p. 1654] has dubbed 'the strong value of content for falsehoods':

$$\text{if } h \text{ and } g \text{ are false, then } h \succ g \text{ iff } \mathrm{Cont}(h) \supset \mathrm{Cont}(g) \tag{11}$$

The proof is straightforward: if h and g are false, then their false content is the same ($\mathrm{Cont}_F(h) = \mathrm{Cont}_F(g) = \{\neg t\}$); thus, $h \succ g$ iff, by definition 3, $\mathrm{Cont}_T(h) \supset \mathrm{Cont}_T(g)$ iff, by definition 2, $\mathrm{Cont}(h) \supset \mathrm{Cont}(g)$. More generally, from Conditions (6) and (11), it follows that:

$$\text{if } h \text{ and } g \text{ have the same truth-value, then } h \succ g \text{ iff } \mathrm{Cont}(h) \supset \mathrm{Cont}(g) \tag{12}$$

a very implausible condition which may be called the strong value of content *simpliciter*.

Note that any theory satisfying principle (11) entails that verisimilitude increases with content among falsehoods. It thus faces the so-called 'child's play objection' [44, p. 157, fn. 2], according to which increasing the truthlikeness of a false theory h is then a child's play: just add to h a falsehood not already entailed by h. For instance, in the example of Fig. 2, $\neg p \wedge \neg q$ is deemed closer to the truth $p \wedge q$ than $\neg p$ (or $\neg q$) alone. More vividly, this would mean that adding to Newton's theory (which is strictly speaking false) the proposition that the moon is made of green cheese would count as an improvement in truth approximation. Virtually all theorists agree that such implication is damning for an adequate theory of truthlikeness, the only exception being David Miller, who, after having first rejected it, has later defended the strong value of content for falsehoods, which is satisfied by his own theory of truthlikeness (cf. [23–25]).

Another unwelcome consequence of the Carnapian account is the 'absolute trivialization of truthlikeness for falsehoods' [32, p. 1652]:

$$\text{if } h \text{ is false and } g \text{ is true, then either } h \text{ is incomparable with } g \text{ or } g \succ h \tag{13}$$

The reason is clear: since the false content of h includes that of g ($\mathrm{Cont}_F(h) = \{\neg t\} \supset \emptyset = \mathrm{Cont}_F(g)$), according to Definition 3 h cannot be more verisimilar than g, notwithstanding how much the true content of h exceeds that of g; at best, the two are incomparable in truthlikeness. On the other hand, it follows from Condition (9) that a true g is more truthlike than a false h just in case g entails the true content of h (i.e, iff $\mathrm{Cont}(g) \supset \mathrm{Cont}_T(h)$). Thus, from Condition (13) it follows that a falsehood can never be more verisimilar than a truth, not even than a tautology. In turn, this implies that 'Newton's theory, for example, is deemed no closer to the truth about motion than the completely trivial proposition that either something is moving or not moving' [32, p. 1652]. While perhaps less disturbing than the strong value of content for falsehoods, this principle should be clearly rejected by an adequate theory of truthlikeness.

Finally, the following features of Carnapian truthlikeness also show the shortcomings of such account. First, due to Condition (11), contradictions are the most verisimilar false propositions;[4]

[4]This explicitly contradicts one of Popper's central desiderata for truthlikeness [36, Addendum 3, p. 393 and 396 ff.], according to which logically false propositions should be minimally truthlike (cf. also [43], p. 419). Note that, in many

moreover, the only true proposition which is more truthlike than a contradiction is the truth itself (cf. Figure 2). This is simply because $\bot$ entails all content elements, including all the true ones; thus, all truths with the exception of t are incomparable with $\bot$, since they entail less false content elements but also less true content elements. (Note that t is exactly the true content $\mathrm{Cont}_T(\bot)$ of the contradictory proposition!)

Secondly, and for related reasons, all false constituents are incomparable in truthlikeness, since their true contents are incomparable in set-theoretical terms. This means that it is impossible, within the Carnapian account, to order possible worlds according to their similarity or closeness to the actual world. For instance, with reference to our toy example in Fig. 2, the constituent $\neg p \wedge \neg q$, which is intuitively farther from the truth $p \wedge q$ than the other false constituents $p \wedge \neg q$ and $\neg p \wedge q$, has the same truthlikeness as the latter. More generally, the 'worst' constituent f—i.e. the conjunction of the negations of the conjuncts of t, i.e. the 'specular' of the truth [11, p. 153]—is not farther from the truth than other constituents, and is more verisimilar than all weaker falsehoods, including of course $\neg t$.

4 Measuring Carnapian truthlikeness

Definition 3 of Carnapian truthlikeness provides a comparative account of verisimilitude, delivering merely a partial ordering of truthlikeness. Accordingly, many propositions turn out to be incomparable as far as their closeness to the truth is concerned. A quantitative account, based on a verisimilitude measure vs, allows for the assessment of the degree of truthlikeness $\mathrm{vs}(h)$ of all propositions of $\mathcal{L}_n$, and hence for a complete comparison. Of course, an adequate quantitative account must extend the corresponding comparative one, in the sense that if $h \succeq g$ according to definition 3 then the verisimilitude measure is such that $\mathrm{vs}(h) \geq \mathrm{vs}(g)$.

There are many different measures which extend the ordering of Carnapian truthlikeness defined in section 3; a simple one is defined as follows. First, we introduce a measure of the (true and false) content of h; then, using Definition 3 as a benchmark, we define a measure vs according to which $\mathrm{vs}(h) \geq \mathrm{vs}(g)$ iff h has more true content and less false content than g. We start by defining the amount of content, $\mathrm{cont}(e_i)$, of a content element e_i of $\mathcal{L}_n$, which, according to Carnap and Bar-Hillel [3, p. 15], is given by:

$$\text{for any } e_i,\ \mathrm{cont}(e_i) = m(z_i) = \frac{1}{k} \tag{14}$$

where m is the logical probability function in the sense of Carnap [1]. In agreement with the inverse relationship principle mentioned in Section 2, the above definition equates the informativeness of each content element with the probability of the constituent it excludes. Note that $\mathrm{cont}(e_i)$ is the same for all content elements of $\mathcal{L}_n$, and it decreases as the number n of the atomic propositions (and hence the number k of content elements) increases.

As far as the amount of content of an arbitrary proposition h is concerned, it can be defined as the sum of the content of all its content elements:

$$\mathrm{cont}(h) = \sum_{e_i \in \mathrm{Cont}(h)} \mathrm{cont}(e_i) \tag{15}$$

accounts of truthlikeness, the verisimilitude of contradictions is undefined since logically false propositions are explicitly excluded from consideration.

TABLE 1. Degrees of Carnapian truthlikeness of the propositions of $\mathcal{L}_2$ assuming $p\wedge q$ as the truth (true propositions are on the left, their false negations on the right).

	h	vs(h)		$\neg h$	vs($\neg h$)
1.	$\top$	0	9.	$\bot$	0.5
2.	$p\vee q$	0.25	10.	$\neg p\wedge\neg q$	0.25
3.	$p\vee\neg q$	0.25	11.	$\neg p\wedge q$	0.25
4.	p	0.5	12.	$\neg p$	0
5.	$\neg p\vee q$	0.25	13.	$p\wedge\neg q$	0.25
6.	q	0.5	14.	$\neg q$	0
7.	$p\leftrightarrow q$	0.5	15.	$p\leftrightarrow\neg q$	0
8.	$p\wedge q$	0.75	16.	$\neg p\vee\neg q$	−0.25

Similarly, summing up the degrees of content of the elements of $\mathrm{Cont}_T(h)$ and $\mathrm{Cont}_F(h)$ one obtains the amount of true and false content of h, denoted, respectively, $\mathrm{cont}_T(h)$ and $\mathrm{cont}_F(h)$. It is easy to check that then $\mathrm{cont}(h)=\mathrm{cont}_T(h)+\mathrm{cont}_F(h)$.

The simplest measure of Carnapian truthlikeness definable on this basis is the following:[5]

DEFINITION 4
For any h, $\mathrm{vs}(h)=\mathrm{cont}_T(h)-\mathrm{cont}_F(h)$.

It is easy to check that measure vs extends Definition 3 in the sense that $h\succeq g$ entails $\mathrm{vs}(h)\geq\mathrm{vs}(g)$. With reference to the toy example in Fig. 2, one can compute, for instance, that $\mathrm{vs}(\neg p\wedge q)=\frac{1}{4}$, while $\mathrm{vs}(\neg p)=0$, in agreement with the ordering $(\neg p\wedge q)\succ\neg p$. However, Definition 4 allows one to order also propositions which are incomparable according to Definition 3: for instance, p and $\neg p$ which in the example have degrees of verisimilitude, respectively, $\mathrm{vs}(p)=0.5$ and $\mathrm{vs}(\neg p)=0$. For the sake of comparison with the ordering in Fig. 2, the degrees of verisimilitude of the 16 propositions of $\mathcal{L}_2$ are displayed in Table 1.

Measure vs inherits all properties of the truthlikeness ordering $\succeq$ discussed in section 3. In particular:

$$\begin{aligned}
&\max\mathrm{vs}(h)=\mathrm{vs}(t);\\
&\min\mathrm{vs}(h)=\mathrm{vs}(\neg t);\\
&\text{if } h \text{ and } g \text{ are true, and } h\vDash g, \text{ then } \mathrm{vs}(h)\geq\mathrm{vs}(g);\\
&\text{if } h \text{ is factually true, } \mathrm{vs}(h)>\mathrm{vs}(\top)=0>\mathrm{vs}(\neg t);\\
&\text{if } h \text{ is factually false, } \mathrm{vs}(h\vee t)>\mathrm{vs}(h).
\end{aligned}\tag{16}$$

Moreover, measure vs avoids both the relative and the absolute trivialization of truthlikeness for falsehoods, i.e. it satisfies, respectively, both the following conditions:

$$\begin{aligned}
&\text{if } h \text{ and } g \text{ are false, it may be that } \mathrm{vs}(h)>\mathrm{vs}(g)\\
&\text{if } h \text{ is false and } g \text{ is true, it may be that } \mathrm{vs}(h)>\mathrm{vs}(g)
\end{aligned}\tag{17}$$

In this respect, the quantitative Definition 4 is an improvement on the comparative Definition 3, which failed on the second requirement above, i.e. suffered from the absolute trivialization of truthlikeness

[5] A formally indentical measure was proposed by Popper [36, p. 396], based however on different explications of the degree of ‘truth’ and ‘falsity content’. For discussion, see [31, sect. 2.3] and [26, sect. 5.7].

for falsehoods (cf. Condition 13).[6] However, quantitative assessments of Carnapian truthlikeness are still basically guided by considerations of content alone, as shown by the following consequences of Definition 4:

$$\begin{array}{l}\text{if } h \text{ and } g \text{ are false, and } h \vDash g, \text{ then } \mathrm{vs}(h) \geq \mathrm{vs}(g);\\ \text{for any } h \neq t,\ \mathrm{vs}(h) \leq \mathrm{vs}(\bot) < \mathrm{vs}(t);\\ \text{if } h \text{ and } g \text{ are false constituents, } \mathrm{vs}(h) = \mathrm{vs}(g).\end{array} \tag{18}$$

The above results show, first, that vs satisfies the strong value of content for falsehoods, and hence faces the child's play objection, making verisimilitude increasing with content even among false propositions. Secondly, contradictions are at least as close to the truth as any other propositions, with the exception of the truth itself. Thirdly, vs is unable to discriminate between the relative truthlikeness of false constituents which are, intuitively, at very different distances from the truth. In sum, the account of truthlikeness given by vs inherits most of the unwelcome features of the ordering $\succeq$ on which it is based.

5 A conjunctive, content-based account of truthlikeness

In the foregoing sections, I showed that Definition 3 of Carnapian truthlikeness avoids the main problem of the Popperian account—i.e. the relative trivialization of falsehoods resulting from the Tichý-Miller theorem—while suffering from serious shortcomings on its own. To understand what the root of these problems is, in this section I discuss the Carnapian account against the background of existing approaches to explicating truthlikeness.

In recent years, verisimilitude theorists have studied how different approaches to truthlikeness can be characterized, distinguished and possibly combined together [29, 32, 33, 40, 43, 45, 46]. For example, Schurz [40] has proposed to classify accounts of truthlikeness according to how they represent theories or propositions in the first place. Within a 'conjunctive' approach to theory representation, propositions are represented as conjunctions of minimal content parts (like their consequences in $\mathcal{L}_n$); within a 'disjunctive' approach, they are instead represented as disjunctions of maximal alternative possibilities (like the constituents, possible worlds, or models of $\mathcal{L}_n$). Schurz also argues that, compared to disjunctive accounts, conjunctive ones are both simpler and more adequate from a cognitive point of view, and can in general deliver intuitively plausible assessments of truthlikeness. In view of this, it is interesting to look at the Carnapian account developed in the present article with Schurz's distinction in mind.

As Schurz [40, pp. 205–206] notes, there are many ways of specifying what a 'content part' is, even within a conjunctive approach to truthlikeness (cf. also [6]). For instance, in Popper's account presented in section 2, a content part of *h* is simply an arbitrary logical consequences of *h*. In other accounts, not all consequences of *h* are relevant in assessing truthlikeness. For instance, both in Schurz's own account and in Gemes' account [13], verisimilitude depends only on some of the logical consequences of *h* (see [13, 42, 43], for relevant definitions). Another, special case is the 'basic feature' account proposed by [5], according to which only the basic propositions (or literals) entailed by *h* are relevant in the calculation of the truthlikeness of *h* (see also [7]).

[6]In this connection, it is worht noting that, in our toy example in Table 1, no falsehood has greater truthlikeness than a factual truth. The reason is that language $\mathcal{L}_2$ is too 'small' to express more interesting verisimilitude assessments. It is easy to check that, in richer frameworks, informative falsehoods can be more verisimilar than uninformative truths. As an example, suppose that $p \wedge q \wedge r$ is the truth in $\mathcal{L}_3$; then it is easy to check that, for instance, $\mathrm{vs}(p \wedge q \wedge \neg r) > \mathrm{vs}(p \vee q)$, although the former proposition is false and the latter is true.

Within the present, Carnapian account, the content parts of h are the content elements entailed by h (recall from Section 3 that any h is logically equivalent to the conjunction of its content elements). Accordingly, this account is clearly conjunctive, although it occupies a very special position among the conjunctive accounts, for two reasons. First, it is in some sense at the opposite extreme of Popper's account. While for Popper truthlikeness depends on the set of *all* the consequences of h, assessments of Carnapian truhtlikeness are only based on the set of the *weakest* consequences of h, i.e. the content elements entailed by h. Between these two extremes, one can arguably place all other conjunctive accounts, according to the different classes of consequences of h they isolate as relevant for truthlikeness comparison. Secondly, the Carnapian account, contrary to virtually all other conjunctive accounts, meets such an implausible principle as the strong value of content for falsehoods. This shows that not all conjunctive accounts guarantee a plausible truthlikeness ordering. Indeed, Schurz and Weingartner [43, p. 424] have already conjectured that:

> an account of verisimilitude based on FCN's [full conjunctive normal forms, i.e. conjunctions of content elements] will probably have similar problems as a possible-world [i.e. disjunctive] account [...] although we do not know of anybody who has worked out such an account.

The results presented in Sections 3 and 4 confirm their conjecture. The root of the problem with the Carnapian account is, as I am going to show in the final part of the article, that it is not only a conjunctive account but also a content-based account in the sense defined below.

In discussions on verisimilitude, it has been customary, starting at least with Oddie [31, pp. x ff.], to distinguish between 'content' and 'likeness' accounts of truthlikeness (cf. also [46], p. 77). Very roughly, according to the former truthlikeness can be defined as 'a mixture of truth and information', as Oddie [31, p. 12] puts it, or more precisely as a function of the truth-value and of the content of a proposition. According to likeness accounts, on the contrary, such an approach is insufficient, and one needs to take seriously the idea of similarity or closeness to the truth, and hence explicitly introduce some measure or relation of distance between propositions. In the most advanced contribution to this debate, Oddie [32, p. 1656] proposes the following two-steps characterization of the content approach.

Definition 5 ([32])
A truthlikeness ordering on the propositions of $\mathcal{L}_n$ is 'content-based' iff:

1. the ordering respects the strong value of content for truths, i.e. if for all h and g:[7]

$$\text{if } h \text{ and } g \text{ are true, } h \vDash g \text{ and } g \nvDash h, \\ \text{then } h \text{ is more verisimilar than } g. \tag{19}$$

2. the ordering is realizable by a 'content-based measure' of truthlikeness, i.e. by a function of the truth-value and of the amount of content of propositions h and g such that:
   (a) if h and g are true then the one with greater content is the more verisimilar; and
   (b) if h and g are equally informative, but h is true and g is false, h is more verisimilar than g.

It is now easy to check that the Carnapian account introduced in this article is an instance of the content approach to truthlikeness as defined by Oddie:

[7]This principle—or at least its weaker version, saying that if h is true and entails g, then h is at least as close to the truth as g—is regarded by most verisimilitude theorists, including Popper himself, as 'an essential desideratum for any theory of truthlikeness' [33, sect. 1]. Indeed, I already mentioned in Section 2 that it is satisfied by Popper's Definition 1, as it is easy to check. Interestingly, Condition 19 is violated by likeness-based accounts of truthlikeness as characterized by Oddie [32, pp. 1668 ff.], and in particular by his preferred 'average' account of truthlikeness.

Remark 6
The Carnapian truthlikeness ordering $\succ$ is content-based.

Proof. We prove, in turn, that $\succ$ (cf. Definition 3) is a content-based ordering and that it is realized by a content-based measure.

(i) $\succ$ meets the strong value of content for truths, as already shown in Section 3 (cf. Condition 6);
(ii) $\succ$ is realized by measure vs, which is a content-based since:
  (a) if h and g are true then $\text{cont}_T(h)=\text{cont}(h)$ and $\text{cont}_T(g)=\text{cont}(g)$; it follows that, if $\text{cont}(h)>\text{cont}(g)$, then $\text{vs}(h)=\text{cont}(h)>\text{cont}(g)=\text{vs}(g)$.
  (b) if $\text{cont}(h)=\text{cont}(g)=\text{cont}_T(g)+\text{cont}_F(g)$ then $\text{cont}_T(g)=\text{cont}(h)-\text{cont}_F(g)$; thus, if h is true and g is false, we have that $\text{vs}(h)=\text{cont}(h)>\text{cont}(h)-2\text{cont}_F(g)=\text{vs}(g)$.

■

In sum, the Carnapian definition of truthlikeness has to be counted among the content-based accounts of truthlikeness. Interestingly, it provides, as far as I know, the only definition of truthlikeness so far proposed in the literature that is both conjunctive and content-based. Indeed, the canonical instance of the content approach to truthlikeness is the well-known 'symmetric difference' or Miller–Kuipers account [18, 20, 23, 25], which is based on a disjunctive representation of theories (for discussion, see [32, 43]). Space limitations prevent us to fully compare the above two accounts: here it suffices to say that both satisfy the strong value of content for falsehoods, and hence face the child's play objection. We conclude that the Carnapian account inherits the weakness of all content-based accounts of truthlikeness proposed so far, despite its natural conjunctive representation of theories.

Acknowledgements

I thank Roberto Festa, Theo Kuipers, Ilkka Niiniluoto, Luca Tambolo, and two anonymous reviewers for useful comments on a previous version of this article. Financial support from the FIRB project *Structures and Dynamics of Knowledge and Cognition* (Italian Ministry of Scientific Research, Turin unit, D11J12000470001) and from the project *Assessing Information Models: Exploring Theories and Applications of Optimal Information Search* (University of Turin and Compagnia San Paolo, D16D15000190005) is gratefully acknowledged.

References

[1] R. Carnap. *Logical Foundations of Probability*. University of Chicago Press, 1950.
[2] R. Carnap. Probability and content measure. In *Mind, Matter and Method: Essays in Philosophy and Science in Honor of Herbert Feigl*, P. K. Feyerabend and G. Maxwell, eds, pp. 248–260. University of Minnesota Press, 1966.
[3] R. Carnap and Y. Bar-Hillel. An outline of a theory of semantic information. *Technical Report 247*, MIT Research Laboratory of Electronics, 1952.
[4] G. Cevolani. Strongly semantic information as information about the truth. In *Recent Trends in Philosophical Logic*, R. Ciuni, H. Wansing, and C. Willkommen, eds, pp. 59–74. Springer, 2014.
[5] G. Cevolani, V. Crupi and R. Festa. Verisimilitude and belief change for conjunctive theories. *Erkenntnis*, **75**, 183–202, 2011.

[6] G. Cevolani and R. Festa. Exploring and extending the landscape of conjunctive approaches to verisimilitude. In *From Arithmetic to Metaphysics: A Path through Philosophical Logic. Studies in Honor of Sergio Galvan.* C. D. Florio and A. Giordani, eds, 2016. Forthcoming.

[7] G. Cevolani, R. Festa and T. A. F. Kuipers. Verisimilitude and belief change for nomic conjunctive theories. *Synthese*, **190**, 3307–3324, 2013.

[8] G. Cevolani and L. Tambolo. Progress as approximation to the truth: a defence of the verisimilitudinarian approach. *Erkenntnis*, **78**, 921–935, 2013.

[9] A. Chakravartty. *A Metaphysics for Scientific Realism: Knowing the Unobservable*. Cambridge University Press, 2007.

[10] V. Crupi and K. Tentori. State of the field: measuring information and confirmation. *Studies in History and Philosophy of Science Part A*, **47**, 81–90, 2014.

[11] R. Festa. Theory of similarity, similarity of theories, and verisimilitude. In *What is Closer-to-the-Truth?*, T. A. F. Kuipers, ed., pp. 145–176. Rodopi, 1987.

[12] L. Floridi. *The Philosophy of Information*. Oxford University Press, 2011.

[13] K. Gemes. Verisimilitude and content. *Synthese*, **154**, 293–306, 2007.

[14] C. G. Hempel. Inductive inconsistencies. *Synthese*, **12**, 439–469, 1960 (Reprinted as ch. 2 of [15]).

[15] Hempel, C. G. *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science*. The Free Press, 1965.

[16] J. Hintikka. The varieties of information and scientific explanation. In *Logic, Methodology and Philosophy of Science III*, vol. 52, B. V. Rootselaar and J. Staal, eds, pp. 311–331. Elsevier, 1968.

[17] T. A. F. Kuipers, ed. *What is Closer-to-the-Truth?* Rodopi, 1987.

[18] T. A. F. Kuipers. *From Instrumentalism to Constructive Realism*. Kluwer Academic Publishers, 2000.

[19] T. A. F. Kuipers. Inductive aspects of confirmation, information and content. In *The philosophy of Jaakko Hintikka*, R. E. Auxier and L. E. Hahn, eds, pp. 855–883. Open Courts, 2006.

[20] T. A. F. Kuipers. Models, postulates, and generalized nomic truth approximation. *Synthese*, 2015 forthcoming. doi: 10.1007/s11229-015-0916-9.

[21] I. Levi. *Gambling with Truth*. MIT Press, 1967.

[22] D. Miller. Popper's qualitative theory of verisimilitude. *The British Journal for the Philosophy of Science*, **25**, 166–177, 1974.

[23] D. Miller. The distance between constituents. *Synthese*, **38**, 197–212, 1978.

[24] D. Miller. *Critical Rationalism: A Restatement and Defence*. Open Court, 1994.

[25] D. Miller. *Out Of Error: Further Essays On Critical Rationalism*. Ashgate Publishing, 2006.

[26] I. Niiniluoto. *Truthlikeness*. Reidel, 1987.

[27] I. Niiniluoto. Verisimilitude: the third period. *The British Journal for the Philosophy of Science*, **49**, 1–29, 1998.

[28] I. Niiniluoto. *Critical Scientific Realism*. Oxford University Press, 1999.

[29] I. Niiniluoto. Content and likeness definitions of truthlikeness. In *Philosophy and Logic: in Search of the Polish Tradition. Essays in Honor of Jan Woleński on the Occasion of His 60th Birthday*, J. Hintikka, T. Czarnecki, K. Kijania-Placek, A. Rojszczak and T. Placek, eds, pp. 27–35. Kluwer Academic Publishers, 2003.

[30] I. Niiniluoto. Scientific progress as increasing verisimilitude. *Studies in History and Philosophy of Science Part A*, **46**, 73–77, 2014.

[31] G. Oddie. *Likeness to Truth*. Reidel, 1986.

[32] G. Oddie. The content, consequence and likeness approaches to verisimilitude: compatibility, trivialization, and underdetermination. *Synthese*, **190**, 1647–1687, 2013.

[33] G. Oddie. Truthlikeness. In *The Stanford Encyclopedia of Philosophy*, Summer 2014 edn, E. N. Zalta, ed., 2014.

[34] G. Oddie. What accuracy could not be. Manuscript, 2015.

[35] R. Pettigrew. *Accuracy and the Laws of Credence*. OUP Oxford, forthcoming in Apr. 2016.

[36] K. R. Popper. *Conjectures and Refutations: the Growth of Scientific Knowledge*, 3rd edn. Routledge and Kegan Paul, 1963.

[37] S. Psillos. *Scientific Realism: How Science Tracks Truth*. Routledge, 1999.

[38] D. Rowbottom. Scientific progress without increasing verisimilitude: in response to Niiniluoto. *Studies in History and Philosophy of Science Part A*, **51**, 100–104, 2015.

[39] W. C. Salmon. Partial entailment as a basis for inductive logic. In *Essays in Honor of Carl G. Hempel*, N. Rescher, ed., pp. 47–82. Springer Netherlands, 1969.

[40] G. Schurz. Verisimilitude and belief revision. With a focus on the relevant element account. *Erkenntnis*, **75**, 203–221, 2011.

[41] G. Schurz and I. Votsis. Editorial introduction to scientific realism quo vadis? Theories, structures, underdetermination and reference. *Synthese*, **180**, 79–85, 2011.

[42] G. Schurz and P. Weingartner. Verisimilitude defined by relevant consequence-elements. *What is Closer-to-the-Truth?*, T. Kuipers, ed., pp. 47–77. Rodopi, 1987.

[43] G. Schurz and P. Weingartner. Zwart and Franssen's impossibility theorem holds for possible-world-accounts but not for consequence-accounts to verisimilitude. *Synthese*, **172**, 415–436, 2010.

[44] P. Tichý. On Popper's definitions of verisimilitude. *The British Journal for the Philosophy of Science*, **25**, 155–160, 1974.

[45] S. D. Zwart. *Refined Verisimilitude*. Kluwer Academic Publishers, 2001.

[46] S. D. Zwart and M. Franssen. An impossibility theorem for verisimilitude. *Synthese*, **158**, 75–92, 2007.

Received 3 December 2015