The Least *at least* Can Do

Daniel Büring
University of California, Los Angeles

1. Data

In this paper I propose a new meaning for *at least n* — basically 'exactly *n* or more than *n*' — which sets it apart from other quantificational expressions such as plain *n* or *more than* $n-1$. Note that on the face of it, the sentences in (1) all have the same truth conditions:

(1) a. Paul has four guitars.
b. Paul has more than three guitars.
c. Paul has at least four guitars.

This might lead one to expect that *four*, *more than three* and *at least four* all have the same lexical entry, e.g. the function $\lambda P.\exists X[4 \leq |X| \wedge P(X)]$ in a generalized quantifier framework.

It has been observed, however, that these three expressions differ in terms of the implicatures they evoke (throughout I use $\rightsquigarrow$ for 'implicates'):

(2) Paul has four guitars. $\rightsquigarrow$ Paul does not have more than four guitars.

(3) Paul has more than three guitars. $\rightsquigarrow$ NO IMPLICATURES

(4) Paul has at least four guitars. $\rightsquigarrow$ Paul may have more than four guitars.

The difference in implicatures between (2) and (3) has recently been tackled in Fox and Hackl (2006). In the next section I will add to this picture a meaning for *at least* that accounts for its implicatures.

2. The proposal

2.1. Literal meaning

I propose that *at least n* is interpreted as 'exactly *n* or more than *n*', cf. (5). For example, (4) is analyzed as in (6):

(5) 'at least *n* p q' := 'exactly *n* p q OR n' p q', where $n < n'$ (preliminary)

(6) Paul has at least four guitars. = Paul has exactly four guitars OR Paul has more than four guitars.

My formal treatment in section 4 will cover non-numerical cases such as (7) as well:

(7) Paul knows at least Alice. = Paul knows only Alice OR Paul knows Alice and someone else.

To keep the discussion simple, though, I will stick to numerical examples in the following sections.

2.2. Implicature I

The proposal in (5) makes an *at least* sentence true exactly when the corresponding sentence with the plain numeral is true. The point of expressing that meaning as in (5) is to trigger a particular implicature, namely that typically associated with disjunction:

(8) If a speaker utters *p or q*, it is implied that (i) in all of the speaker's doxastic alternatives $q \vee p$, and (ii-a) not in all *p*, and (ii-b) not in all *q*.

*I'd like to thank the audiences at WCCFL 2007 at UC Berkeley, at SFIPtip at the Dept. of Spanish, French, Italian & Portuguese at the University of Illinois, Chicago, my UCLA Semantics II class, and in particular Elena Guerzoni and Martin Hackl for helpful comments and pointers. Thanks, too, to Bart Geurts for making the manuscript of Geurts and Nouwen (2007) available to me.

(8), in particular (ii-a/b), plainly expresses the Gricean intuition that one doesn't use a disjunction if, roughly, one is certain of the truth of any individual conjunct. This is a special instance of the maxim of Quantity (and perhaps Manner; (8i) on the other hand is simply Quality).

Applied to (4), we get (9):

(9) Paul has at least four guitars.
   a. lit.: Paul has exactly four guitars or Paul has more than four guitars
   b. $\rightsquigarrow$ (i) the speaker is certain that Paul has (at least) four guitars, and (ii-a) she is not certain that Paul has exactly four guitars, and (ii-b) she is not certain that Paul has more than four guitars

Before we go on, it should be noticed that in case *q* and *p* in *q or p* are incompatible, *q* must hold in any world in which *p* doesn't, and *vice versa*. Since 'exactly *n*' and 'more than *n*' are always incompatible, this allows us to simplify our implicature schema (8) to:

(10) . . . and (ii-a) in some of the speaker's doxastic alternatives *p*, and (ii-b) in some *q*.

For (4), this yields the more easily understood (11):

(11) Paul has at least four guitars.
   a. lit.: Paul has exactly four guitars or Paul has more than four guitars
   b. $\rightsquigarrow$ the speaker is certain that Paul has four guitars, and considers it possible that Paul has exactly four guitars, and considers it possible that Paul has more than four guitars

I submit that (11b) accurately captures the entailments and implicatures of an *at least* sentence. We will now turn to cases that provide additional evidence for this particular way of deriving this meaning.

3. Modal implicatures

3.1. Two readings for at least + *modal*

When combined with a universal modal, *at least* gives rise to what I will argue are two different readings. (12), encountered on a computer screen while creating a new account, illustrates the *authoritative reading*:

(12) The password must be at least five characters long.

If it turns out that the password must in fact be seven (or more) characters long, we would object to (12). It is worth noting, though, that it would still literally be true: in every world that accords to the regulations, the password is five or more characters long.

Contrast this with what I'll call the *speaker insecurity* (SI) reading: As we walk by a fancy country club, I peek over the fence and spot the most lavish club house ever seen. I say to you:

(13) To become a member of this club, you have to pay at least $200,000.

If it turns out that no donation of less than $250,000 gets you into the club's rank and file, what I said wasn't false, nor infelicitous. I merely claimed that the minimum contribution was 200K, or more.

To a first approximation, we can render the implicatures of these readings as in (14):

(14) a. (12) $\rightsquigarrow$ the password must be (at least) 5 characters long AND it is allowed to be exactly 5 AND it is allowed to be more than 5 characters long
   b. (13) $\rightsquigarrow$ to become a member of this club you have to pay (at least) 200K AND perhaps you have to pay exactly 200K AND perhaps you have to pay more than that

(14b) already looks like our implicature schema from above: the speaker considers it possible ('perhaps') that exactly *n*, and she considers it possible that more than *n*. In (14a), on the other hand, no epistemic or doxastic possibilities are involved. Rather, we appear to get *deontic* possibilities ('is allowed to be'), in keeping, it appears, with the epistemic modal *must*. We will return to this in section 3.3 below.

3.2. A matter of scope?

The examples in (12) and (13) were chosen in such a way as to radically favor one of the two readings. Pragmatic factors aside, I claim that any regular 'must at least' sentence is ambiguous between the two readings. (15) is an example that isn't pragmatically biased and hence can be understood either way:

(15) John has to read at least 3 books.
  a. It has to be the case that John reads three or more books. (AUTHORITATIVE)
  b. Three or more is such that John has to read that many books. (SI)

(15a/b) provide reasonable paraphrases for the two readings, and they suggest that the difference between the authoritative and the speaker insecurity reading is one of scope: In the authoritative (15a), 'three or more' scopes below the modal; in the SI (15b) it scopes above it.

In order to cash out this intuition, we need a meaning for *at least three* that lets us scope it. (16) defines such a meaning, which will serve us throughout the rest of this paper (but see section 4 for the 'official' definition):

(16) for any set D of numbers/degrees, $[\![\text{at least } 3]\!](D) = 1$ iff $3 = max(D) \vee 3 > max(D)$

We assume that *at least n* is generated as the Degree Phrase specifier to an AP and moved to a propositional position. For our simple (4) this gives us the LF in (17):

(17) at least three [λd [John owns [[$_{\text{AP}}$ d-many] guitars]]]

Now, in (15), *at least three* can scope above *to read d-many books* but underneath *has to*, or above everything, including the modal. The LFs and their interpretations are sketched in (18) and (19):

(18) a. John has to [[at least three] [λd [t_{John} read d-many books]]]
  b. $\Box[3 = max(\lambda d.$John reads d-many books) $\vee$
    $3 > max(\lambda d.$John reads d-many books)]
    'in every permitted world, the maximum number of books John reads is greater than or equal to 3'

(19) a. [at least three] [λd [John has to [t_{John} read d-many books]]]
  b. $3 = max(\lambda d.\Box[$John reads d-many books]) $\vee$
    $3 > max(\lambda d.\Box[$John reads d-many books])
    'the highest number n s.t. in every permitted world, John reads that n (or more) books is greater than or equal to three'

Without further ado, (19) gives us the speaker insecurity reading of (15) using the implicature schema (10): the speaker is convinced that John has to read three or more books, and she considers it possible that he has to read exactly three, and she considers it possible that he has to read more than three. (18) is trickier and involves an extra step, to which we now turn.

3.3. Implicature II

In (18), the disjunction introduced by *at least* is embedded under the universal modal *have to*. In this case, we employ (20):

(20) Local Implicature Schema: $\forall w \in R[p(w) \vee q(w)] \quad \rightsquigarrow \quad \neg\forall w \in R[p(w)]$ and $\neg\forall w \in R[q(w)]$ (Klinedinst 2007)

Note that (20) follows our earlier implicature schema in (8)/(10), and in fact the latter can be subsumed under (20): Suppose that you utter *q or p*, then by Quality we deduce that in all your doxastic alternatives w, $q'(w) = 1$ or $p'(w) = 1$ (where p', q' are the propositions expressed by p and q); that is, R in (20) is the set of your doxastic alternatives. In (15), on the other hand, R is the set of deontically accessible worlds, i.e. $\forall x \in R$ is instantiated as $\forall w' \in Deon(w)$, or simply: $\Box$.

As before, we can simplify (20) to (21) since q and p will always be exclusive:

(21) $\forall w \in R[p(w) \vee q(w)] \quad \rightsquigarrow \quad \exists w \in R[p(w)]$ and $\exists w \in R[q(w)]$

Applied to (15) we get the following meaning:

(22) a. lit.: in all deontically accessible worlds, the password is five or more characters long
b. $\rightsquigarrow$ in some deontically accessible worlds, the password is exactly five characters long and in some it is more than 5 characters long

In other words, it is necessarily five or more, it may be five, and it may be more than that. Note that from that it follows that the password doesn't have to be, say, seven characters long: five is sufficient. This is the implicature we set out to derive.

3.4. Independent motivation for (20)

I've argued above that (20) is localized generalization of (8). Local implicatures *per se* have become presentable in recent years, but is there evidence for this particular species? (20) is proposed in Klinedinst (2007) to account, among other things, for so-called *free choice permission* sentences such as (23):

(23) (If you want to take my class for four units) You must do a presentation or write a term paper.

Extremely distracted professors may use (23) on a speaker insecurity reading ('I forget which'), but most likely (23) is used authoritatively, and will be taken to imply that it is sufficient to do a presentation (but no paper), and that it is sufficient to write a paper (but skip the presentation). This does not follow literally from (23), however, since semantically, if A is the thing you have to do, you also have to do A or B. Clearly, pragmatics is called upon here.

(20) provides the remedy: it is necessary to do a presentation or a paper, and it is possible to not do the presentation (and hence do the paper), and it is possible to not do the paper (and hence do the presentation).[1]

Klinedinst (2007) carefully motivates (20) in many other instances, and I refer the reader to that work. What's important for our purposes is that, according to the present analysis, authoritative readings are just another instance of (20) combined with our basic meaning for *at least*.

3.5. Moving evidence

Given the local implicature schema (20), it should now be clear why it is important to move *at least* out of the scope of the modal for the speaker insecurity reading. Otherwise, we couldn't apply the global implicature schema (8) since the local one would bleed its application, as it were.[2] It is predicted by the present analysis, then, that the speaker insecurity reading will be absent if raising of *at least* out of the scope of the modal is independently blocked. As far as I can tell, the following contrast bears out that prediction:[3]

(24) a. We had to pay at least 3% to the agent. EITHER
b. It was required that we pay at least 3% to the agent. AUTHORITATIVE ONLY

If I forgot exactly how much my agent charged (speaker insecurity), I can use (24a), but hardly (24b). Instead (24b) implies that my agent left it up to me to pay her 3%, or more (i.e. authoritative). Arguably this is so because *at least 3%* is contained within a finite clause in (24b) (but not (24a)), which prevents it from scoping over the modal in the matrix clause.

I observe the same contrast for the following German examples. In (25a), *wenigstens 20K* can scope out of an infinitival clause and over the modal *muss*, yielding a SI reading. In (25b) the embedded clause is finite and extraposed, making it an island for scoping, and only the authoritative reading is possible:

[1] This readings is different, too, from what we get if we assume (23) to be underlyingly a wide scope disjunction as in (i) (Larson 1985, Schwarz 1999):

(i) (If you want to take my class for four units) You must do a presentation or you must write a term paper.

(i) is the counterpart to the speaker insecurity reading ('but I don't know which').

[2] Or put differently, the schema (20) would be instantiated with $R = Deon(w)$ rather than $R = Dox(speaker)$.

[3] Two things to keep in mind here: (i) we are not dealing with a *de re* reading in (24), since there is no particular three percent we have to pay the agent, and (ii) I made sure not to put the *at least* phrase sentence-final, since it is known that sentence-final quantifiers can occasionally scope out of finite clauses in English.

(25) a. *Er muss wenigstens 20,000 zurückzahlen*
he must at least 20K return
'he has to return at least 20K.' EITHER

b. *Es ist erforderlich, dass er wenigstens 20,000 zurückzahlt.*
it is necessary that he at least 20K returns
'It is necessary that he pay back at least 20K.' AUTHORITATIVE ONLY

In sum, then, it seems that the SI reading is possible only where *at least* can plausibly be assumed to be able to scope over the modal, lending further evidence to the present proposal that that reading is the result of scoping *at least* above the modal and thus enabling it to trigger a root implicature according to (8).

3.6. Authoritative possibility?

Above I claimed that the authoritative reading of, say, (15) is possible because the disjunction is 'trapped' underneath the universal modal, which then triggers the (20)-implicature locally. This predicts that no parallel authoritative reading will emerge under a possibility modal: the possibility modal is not a universal, and hence doesn't trigger the (20)-implicature. This is indeed what we find. The sentences in (26) do not mean that the password can be 10 characters or longer, or that I give you permission to eat two or three or four candy bars, but instead only display a SI-type reading (which is weird for transparent pragmatic reasons in these examples):

(26) a. The password can be at least 10 characters long. (odd as website instruction)
b. I give you permission to eat at least two candy bars. (plain odd)

Lack of space prevents a more thorough discussion of these cases here, but it should be clear that the absence of an 'authoritative possibility' reading in these examples is expected under the present proposal.

4. Implementation

As mentioned in connection with (7) above, *at least* is not restricted to modifying numerals. Following Krifka (1999) I assume that the meaning of *[at least] A* is defined using the ordinary meaning of *A*, $[\![\mathrm{A}]\!]$, and its *scalar alternatives*, $[\![\mathrm{A}]\!]_A$. Each constituent *A* is associated with an ordered set $[\![\mathrm{A}]\!]_A$ of scalar alternatives, (27a). Many elements, among them numerals, are lexically associated with a scale, as in (28a). Others might be associated with one by focussing, as in (28b). Elements that don't intuitively have scalar alternatives are associated with a trivial scale, (27b):

(27) Scalar alternatives:
a. for any scalar item E, $[\![\mathrm{E}]\!]_A$ is a transitive ordering of E and its scalar alternatives ('the scale')
b. for any non-scalar item E, $[\![\mathrm{E}]\!]_A =_{\mathrm{def}} \{\langle [\![\mathrm{E}]\!], [\![\mathrm{E}]\!]\rangle\}$

Scalar alternatives to complex expressions are formed by pointwise combination of the scalar alternatives of the immediate constituents, exemplified in (28c):

(28) a. $[\![\mathrm{four}]\!]_A = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \langle 1,4\rangle, \langle 2,4\rangle, \langle 3,4\rangle, \langle 1,5\rangle, \langle 2,5\rangle, \langle 3,5\rangle, \langle 4,5\rangle, \ldots\}$
b. $[\![\mathrm{Mary_F}]\!]_A = \{\langle \mathit{Mary}, \mathit{Mary} \oplus \mathit{Sue}\rangle, \langle \mathit{Mary}, \mathit{Mary} \oplus \mathit{Pete}\rangle, \langle \mathit{Mary}, \mathit{Mary} \oplus \mathit{Pete} \oplus \mathit{Karen}\rangle, \langle \mathit{Mary} \oplus \mathit{Pete}, \mathit{Mary} \oplus \mathit{Pete} \oplus \mathit{Karen}\rangle, \ldots\}$
c. $[\![\mathrm{second\ year\ student}]\!]_A$ = {⟨first year student, second year student⟩, ⟨first year student, third year student⟩, ⟨second year student, third year student⟩,...}

To define the meaning of *at least*, we first define, for any expression *E*, ABOVE(E), the set of scalar alternatives that are strictly higher than the meaning of A:

(29) for any expression E, $\mathrm{ABOVE(E)} =_{\mathrm{def}} \cup\{O' \mid \langle [\![\mathrm{E}]\!], O'\rangle \in [\![E]\!]_A\}$

We then define *at least A* to be true if either $[\![\mathrm{A}]\!]$ but none of its higher alternatives is true ('exactly/only A') or one of its higher alternatives is true ('or more than A'):

(30) (for any q of type <s,t>), ⟦at least q⟧ = $[[\![q]\!] - \cup(ABOVE(q))] \vee \cup(ABOVE(q))$

The following examples illustrate (keep in mind that these are the LFs after raising of *at least* from its position adjacent to *three/Alice*, not the sentences starting with *at least*):[4]

(31) ⟦at least [he reads three books]⟧ = [he reads three books and not four, five, six. . .] OR [he reads four or five or six. . . books]
≡ he reads exactly three books or he reads more than three books

(32) ⟦at least [he knows $Alice_F$]⟧ = [he knows Alice and not Alice and Sue, Alice and Steve, or Alice, Steve and Bill] OR [he knows Alice and Sue, or Alice and Steve, or Alice and Steve and Bill, or . . .]
≡ he knows only Alice, or he knows Alice and someone else

The authoritative reading of, say, (15) is achieved simply by embedding (31) under *has to*. The SI reading is derived in (33):

(33) ⟦at least [he has to read three books]⟧ = [he has to read three and not four, five. . . books] OR [he has to read four, five, six. . . books]

5. Comparison, summary, and outlook

5.1. Comparison with Krifka (1999) and Geurts and Nouwen (2007)

In closing, I want to add a few words of comparison with two proposals found in the literature. I have adopted the semantics for *at least* proposed in Krifka (1999) almost verbatim in section 4. Crucially, however, Krifka explicitly stipulates that *at least* cancels all scalar implicatures. This correctly explains why *at least four* (and *more than three*), unlike bare *four*, has no 'exactly four' implicature, but by the same token it fails to predict *any* implicatures for *at least* sentences, unlike the present proposal.

As we have seen, *at least* — unlike *more than* — does induce implicatures. Moreover, I have argued that the structural ambiguity of sentences involving *at least* and a universal modal, while truth-conditionally spurious, results in different implicatures and hence different readings (authoritative vs. SI). No such account of the two readings (or the difference between *at least* and *more than*) can be given under the assumption that *at least* in fact annihilates all implicatures.

The proposal in Geurts and Nouwen (2007), too, is based in Krifka's semantics, but they give *at least* an explicitly modal meaning, roughly (34), exemplified in (35) (where $\Box/\Diamond_{epi}$ stands for epistemic necessity/possibility):

(34) *at least n p q*: 'it is certain that n p q, and it is possible that n+m p q'

(35) Paul has at least four guitars. = $\Box_{epi}$[Paul has 4 guitars] $\wedge \Diamond_{epi}$[Paul has more than 4 guitars]

The net result of this in a simple sentence like (35) is virtually identical to the present proposal. However, unlike Geurts and Nouwen (2007) I proposed to derive all modal aspects of the meaning of *at least* from pragmatic implicatures. That is, according to Geurts and Nouwen (2007), a speaker who utters (35) literally asserts a modal proposition about their own beliefs, whereas according to the present proposal they assert a non-modal disjunction, which gets 'modalized' due to general pragmatic principles.

One crucial argument for the pragmatic nature of the modal component was that the implicature (20) piggybacks, as it were, on the modal force of the nearest universal modal, as evidenced by the authoritative readings discussed. Indeed, Geurts and Nouwen (2007) on the face of it predict that a sentence of the form . . . *must* . . . *at least* involves stacked modals, one (or two) contributed by *at least*, the other by the modal. That is, Geurts and Nouwen's system straightforwardly predicts the speaker insecurity reading. But to derive the authoritative reading, they have to assume an additional rule of 'modal concord' which can optionally turn the epistemic modals contributed by *at least* into deontic ones and subsequently delete the original deontic modal(s) corresponding to the matrix predicate. This is illustrated in (36):

[4] Note that we now assume that *at least* adjoins to a proposition-denoting constituent, i.e. *at least three letters arrived* has the constituency *at least [three letters arrived]*.

(36) John needs to pay at least \$500.

a. normal reading: $\underline{\Box_{epi}[\Box_{deon}}$[John pays \$500]] $\wedge$ [$\underline{\Diamond_{epi}[\Box_{deon}}$[John pays more than \$500]]]

b. concord reading: $\underline{\Box_{deon}}$[John pays \$500] $\wedge$[$\underline{\Diamond_{deon}}$[John pays more than \$500]]

Unlike the implicature schemata (8) and (20), there doesn't seem to be independent motivation for a rule of modal concord, and it seems to be an attractive feature of the present proposal that it doesn't need such a rule.

5.2. Summary and outlook

In this paper I have proposed that *at least* is interpreted as a disjunction operator over scalar alternatives. In conjunction with general implicature schemata for disjunctions this derives the basic entailments and implicatures of sentences with *at least*. It also derives the contribution of *at least* to modal sentences, in particular the presence of two truth-conditionally identical, but distinguishable readings for sentences with *at least* and universal modals (SI and authoritative).

While the present paper builds substantially on previous approaches, it was demonstrated that it differs from them in terms of empirical coverage and its division of labor between semantics and pragmatics. A serious comparison, however, has to be left for a later occasion. In particular, I have not talked about *at least*'s direct counterpart, *at most*. Intuitively, the correct analysis for *at least* should *mutatis mutandis* carry over to *at most*, and this is precisely what the analysis in Geurts and Nouwen (2007) does. The present analysis is crucially in need of refinement to do so, a task I hope to carry out in a more comprehensive version of the present paper.

Another issue ignored here is what it *means* for *at least* to be interpreted as 'exactly or more'. Our claim, to be sure, is that *at least A* triggers the same implicature about 'only A' and '$\cup$ABOVE(A)' that *B or C* triggers about 'B' and 'C'. But I haven't been explicit at all about what it is to trigger an implicature, and at least the arguably most straightforward, Gricean idea — that the implicature is attached to the very expression *or* — seems inapplicable here. Put differently, the present proposal is committed to a view of implicatures that allows a given implicature to be triggered by a class of expression (or some more abstract property characteristic of that class), rather than particular expressions. The ramifications of such a view will have to be explored in the context of a larger theory of implicatures.

References

Fox, Danny and Martin Hackl (2006). "The Universal Density of Measurement." *Linguistics & Philosophy* 29(5):537–586.

Geurts, Bart and Rick Nouwen (2007). "At Least *et al*: The Semantics of Scalar Modifiers." *Language* 83:533–559.

Klinedinst, Nathan (2007). *Plurality and Possibility*. Ph.D. thesis, UCLA.

Krifka, Manfred (1999). "At Least Some Determiners Aren't Determiners." In Ken Turner, ed., *The Semantics/Pragmatics Interface from Different Points of View (= Current Research in the Semantics/Pragmatics Interface Vol. 1)*, 257–291. Elsevier Science.

Larson, Richard K. (1985). "On the Syntax of Disjunction Scope." *Natural Language & Linguistic Theory* 3:217–264.

Schwarz, Bernhard (1999). "On the Syntax of either…or." *Natural Language & Linguistic Theory* 17:339–370.

Proceedings of the 26th West Coast Conference on Formal Linguistics

edited by Charles B. Chang
and Hannah J. Haynie

Cascadilla Proceedings Project Somerville, MA 2008

ISBN 1-57473-423-2 library binding

Ordering information

Orders for the library binding edition are handled by Cascadilla Press.
To place an order, go to www.lingref.com or contact:

Cascadilla Press, P.O. Box 440355, Somerville, MA 02144, USA
phone: 1-617-776-2370, fax: 1-617-776-2271, e-mail: sales@cascadilla.com

Web access and citation information

This entire proceedings can also be viewed on the web at www.lingref.com. Each paper has a unique document # which can be added to citations to facilitate access. The document # should not replace the full citation.

This paper can be cited as:

Büring, Daniel. 2008. The Least *at least* Can Do. In *Proceedings of the 26th West Coast Conference on Formal Linguistics,* ed. Charles B. Chang and Hannah J. Haynie, 114-120. Somerville, MA: Cascadilla Proceedings Project.

or:

Büring, Daniel. 2008. The Least *at least* Can Do. In *Proceedings of the 26th West Coast Conference on Formal Linguistics,* ed. Charles B. Chang and Hannah J. Haynie, 114-120. Somerville, MA: Cascadilla Proceedings Project. www.lingref.com, document #1662.